Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.1-Sine/68-4.1.11-e-x-^m-a+b-x^n-^p-sin

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [113]. This is test number [68].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (113)	0.00 (0)
Mathematica	100.00 (113)	0.00 (0)
Fricas	100.00 (113)	0.00 (0)
Maple	100.00 (113)	0.00 (0)
Giac	62.83 (71)	37.17 (42)
Maxima	46.90 (53)	53.10 (60)
Sympy	23.01 (26)	76.99 (87)
Mupad	17.70 (20)	82.30 (93)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

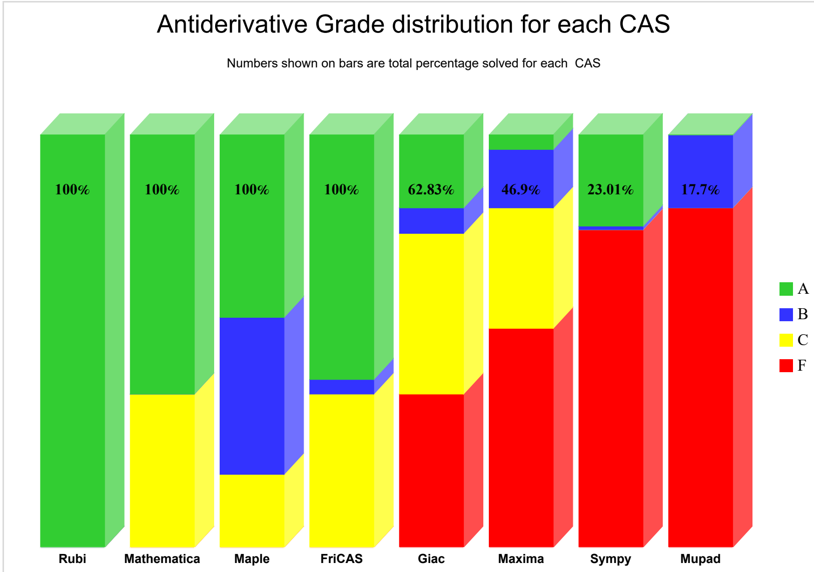
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

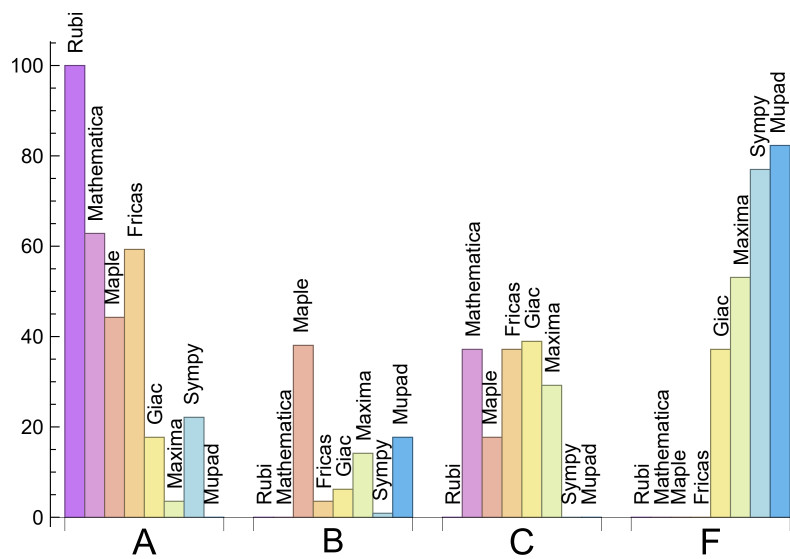
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	62.83	0.00	37.17	0.00
Fricas	59.29	3.54	37.17	0.00
Maple	44.25	38.05	17.70	0.00
Sympy	22.12	0.88	0.00	76.99
Giac	17.70	6.19	38.94	37.17
Maxima	3.54	14.16	29.20	53.10
Mupad	N/A	17.70	0.00	82.30

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	0	0.00 %	0.00 %	0.00 %
Giac	42	100.00 %	0.00 %	0.00 %
Maxima	60	100.00 %	0.00 %	0.00 %
Sympy	87	86.21 %	13.79 %	0.00 %
Mupad	93	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

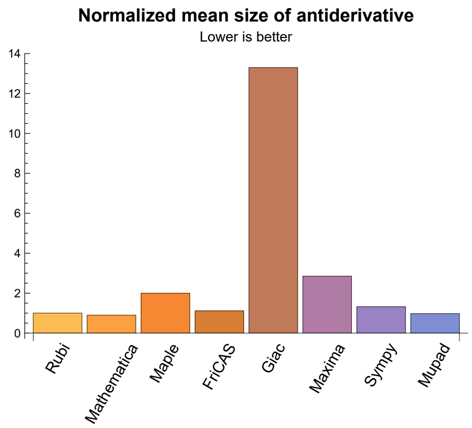
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

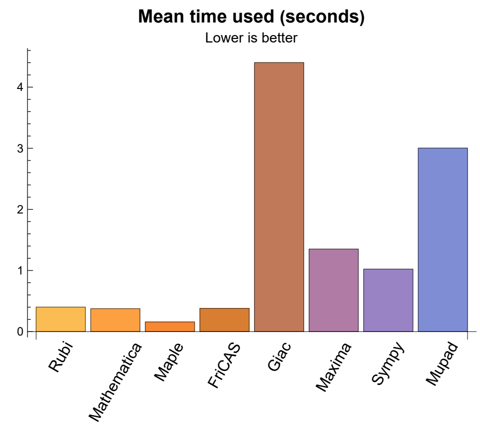
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.40	281.22	1.00	181.00	1.00
Mathematica	0.38	260.06	0.90	145.00	0.85
Maple	0.16	504.96	2.00	281.00	1.46
Maxima	1.35	249.96	2.85	164.00	1.80
Fricas	0.38	295.67	1.11	186.00	1.04
Sympy	1.02	144.08	1.32	142.50	1.23
Giac	4.40	2408.20	13.30	834.00	8.14
Mupad	3.00	116.40	0.98	119.50	0.95

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
14
12
10
8
6
4
2
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
4
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {111}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

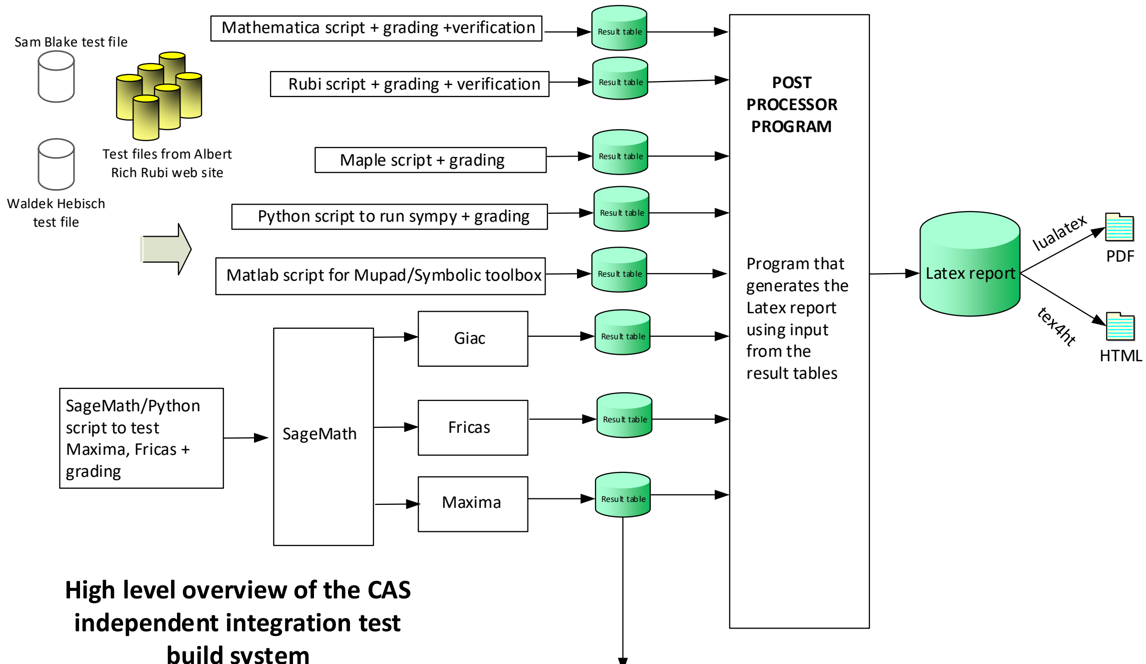
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

The following fields are present only in ***Rubi*** *Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { }

C grade: { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F grade: { }

2.1.3 Maple

A grade: { 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 22, 23, 24, 25, 30, 31, 32, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 61, 62, 63, 64, 69, 70, 71, 75, 76, 78, 83, 84, 85, 86, 91, 92, 93 }

B grade: { 1, 2, 10, 11, 12, 18, 19, 20, 21, 26, 27, 28, 29, 33, 34, 35, 40, 41, 42, 49, 50, 51, 52, 57, 58, 59, 60, 65, 66, 67, 68, 72, 73, 74, 77, 79, 80, 81, 82, 87, 88, 89, 90 }

C grade: { 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F grade: { }

2.1.4 Maxima

A grade: { 3, 4, 11, 43 }

B grade: { 1, 2, 10, 12, 40, 41, 42, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

C grade: { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 21, 22, 30, 36, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 83, 84, 85, 86, 89, 90, 91, 92, 93 }

F grade: { 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 37, 38, 39, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { 36, 37, 38, 39 }

C grade: { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F grade: { }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 10, 11, 13, 40, 41, 42, 43, 44, 49, 50, 51, 52, 79, 80, 81, 82, 83, 87, 88, 89 }

B grade: { 12 }

C grade: { }

F grade: { 6, 7, 8, 9, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 10, 11, 12, 40, 41, 42, 43, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

B grade: { 26, 27, 28, 29, 30, 31, 32 }

C grade: { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 83, 84, 85, 86, 89, 90, 91, 92, 93 }

F grade: { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 10, 11, 12, 40, 41, 42, 43, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

C grade: { }

F grade: { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	82	359	306	85	151	86	122
N.S.	1	1.00	0.65	2.85	2.43	0.67	1.20	0.68	0.97
time (sec)	N/A	0.226	0.113	0.052	0.298	0.346	0.354	3.710	0.279

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	65	225	201	67	117	68	92
N.S.	1	1.00	0.68	2.34	2.09	0.70	1.22	0.71	0.96
time (sec)	N/A	0.149	0.101	0.037	0.314	0.367	0.193	3.336	4.619

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	121	117	48	82	49	62
N.S.	1	1.00	0.69	1.86	1.80	0.74	1.26	0.75	0.95
time (sec)	N/A	0.073	0.074	0.035	0.308	0.343	0.136	3.652	4.499

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	52	53	30	46	31	35
N.S.	1	1.00	0.96	1.86	1.89	1.07	1.64	1.11	1.25
time (sec)	N/A	0.012	0.056	0.033	0.294	0.359	0.080	4.509	4.485

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	40	31	522	44	37	339	-1
N.S.	1	1.00	1.38	1.07	18.00	1.52	1.28	11.69	-0.03
time (sec)	N/A	0.105	0.025	0.068	0.353	0.338	3.116	5.447	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	60	56	108	75	0	569	-1
N.S.	1	1.00	1.25	1.17	2.25	1.56	0.00	11.85	-0.02
time (sec)	N/A	0.156	0.102	0.088	0.483	0.352	0.000	5.791	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	76	88	112	111	0	796	-1
N.S.	1	1.00	0.85	0.99	1.26	1.25	0.00	8.94	-0.01
time (sec)	N/A	0.191	0.183	0.090	0.563	0.342	0.000	4.467	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	117	111	137	0	961	-1
N.S.	1	1.00	0.83	0.89	0.84	1.04	0.00	7.28	-0.01
time (sec)	N/A	0.224	0.229	0.122	0.651	0.334	0.000	3.245	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	138	145	110	154	0	1108	-1
N.S.	1	1.00	0.83	0.87	0.66	0.93	0.00	6.67	-0.01
time (sec)	N/A	0.249	0.186	0.116	0.575	0.346	0.000	2.893	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	101	468	406	126	228	128	172
N.S.	1	1.00	0.54	2.52	2.18	0.68	1.23	0.69	0.92
time (sec)	N/A	0.214	0.167	0.069	0.358	0.355	0.322	2.742	0.295

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	87	281	259	95	172	95	128
N.S.	1	1.00	0.64	2.08	1.92	0.70	1.27	0.70	0.95
time (sec)	N/A	0.127	0.122	0.067	0.293	0.337	0.203	4.630	4.763

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	57	148	141	63	112	65	84
N.S.	1	1.00	1.14	2.96	2.82	1.26	2.24	1.30	1.68
time (sec)	N/A	0.028	0.105	0.062	0.327	0.354	0.135	5.151	4.700

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	51	79	80	78	92	551	-1
N.S.	1	1.00	0.82	1.27	1.29	1.26	1.48	8.89	-0.02
time (sec)	N/A	0.129	0.188	0.102	0.540	0.349	2.245	5.818	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	74	122	111	0	743	-1
N.S.	1	1.00	0.89	1.03	1.69	1.54	0.00	10.32	-0.01
time (sec)	N/A	0.168	0.156	0.089	0.896	0.350	0.000	5.908	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	95	114	189	147	0	1182	-1
N.S.	1	1.00	0.79	0.94	1.56	1.21	0.00	9.77	-0.01
time (sec)	N/A	0.240	0.259	0.104	1.397	0.375	0.000	4.325	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	154	158	187	186	0	1400	-1
N.S.	1	1.00	0.88	0.90	1.07	1.06	0.00	8.00	-0.01
time (sec)	N/A	0.274	0.322	0.122	1.417	0.362	0.000	6.543	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	204	201	186	222	0	1712	-1
N.S.	1	1.00	0.82	0.81	0.75	0.90	0.00	6.90	-0.00
time (sec)	N/A	0.320	0.252	0.138	1.577	0.357	0.000	4.043	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	158	784	0	213	0	3337	-1
N.S.	1	1.00	0.72	3.60	0.00	0.98	0.00	15.31	-0.00
time (sec)	N/A	0.320	0.416	0.230	0.000	0.361	0.000	3.536	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	117	516	0	167	0	2709	-1
N.S.	1	1.00	0.77	3.39	0.00	1.10	0.00	17.82	-0.01
time (sec)	N/A	0.218	0.347	0.066	0.000	0.345	0.000	3.554	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	87	318	0	133	0	2205	-1
N.S.	1	1.00	0.88	3.21	0.00	1.34	0.00	22.27	-0.01
time (sec)	N/A	0.180	0.191	0.059	0.000	0.354	0.000	3.296	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	63	180	776	99	0	1647	-1
N.S.	1	1.00	0.91	2.61	11.25	1.43	0.00	23.87	-0.01
time (sec)	N/A	0.115	0.101	0.060	0.424	0.346	0.000	4.389	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	73	141	78	0	597	-1
N.S.	1	1.00	0.96	1.43	2.76	1.53	0.00	11.71	-0.02
time (sec)	N/A	0.050	0.054	0.053	0.388	0.347	0.000	3.467	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	99	0	99	0	838	-1
N.S.	1	1.00	0.86	1.36	0.00	1.36	0.00	11.48	-0.01
time (sec)	N/A	0.175	0.087	0.069	0.000	0.386	0.000	3.453	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	101	144	0	157	0	2897	-1
N.S.	1	1.00	0.89	1.26	0.00	1.38	0.00	25.41	-0.01
time (sec)	N/A	0.239	0.252	0.099	0.000	0.353	0.000	3.826	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	176	202	0	245	0	4565	-1
N.S.	1	1.00	0.93	1.07	0.00	1.30	0.00	24.15	-0.01
time (sec)	N/A	0.335	0.382	0.135	0.000	0.386	0.000	3.894	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	177	1216	0	357	0	1973	-1
N.S.	1	1.00	0.76	5.22	0.00	1.53	0.00	8.47	-0.00
time (sec)	N/A	0.358	0.648	0.236	0.000	0.362	0.000	3.523	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	153	851	0	316	0	1474	-1
N.S.	1	1.00	0.85	4.70	0.00	1.75	0.00	8.14	-0.01
time (sec)	N/A	0.275	0.578	0.092	0.000	0.351	0.000	3.787	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	117	553	0	264	0	1120	-1
N.S.	1	1.00	0.79	3.71	0.00	1.77	0.00	7.52	-0.01
time (sec)	N/A	0.253	0.546	0.072	0.000	0.350	0.000	4.119	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	96	315	0	208	0	951	-1
N.S.	1	1.00	0.77	2.54	0.00	1.68	0.00	7.67	-0.01
time (sec)	N/A	0.197	0.308	0.063	0.000	0.359	0.000	5.016	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	107	164	123	0	518	-1
N.S.	1	1.00	0.92	1.49	2.28	1.71	0.00	7.19	-0.01
time (sec)	N/A	0.067	0.161	0.054	0.368	0.338	0.000	5.987	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	138	210	0	260	0	1281	-1
N.S.	1	1.00	0.93	1.41	0.00	1.74	0.00	8.60	-0.01
time (sec)	N/A	0.276	0.701	0.110	0.000	0.356	0.000	4.154	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	184	256	0	355	0	3180	-1
N.S.	1	1.00	0.98	1.36	0.00	1.89	0.00	16.91	-0.01
time (sec)	N/A	0.353	1.223	0.122	0.000	0.381	0.000	3.935	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	235	1208	0	515	0	16724	-1
N.S.	1	1.00	0.89	4.56	0.00	1.94	0.00	63.11	-0.00
time (sec)	N/A	0.419	0.644	0.240	0.000	0.381	0.000	3.247	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	154	779	0	438	0	15410	-1
N.S.	1	1.00	0.64	3.23	0.00	1.82	0.00	63.94	-0.00
time (sec)	N/A	0.366	0.796	0.093	0.000	0.378	0.000	5.157	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	157	419	0	346	0	10535	-1
N.S.	1	1.00	0.88	2.34	0.00	1.93	0.00	58.85	-0.01
time (sec)	N/A	0.243	0.364	0.073	0.000	0.352	0.000	4.500	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	87	145	199	210	0	5727	-1
N.S.	1	1.00	0.84	1.39	1.91	2.02	0.00	55.07	-0.01
time (sec)	N/A	0.090	0.525	0.054	0.387	0.364	0.000	3.039	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	449	359	0	532	0	17806	-1
N.S.	1	1.00	1.72	1.38	0.00	2.04	0.00	68.22	-0.00
time (sec)	N/A	0.374	0.620	0.138	0.000	0.404	0.000	3.619	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	540	405	0	689	0	20808	-1
N.S.	1	1.00	1.81	1.35	0.00	2.30	0.00	69.59	-0.00
time (sec)	N/A	0.455	1.312	0.174	0.000	0.398	0.000	5.155	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	630	466	0	816	0	24116	-1
N.S.	1	1.00	1.67	1.24	0.00	2.16	0.00	63.97	-0.00
time (sec)	N/A	0.543	1.184	0.207	0.000	0.410	0.000	4.112	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	92	449	372	95	168	97	121
N.S.	1	1.00	0.65	3.18	2.64	0.67	1.19	0.69	0.86
time (sec)	N/A	0.142	0.132	0.051	0.291	0.359	0.436	3.486	0.344

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	75	302	258	77	134	79	97
N.S.	1	1.00	0.68	2.72	2.32	0.69	1.21	0.71	0.87
time (sec)	N/A	0.109	0.104	0.049	0.300	0.417	0.321	2.885	4.733

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	57	181	165	60	99	60	73
N.S.	1	1.00	0.71	2.26	2.06	0.75	1.24	0.75	0.91
time (sec)	N/A	0.070	0.090	0.038	0.282	0.358	0.206	3.401	0.144

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	41	99	91	41	65	42	49
N.S.	1	1.00	0.77	1.87	1.72	0.77	1.23	0.79	0.92
time (sec)	N/A	0.040	0.063	0.036	0.286	0.354	0.143	5.311	4.692

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	54	60	66	61	65	432	-1
N.S.	1	1.00	1.32	1.46	1.61	1.49	1.59	10.54	-0.02
time (sec)	N/A	0.061	0.097	0.047	0.501	0.398	2.598	5.304	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	48	937	68	0	411	-1
N.S.	1	1.00	1.00	1.09	21.30	1.55	0.00	9.34	-0.02
time (sec)	N/A	0.069	0.071	0.068	0.461	0.393	0.000	4.033	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	82	73	123	85	0	766	-1
N.S.	1	1.00	1.11	0.99	1.66	1.15	0.00	10.35	-0.01
time (sec)	N/A	0.109	0.135	0.079	0.768	0.362	0.000	3.725	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	95	102	121	105	0	834	-1
N.S.	1	1.00	0.90	0.96	1.14	0.99	0.00	7.87	-0.01
time (sec)	N/A	0.146	0.133	0.111	0.953	0.380	0.000	5.391	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	125	131	121	127	0	1086	-1
N.S.	1	1.00	0.84	0.88	0.81	0.85	0.00	7.29	-0.01
time (sec)	N/A	0.186	0.163	0.111	0.888	0.358	0.000	5.900	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	139	746	612	154	286	162	186
N.S.	1	1.00	0.59	3.16	2.59	0.65	1.21	0.69	0.79
time (sec)	N/A	0.221	0.240	0.072	0.328	0.377	0.675	6.184	0.581

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	113	514	438	126	226	129	151
N.S.	1	1.00	0.61	2.78	2.37	0.68	1.22	0.70	0.82
time (sec)	N/A	0.165	0.153	0.072	0.301	0.373	0.487	4.866	4.928

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	86	336	292	97	172	99	118
N.S.	1	1.00	0.62	2.43	2.12	0.70	1.25	0.72	0.86
time (sec)	N/A	0.112	0.117	0.062	0.299	0.367	0.338	4.079	4.834

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	82	236	116	114	160	725	-1
N.S.	1	1.00	0.74	2.13	1.05	1.03	1.44	6.53	-0.01
time (sec)	N/A	0.118	0.254	0.136	1.727	0.370	4.353	3.527	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	156	97	113	0	1638	-1
N.S.	1	1.00	1.00	1.61	1.00	1.16	0.00	16.89	-0.01
time (sec)	N/A	0.112	0.168	0.149	1.438	0.350	0.000	3.560	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	99	124	150	136	0	1058	-1
N.S.	1	1.00	0.87	1.09	1.32	1.19	0.00	9.28	-0.01
time (sec)	N/A	0.142	0.265	0.151	2.378	0.374	0.000	4.144	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	114	120	140	145	0	1032	-1
N.S.	1	1.00	0.85	0.90	1.04	1.08	0.00	7.70	-0.01
time (sec)	N/A	0.159	0.269	0.161	1.816	0.357	0.000	3.817	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	122	157	222	162	0	1497	-1
N.S.	1	1.00	0.69	0.89	1.25	0.92	0.00	8.46	-0.01
time (sec)	N/A	0.224	0.259	0.227	9.934	0.371	0.000	4.494	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	275	1630	0	240	0	0	-1
N.S.	1	1.00	1.01	5.97	0.00	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.541	0.314	0.303	0.000	0.385	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	202	1164	0	185	0	0	-1
N.S.	1	1.00	0.97	5.57	0.00	0.89	0.00	0.00	-0.00
time (sec)	N/A	0.249	0.264	0.099	0.000	0.400	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	216	782	0	195	0	0	-1
N.S.	1	1.00	0.95	3.44	0.00	0.86	0.00	0.00	-0.00
time (sec)	N/A	0.301	0.233	0.071	0.000	0.374	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	163	486	0	146	0	0	-1
N.S.	1	1.00	0.92	2.75	0.00	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.129	0.059	0.000	0.361	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	172	225	0	187	0	0	-1
N.S.	1	1.00	0.81	1.06	0.00	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.209	0.131	0.045	0.000	0.382	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	179	200	0	168	0	0	-1
N.S.	1	1.00	0.91	1.02	0.00	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.266	0.226	0.093	0.000	0.375	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	238	266	0	240	0	0	-1
N.S.	1	1.00	0.95	1.06	0.00	0.96	0.00	0.00	-0.00
time (sec)	N/A	0.400	0.345	0.118	0.000	0.377	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	247	259	0	231	0	0	-1
N.S.	1	1.00	0.91	0.96	0.00	0.86	0.00	0.00	-0.00
time (sec)	N/A	0.348	0.434	0.146	0.000	0.390	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	632	3411	0	351	0	0	-1
N.S.	1	1.00	1.40	7.58	0.00	0.78	0.00	0.00	-0.00
time (sec)	N/A	0.607	0.698	0.570	0.000	0.373	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	583	2531	0	291	0	0	-1
N.S.	1	1.00	1.35	5.87	0.00	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.521	0.505	0.164	0.000	0.383	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	583	1780	0	333	0	0	-1
N.S.	1	1.00	1.40	4.28	0.00	0.80	0.00	0.00	-0.00
time (sec)	N/A	0.425	0.468	0.129	0.000	0.364	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	309	1093	0	244	0	0	-1
N.S.	1	1.00	1.29	4.57	0.00	1.02	0.00	0.00	-0.00
time (sec)	N/A	0.254	0.286	0.084	0.000	0.358	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	585	491	0	333	0	0	-1
N.S.	1	1.00	1.23	1.03	0.00	0.70	0.00	0.00	-0.00
time (sec)	N/A	0.590	0.454	0.060	0.000	0.379	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	650	478	0	332	0	0	-1
N.S.	1	1.00	1.49	1.10	0.00	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.632	0.535	0.157	0.000	0.399	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	768	761	0	406	0	0	-1
N.S.	1	1.00	1.53	1.52	0.00	0.81	0.00	0.00	-0.00
time (sec)	N/A	0.999	0.617	0.191	0.000	0.387	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	647	3353	0	492	0	0	-1
N.S.	1	1.00	1.36	7.04	0.00	1.03	0.00	0.00	-0.00
time (sec)	N/A	0.804	1.348	0.815	0.000	0.384	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	927	2284	0	604	0	0	-1
N.S.	1	1.00	1.24	3.06	0.00	0.81	0.00	0.00	-0.00
time (sec)	N/A	0.893	1.859	0.188	0.000	0.404	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	634	1360	0	483	0	0	-1
N.S.	1	1.00	1.24	2.66	0.00	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.599	1.238	0.127	0.000	0.377	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	856	856	932	598	0	611	0	0	-1
N.S.	1	1.00	1.09	0.70	0.00	0.71	0.00	0.00	-0.00
time (sec)	N/A	0.889	1.708	0.090	0.000	0.399	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	924	580	0	647	0	0	-1
N.S.	1	1.00	1.27	0.79	0.00	0.89	0.00	0.00	-0.00
time (sec)	N/A	1.256	1.680	0.313	0.000	0.505	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	875	875	1177	1363	0	720	0	0	-1
N.S.	1	1.00	1.35	1.56	0.00	0.82	0.00	0.00	-0.00
time (sec)	N/A	2.031	1.811	0.371	0.000	0.483	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	791	791	995	697	0	768	0	0	-1
N.S.	1	1.00	1.26	0.88	0.00	0.97	0.00	0.00	-0.00
time (sec)	N/A	1.385	1.718	1.836	0.000	0.470	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	101	556	449	104	185	106	151
N.S.	1	1.00	0.65	3.56	2.88	0.67	1.19	0.68	0.97
time (sec)	N/A	0.162	0.135	0.056	0.325	0.357	0.677	5.043	0.586

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	84	392	326	87	151	88	121
N.S.	1	1.00	0.67	3.11	2.59	0.69	1.20	0.70	0.96
time (sec)	N/A	0.128	0.112	0.047	0.329	0.376	0.443	4.726	4.954

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	66	258	224	68	116	69	92
N.S.	1	1.00	0.69	2.72	2.36	0.72	1.22	0.73	0.97
time (sec)	N/A	0.088	0.090	0.042	0.313	0.398	0.292	5.101	4.795

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	159	141	52	82	54	65
N.S.	1	1.00	0.74	2.34	2.07	0.76	1.21	0.79	0.96
time (sec)	N/A	0.056	0.067	0.036	0.296	0.362	0.181	5.117	0.111

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	50	112	76	72	83	510	-1
N.S.	1	1.00	0.88	1.96	1.33	1.26	1.46	8.95	-0.02
time (sec)	N/A	0.078	0.139	0.056	0.753	0.425	2.900	4.641	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	79	69	79	0	489	-1
N.S.	1	1.00	1.00	1.41	1.23	1.41	0.00	8.73	-0.02
time (sec)	N/A	0.080	0.098	0.083	0.695	0.373	0.000	4.685	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	1146	84	0	564	-1
N.S.	1	1.00	0.94	0.93	16.37	1.20	0.00	8.06	-0.01
time (sec)	N/A	0.084	0.111	0.091	0.543	0.371	0.000	5.133	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	104	87	132	114	0	796	-1
N.S.	1	1.00	1.14	0.96	1.45	1.25	0.00	8.75	-0.01
time (sec)	N/A	0.130	0.151	0.085	1.085	0.375	0.000	5.283	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	139	822	662	161	284	161	225
N.S.	1	1.00	0.59	3.50	2.82	0.69	1.21	0.69	0.96
time (sec)	N/A	0.221	0.240	0.094	0.338	0.353	0.944	5.483	5.086

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	112	599	489	129	226	131	184
N.S.	1	1.00	0.60	3.19	2.60	0.69	1.20	0.70	0.98
time (sec)	N/A	0.166	0.196	0.075	0.340	0.349	0.644	4.838	0.619

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	108	487	147	145	207	921	-1
N.S.	1	1.00	0.67	3.02	0.91	0.90	1.29	5.72	-0.01
time (sec)	N/A	0.167	0.326	0.177	6.407	0.367	4.155	5.587	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	145	365	129	145	0	2038	-1
N.S.	1	1.00	1.00	2.52	0.89	1.00	0.00	14.06	-0.01
time (sec)	N/A	0.151	0.227	0.215	7.430	0.346	0.000	4.969	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	138	251	110	142	0	2171	-1
N.S.	1	1.00	0.97	1.77	0.77	1.00	0.00	15.29	-0.01
time (sec)	N/A	0.139	0.234	0.247	2.459	0.359	0.000	3.861	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	135	196	173	176	0	1181	-1
N.S.	1	1.00	0.89	1.30	1.15	1.17	0.00	7.82	-0.01
time (sec)	N/A	0.169	0.354	0.244	8.164	0.364	0.000	4.545	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	148	167	164	186	0	1255	-1
N.S.	1	1.00	0.89	1.00	0.98	1.11	0.00	7.51	-0.01
time (sec)	N/A	0.196	0.349	0.345	6.919	0.368	0.000	4.380	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	231	558	0	397	0	0	-1
N.S.	1	1.00	0.62	1.50	0.00	1.07	0.00	0.00	-0.00
time (sec)	N/A	0.609	0.329	0.175	0.000	0.415	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	216	392	0	393	0	0	-1
N.S.	1	1.00	0.61	1.10	0.00	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.420	0.209	0.062	0.000	0.390	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	186	266	0	292	0	0	-1
N.S.	1	1.00	0.66	0.95	0.00	1.04	0.00	0.00	-0.00
time (sec)	N/A	0.291	0.174	0.052	0.000	0.425	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	196	176	0	379	0	0	-1
N.S.	1	1.00	0.57	0.51	0.00	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.256	0.157	0.057	0.000	0.401	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	196	85	0	385	0	0	-1
N.S.	1	1.00	0.57	0.25	0.00	1.12	0.00	0.00	-0.00
time (sec)	N/A	0.256	0.101	0.050	0.000	0.411	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	206	88	0	314	0	0	-1
N.S.	1	1.00	0.68	0.29	0.00	1.04	0.00	0.00	-0.00
time (sec)	N/A	0.344	0.221	0.094	0.000	0.399	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	233	116	0	454	0	0	-1
N.S.	1	1.00	0.61	0.31	0.00	1.19	0.00	0.00	-0.00
time (sec)	N/A	0.392	0.260	0.125	0.000	0.430	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	253	136	0	491	0	0	-1
N.S.	1	1.00	0.62	0.33	0.00	1.20	0.00	0.00	-0.00
time (sec)	N/A	0.433	0.277	0.155	0.000	0.408	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	714	714	383	1184	0	670	0	0	-1
N.S.	1	1.00	0.54	1.66	0.00	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.662	0.153	0.434	0.000	0.401	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	214	823	0	482	0	0	-1
N.S.	1	1.00	0.58	2.22	0.00	1.30	0.00	0.00	-0.00
time (sec)	N/A	0.374	0.126	0.095	0.000	0.406	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	691	691	408	508	0	655	0	0	-1
N.S.	1	1.00	0.59	0.74	0.00	0.95	0.00	0.00	-0.00
time (sec)	N/A	0.856	0.155	0.073	0.000	0.411	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	735	735	406	248	0	669	0	0	-1
N.S.	1	1.00	0.55	0.34	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.822	0.160	0.059	0.000	0.399	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	693	693	446	233	0	588	0	0	-1
N.S.	1	1.00	0.64	0.34	0.00	0.85	0.00	0.00	-0.00
time (sec)	N/A	0.953	0.370	0.178	0.000	0.423	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	712	712	445	284	0	726	0	0	-1
N.S.	1	1.00	0.62	0.40	0.00	1.02	0.00	0.00	-0.00
time (sec)	N/A	0.996	0.488	0.253	0.000	0.413	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	800	800	470	388	0	908	0	0	-1
N.S.	1	1.00	0.59	0.48	0.00	1.14	0.00	0.00	-0.00
time (sec)	N/A	1.149	0.600	0.332	0.000	0.437	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	772	772	457	2035	0	890	0	0	-1
N.S.	1	1.00	0.59	2.64	0.00	1.15	0.00	0.00	-0.00
time (sec)	N/A	1.719	0.435	1.343	0.000	0.425	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	777	777	449	1396	0	935	0	0	-1
N.S.	1	1.00	0.58	1.80	0.00	1.20	0.00	0.00	-0.00
time (sec)	N/A	0.927	0.310	0.188	0.000	0.415	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1141	1141	698	847	0	1319	0	0	-1
N.S.	1	1.00	0.61	0.74	0.00	1.16	0.00	0.00	-0.00
time (sec)	N/A	1.969	0.408	0.136	0.000	0.464	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1161	1161	675	392	0	1223	0	0	-1
N.S.	1	1.00	0.58	0.34	0.00	1.05	0.00	0.00	-0.00
time (sec)	N/A	2.084	0.320	0.106	0.000	0.435	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1163	1163	2109	363	0	1125	0	0	-1
N.S.	1	1.00	1.81	0.31	0.00	0.97	0.00	0.00	-0.00
time (sec)	N/A	2.426	0.510	0.540	0.000	0.473	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [49] had the largest ratio of [19]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	11	4	1.00	15	0.267
2	A	9	4	1.00	15	0.267
3	A	7	4	1.00	13	0.308
4	A	2	2	1.00	12	0.167
5	A	6	5	1.00	15	0.333
6	A	9	5	1.00	15	0.333
7	A	11	5	1.00	15	0.333
8	A	13	5	1.00	15	0.333
9	A	15	5	1.00	15	0.333
10	A	14	4	1.00	17	0.235
11	A	11	4	1.00	15	0.267
12	A	3	2	1.00	14	0.143
13	A	8	7	1.00	17	0.412
14	A	10	6	1.00	17	0.353
15	A	14	5	1.00	17	0.294
16	A	17	5	1.00	17	0.294
17	A	20	5	1.00	17	0.294
18	A	15	7	1.00	17	0.412
19	A	11	7	1.00	17	0.412
20	A	8	7	1.00	17	0.412
21	A	6	5	1.00	15	0.333
22	A	3	3	1.00	14	0.214
23	A	8	4	1.00	17	0.235
24	A	12	5	1.00	17	0.294
25	A	17	5	1.00	17	0.294

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	15	8	1.00	17	0.471
27	A	12	8	1.00	17	0.471
28	A	10	6	1.00	17	0.353
29	A	9	5	1.00	15	0.333
30	A	4	4	1.00	14	0.286
31	A	12	5	1.00	17	0.294
32	A	16	5	1.00	17	0.294
33	A	15	6	1.00	17	0.353
34	A	14	5	1.00	17	0.294
35	A	11	5	1.00	15	0.333
36	A	5	4	1.00	14	0.286
37	A	17	5	1.00	17	0.294
38	A	21	5	1.00	17	0.294
39	A	26	5	1.00	17	0.294
40	A	12	3	1.00	17	0.176
41	A	10	3	1.00	17	0.176
42	A	8	3	1.00	15	0.200
43	A	6	3	1.00	14	0.214
44	A	7	6	1.00	17	0.353
45	A	7	6	1.00	17	0.353
46	A	10	5	1.00	17	0.294
47	A	12	5	1.00	17	0.294
48	A	14	5	1.00	17	0.294
49	A	17	3	1.00	19	0.158
50	A	14	3	1.00	17	0.176
51	A	11	3	1.00	16	0.188
52	A	11	6	1.00	19	0.316
53	A	10	7	1.00	19	0.368
54	A	12	7	1.00	19	0.368
55	A	13	6	1.00	19	0.316
56	A	17	5	1.00	19	0.263
57	A	14	7	1.00	19	0.368
58	A	12	6	1.00	19	0.316
59	A	11	6	1.00	19	0.316
60	A	8	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	8	4	1.00	16	0.250
62	A	13	4	1.00	19	0.210
63	A	14	6	1.00	19	0.316
64	A	18	5	1.00	19	0.263
65	A	24	9	1.00	19	0.474
66	A	20	8	1.00	19	0.421
67	A	17	6	1.00	19	0.316
68	A	9	5	1.00	17	0.294
69	A	18	5	1.00	16	0.312
70	A	22	6	1.00	19	0.316
71	A	32	6	1.00	19	0.316
72	A	27	8	1.00	19	0.421
73	A	28	7	1.00	19	0.368
74	A	19	6	1.00	17	0.353
75	A	28	5	1.00	16	0.312
76	A	41	7	1.00	19	0.368
77	A	60	6	1.00	19	0.316
78	A	46	7	1.00	19	0.368
79	A	13	4	1.00	17	0.235
80	A	11	4	1.00	17	0.235
81	A	9	4	1.00	15	0.267
82	A	7	4	1.00	14	0.286
83	A	8	6	1.00	17	0.353
84	A	8	7	1.00	17	0.412
85	A	8	6	1.00	17	0.353
86	A	11	5	1.00	17	0.294
87	A	17	4	1.00	17	0.235
88	A	14	4	1.00	16	0.250
89	A	14	7	1.00	19	0.368
90	A	13	8	1.00	19	0.421
91	A	12	8	1.00	19	0.421
92	A	14	7	1.00	19	0.368
93	A	15	7	1.00	19	0.368
94	A	15	6	1.00	19	0.316
95	A	14	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	11	4	1.00	19	0.210
97	A	11	4	1.00	17	0.235
98	A	11	4	1.00	16	0.250
99	A	16	4	1.00	19	0.210
100	A	17	5	1.00	19	0.263
101	A	18	6	1.00	19	0.316
102	A	23	6	1.00	19	0.316
103	A	12	5	1.00	19	0.263
104	A	34	7	1.00	17	0.412
105	A	36	8	1.00	16	0.500
106	A	41	8	1.00	19	0.421
107	A	47	7	1.00	19	0.368
108	A	51	8	1.00	19	0.421
109	A	71	10	1.00	19	0.526
110	A	37	9	1.00	19	0.474
111	A	89	9	1.00	17	0.529
112	A	99	10	1.00	16	0.625
113	A	110	9	1.00	19	0.474

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^3(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=126

$$-\frac{24b\cos(c+dx)}{d^5}+\frac{6ax\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}-\frac{6a\sin(c+dx)}{d^4}$$

```
[Out] -24*b*cos(d*x+c)/d^5+6*a*x*cos(d*x+c)/d^3+12*b*x^2*cos(d*x+c)/d^3-a*x^3*cos
(d*x+c)/d-b*x^4*cos(d*x+c)/d-6*a*sin(d*x+c)/d^4-24*b*x*sin(d*x+c)/d^4+3*a*x
^2*sin(d*x+c)/d^2+4*b*x^3*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.23, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6874, 3377, 2717, 2718}

$$-\frac{6a\sin(c+dx)}{d^4}+\frac{6ax\cos(c+dx)}{d^3}+\frac{3ax^2\sin(c+dx)}{d^2}-\frac{ax^3\cos(c+dx)}{d}-\frac{24b\cos(c+dx)}{d^5}-\frac{24bx\sin(c+dx)}{d^4}+\frac{12bx^2\cos(c+dx)}{d^3}+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{bx^4\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (-24*b*Cos[c + d*x])/d^5 + (6*a*x*Cos[c + d*x])/d^3 + (12*b*x^2*Cos[c + d*x
])/d^3 - (a*x^3*Cos[c + d*x])/d - (b*x^4*Cos[c + d*x])/d - (6*a*Sin[c + d*x
])/d^4 - (24*b*x*Sin[c + d*x])/d^4 + (3*a*x^2*Sin[c + d*x])/d^2 + (4*b*x^3*
Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int x^3(a+bx)\sin(c+dx)\,dx &= \int \left(ax^3\sin(c+dx)+bx^4\sin(c+dx)\right)\,dx\\
&= a\int x^3\sin(c+dx)\,dx + b\int x^4\sin(c+dx)\,dx\\
&= -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}+\frac{(3a)\int x^2\cos(c+dx)\,dx}{d}+\frac{(4b)\int x^3 \text{co}}{}\\
&= -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{4bx^3\sin(c+dx)}{d^2}-\\
&= \frac{6ax\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}+\\
&= \frac{6ax\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}-\\
&= -\frac{24b\cos(c+dx)}{d^5}+\frac{6ax\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 82, normalized size = 0.65

$$\frac{-((ad^2x(-6+d^2x^2)+b(24-12d^2x^2+d^4x^4))\cos(c+dx))+d(4bx(-6+d^2x^2)+3a(-2+d^2x^2))\sin(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (-((a*d^2*x*(-6 + d^2*x^2) + b*(24 - 12*d^2*x^2 + d^4*x^4))*Cos[c + d*x]) +
 d*(4*b*x*(-6 + d^2*x^2) + 3*a*(-2 + d^2*x^2))*Sin[c + d*x])/d^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(126) = 252.
time = 0.05, size = 359, normalized size = 2.85

method	result
risch	$-\frac{(b\,x^4d^4+a\,d^4x^3-12d^2x^2b-6a\,d^2x+24b)\cos(dx+c)}{d^5}+\frac{(4b\,d^2x^3+3a\,d^2x^2-24bx-6a)\sin(dx+c)}{d^4}$
norman	$\frac{\frac{a\,x^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b\,x^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{48b}{d^5}-\frac{12a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{6ax}{d^3}-\frac{a\,x^3}{d}+\frac{12b\,x^2}{d^3}-\frac{b\,x^4}{d}-\frac{6ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}+\frac{6a\,x^2\tan(}{d^2}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
meijerg	$\frac{16b\sqrt{\pi}\,\sin(c)\left(-\frac{x\left(d^2\right)^{\frac{5}{2}}\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{10\sqrt{\pi}\,d^4}+\frac{\left(d^2\right)^{\frac{5}{2}}\left(\frac{5}{8}d^4x^4-\frac{15}{2}d^2x^2+15\right)\sin(dx)}{10\sqrt{\pi}\,d^5}\right)}{d^4\sqrt{d^2}}+\frac{16b\sqrt{\pi}\,\cos(c)\left(\frac{3}{2\sqrt{\pi}}-\right.}{}$

derivativedivides	$\frac{a\,c^3\cos(dx+c)+3a\,c^2(\sin(dx+c)-(dx+c)\cos(dx+c))-3ac\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)+}{}$
default	$\frac{a\,c^3\cos(dx+c)+3a\,c^2(\sin(dx+c)-(dx+c)\cos(dx+c))-3ac\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(a*c^3*cos(d*x+c)+3*a*c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-3*a*c*(-(d*
x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+a*(-(d*x+c)^3*cos(d*x+
c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))-1/d*b*c^4*cos(
d*x+c)-4/d*b*c^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+6/d*b*c^2*(-(d*x+c)^2*cos(
d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-4/d*b*c*(-(d*x+c)^3*cos(d*x+c)+3*
(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+1/d*b*(-(d*x+c)^4*c
os(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)-24*(
d*x+c)*sin(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(126) = 252$.
time = 0.30, size = 306, normalized size = 2.43

$$\frac{ac^3\cos(dx+c) - \frac{bc^4\cos(dx+c)}{d} - 3\left((dx+c)\cos(dx+c) - \sin(dx+c)\right)ac^2 + \frac{4\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)bc^3}{d} + 3\left(\left((dx+c)^2-2\right)\cos(dx+c) - 2(dx+c)\sin(dx+c)\right)ac - \frac{6\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)bc^2}{d} - \left(\left((dx+c)^3-6dx-6c\right)\cos(dx+c) - 3\left((dx+c)^2-2\right)\sin(dx+c)\right)a + \frac{4\left(\left((dx+c)^3-6dx-6c\right)\cos(dx+c)-3\left((dx+c)^2-2\right)\sin(dx+c)\right)bc}{d} - \frac{\left(\left((dx+c)^4-12(dx+c)^2+24\right)\cos(dx+c)-4\left((dx+c)^3-6dx-6c\right)\sin(dx+c)\right)b}{d}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c^3*cos(d*x + c) - b*c^4*cos(d*x + c)/d - 3*((d*x + c)*cos(d*x + c) - si
n(d*x + c))*a*c^2 + 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^3/d + 3*(
((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*c - 6*(((d*x +
 c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^2/d - (((d*x + c)^3
 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a + 4*(((d
*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b
*c/d - (((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 -
 6*d*x - 6*c)*sin(d*x + c))*b/d)/d^4
```

Fricas [A]
time = 0.35, size = 85, normalized size = 0.67

$$-\frac{\left(bd^4x^4 + ad^4x^3 - 12\,bd^2x^2 - 6\,ad^2x + 24\,b\right)\cos(dx+c) - \left(4\,bd^3x^3 + 3\,ad^3x^2 - 24\,bdx - 6\,ad\right)\sin(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^4*x^4 + a*d^4*x^3 - 12*b*d^2*x^2 - 6*a*d^2*x + 24*b)*cos(d*x + c) -
(4*b*d^3*x^3 + 3*a*d^3*x^2 - 24*b*d*x - 6*a*d)*sin(d*x + c))/d^5
```

Sympy [A]
time = 0.35, size = 151, normalized size = 1.20

$$\begin{cases} -\frac{ax^3\cos(c+dx)}{d} + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{6ax\cos(c+dx)}{d^3} - \frac{6a\sin(c+dx)}{d^4} - \frac{bx^4\cos(c+dx)}{d} + \frac{4bx^3\sin(c+dx)}{d^2} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{24bx\sin(c+dx)}{d^4} - \frac{24b\cos(c+dx)}{d^5} & \text{for } d \neq 0 \\ \left(\frac{ax^4}{4} + \frac{bx^5}{5}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**3*cos(c + d*x)/d + 3*a*x**2*sin(c + d*x)/d**2 + 6*a*x*cos(
c + d*x)/d**3 - 6*a*sin(c + d*x)/d**4 - b*x**4*cos(c + d*x)/d + 4*b*x**3*si
n(c + d*x)/d**2 + 12*b*x**2*cos(c + d*x)/d**3 - 24*b*x*sin(c + d*x)/d**4 -
24*b*cos(c + d*x)/d**5, Ne(d, 0)), ((a*x**4/4 + b*x**5/5)*sin(c), True))
```

Giac [A]
time = 3.71, size = 86, normalized size = 0.68

$$-\frac{(bd^4x^4 + ad^4x^3 - 12\,bd^2x^2 - 6\,ad^2x + 24\,b)\cos(dx+c)}{d^5} + \frac{(4\,bd^3x^3 + 3\,ad^3x^2 - 24\,bdx - 6\,ad)\sin(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^4*x^4 + a*d^4*x^3 - 12*b*d^2*x^2 - 6*a*d^2*x + 24*b)*cos(d*x + c)/d^5
 + (4*b*d^3*x^3 + 3*a*d^3*x^2 - 24*b*d*x - 6*a*d)*sin(d*x + c)/d^5
```

Mupad [B]
time = 0.28, size = 122, normalized size = 0.97

$$\frac{6\,a\,x\,\cos(c+d\,x)+12\,b\,x^2\,\cos(c+d\,x)}{d^3}-\frac{6\,a\,\sin(c+d\,x)+24\,b\,x\,\sin(c+d\,x)}{d^4}-\frac{a\,x^3\,\cos(c+d\,x)+b\,x^4\,\cos(c+d\,x)}{d}+\frac{3\,a\,x^2\,\sin(c+d\,x)+4\,b\,x^3\,\sin(c+d\,x)}{d^2}-\frac{24\,b\,\cos(c+d\,x)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(c + d*x)*(a + b*x),x)
```

```
[Out] (6*a*x*cos(c + d*x) + 12*b*x^2*cos(c + d*x))/d^3 - (6*a*sin(c + d*x) + 24*b
*x*sin(c + d*x))/d^4 - (a*x^3*cos(c + d*x) + b*x^4*cos(c + d*x))/d + (3*a*x
^2*sin(c + d*x) + 4*b*x^3*sin(c + d*x))/d^2 - (24*b*cos(c + d*x))/d^5
```

3.2 $\int x^2(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=96

$$\frac{2a\cos(c+dx)}{d^3}+\frac{6bx\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{2ax\sin(c+dx)}{d^2}+3bx$$

```
[Out] 2*a*cos(d*x+c)/d^3+6*b*x*cos(d*x+c)/d^3-a*x^2*cos(d*x+c)/d-b*x^3*cos(d*x+c)
/d-6*b*sin(d*x+c)/d^4+2*a*x*sin(d*x+c)/d^2+3*b*x^2*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.15, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6874, 3377, 2718, 2717}

$$\frac{2a\cos(c+dx)}{d^3}+\frac{2ax\sin(c+dx)}{d^2}-\frac{ax^2\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{6bx\cos(c+dx)}{d^3}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{bx^3\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (2*a*Cos[c + d*x])/d^3 + (6*b*x*Cos[c + d*x])/d^3 - (a*x^2*Cos[c + d*x])/d
- (b*x^3*Cos[c + d*x])/d - (6*b*Sin[c + d*x])/d^4 + (2*a*x*Sin[c + d*x])/d^
2 + (3*b*x^2*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2(a+bx)\sin(c+dx)\,dx = \int \left(ax^2\sin(c+dx)+bx^3\sin(c+dx)\right)\,dx$$
$$= a\int x^2\sin(c+dx)\,dx + b\int x^3\sin(c+dx)\,dx$$
$$= -\frac{ax^2\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} + \frac{(2a)\int x\cos(c+dx)\,dx}{d} + \frac{(3b)\int x^2\cos\cdots}{\cdots}$$
$$= -\frac{ax^2\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} + \frac{2ax\sin(c+dx)}{d^2} + \frac{3bx^2\sin(c+dx)}{d^2} - \cdots$$
$$= \frac{2a\cos(c+dx)}{d^3} + \frac{6bx\cos(c+dx)}{d^3} - \frac{ax^2\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} + \frac{2ax\cdots}{\cdots}$$
$$= \frac{2a\cos(c+dx)}{d^3} + \frac{6bx\cos(c+dx)}{d^3} - \frac{ax^2\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} - \frac{6b\,\mathrm{s}\cdots}{\cdots}$$

Mathematica [A]
time = 0.10, size = 65, normalized size = 0.68

$$\frac{-d(bx(-6+d^2x^2)+a(-2+d^2x^2))\cos(c+dx)+(2ad^2x+3b(-2+d^2x^2))\sin(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (-(d*(b*x*(-6 + d^2*x^2) + a*(-2 + d^2*x^2))*Cos[c + d*x]) + (2*a*d^2*x + 3
*b*(-2 + d^2*x^2))*Sin[c + d*x])/d^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(96) = 192$.
time = 0.04, size = 225, normalized size = 2.34

method	result
risch	$-\frac{\left(b\,d^2x^3+a\,d^2x^2-6bx-2a\right)\cos(dx+c)}{d^3}+\frac{\left(3d^2x^2b+2a\,d^2x-6b\right)\sin(dx+c)}{d^4}$
norman	$\frac{\frac{4a}{d^3}+\frac{a\,x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b\,x^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\,x^2}{d}-\frac{12b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{6bx}{d^3}-\frac{b\,x^3}{d}+\frac{4ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{6bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}+\cdots}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
meijerg	$\frac{8b\sqrt{\pi}\,\sin(c)\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3d^2x^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}}-\frac{dx\left(-\frac{d^2x^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4}+\frac{8b\sqrt{\pi}\,\cos(c)\left(\frac{xd\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{20\sqrt{\pi}}-\cdots\right)}{d^4}$
derivativedivides	$\frac{-a\,c^2\cos(dx+c)-2ac(\sin(dx+c)-(dx+c)\cos(dx+c))+a\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)+\frac{b\,c^3\cdots}{\cdots}}{\cdots}$

default	$\frac{-a\,c^2\cos(dx+c)-2ac(\sin(dx+c)-(dx+c)\cos(dx+c))+a\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)+\frac{b\,c}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(-a*c^2*cos(d*x+c)-2*a*c*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+a*(-(d*x+c)^
2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+1/d*b*c^3*cos(d*x+c)+3/d*b*
c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-3/d*b*c*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*
x+c)+2*(d*x+c)*sin(d*x+c))+1/d*b*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x
+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(96) = 192$.
time = 0.31, size = 201, normalized size = 2.09

$$-\frac{ac^2\cos(dx+c)-\frac{bc^3\cos(dx+c)}{d}-2\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)ac+\frac{3((dx+c)\cos(dx+c)-\sin(dx+c))bc^2}{d}+\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)a-\frac{3\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)bc}{d}+\frac{\left(\left((dx+c)^3-6\,dx-6\,c\right)\cos(dx+c)-3\left((dx+c)^2-2\right)\sin(dx+c)\right)b}{d}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*c^2*cos(d*x + c) - b*c^3*cos(d*x + c)/d - 2*((d*x + c)*cos(d*x + c) - s
in(d*x + c))*a*c + 3*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^2/d + (((d
*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a - 3*(((d*x + c)^2
 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c/d + (((d*x + c)^3 - 6*d*
x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b/d)/d^3
```

Fricas [A]
time = 0.37, size = 67, normalized size = 0.70

$$-\frac{(bd^3x^3+ad^3x^2-6\,bdx-2\,ad)\cos(dx+c)-(3\,bd^2x^2+2\,ad^2x-6\,b)\sin(dx+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^3*x^3 + a*d^3*x^2 - 6*b*d*x - 2*a*d)*cos(d*x + c) - (3*b*d^2*x^2 + 2
*a*d^2*x - 6*b)*sin(d*x + c))/d^4
```

Sympy [A]
time = 0.19, size = 117, normalized size = 1.22

$$\begin{cases} -\frac{ax^2\cos(c+dx)}{d}+\frac{2ax\sin(c+dx)}{d^2}+\frac{2a\cos(c+dx)}{d^3}-\frac{bx^3\cos(c+dx)}{d}+\frac{3bx^2\sin(c+dx)}{d^2}+\frac{6bx\cos(c+dx)}{d^3}-\frac{6b\sin(c+dx)}{d^4} & \text{for } d\neq 0 \\ \left(\frac{ax^3}{3}+\frac{bx^4}{4}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**2*cos(c + d*x)/d + 2*a*x*sin(c + d*x)/d**2 + 2*a*cos(c + d
*x)/d**3 - b*x**3*cos(c + d*x)/d + 3*b*x**2*sin(c + d*x)/d**2 + 6*b*x*cos(c
 + d*x)/d**3 - 6*b*sin(c + d*x)/d**4, Ne(d, 0)), ((a*x**3/3 + b*x**4/4)*sin
(c), True))
```

Giac [A]
time = 3.34, size = 68, normalized size = 0.71

$$-\frac{(bd^3x^3 + ad^3x^2 - 6\,bdx - 2\,ad)\cos(dx + c)}{d^4} + \frac{(3\,bd^2x^2 + 2\,ad^2x - 6\,b)\sin(dx + c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^3*x^3 + a*d^3*x^2 - 6*b*d*x - 2*a*d)*cos(d*x + c)/d^4 + (3*b*d^2*x^2
+ 2*a*d^2*x - 6*b)*sin(d*x + c)/d^4
```

Mupad [B]
time = 4.62, size = 92, normalized size = 0.96

$$\frac{3\,b\,x^2\,\sin(c + d\,x) + 2\,a\,x\,\sin(c + d\,x)}{d^2} + \frac{2\,a\,\cos(c + d\,x) + 6\,b\,x\,\cos(c + d\,x)}{d^3} - \frac{a\,x^2\,\cos(c + d\,x) + b\,x^3\,\cos(c + d\,x)}{d} - \frac{6\,b\,\sin(c + d\,x)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(c + d*x)*(a + b*x),x)
```

```
[Out] (3*b*x^2*sin(c + d*x) + 2*a*x*sin(c + d*x))/d^2 + (2*a*cos(c + d*x) + 6*b*x
*cos(c + d*x))/d^3 - (a*x^2*cos(c + d*x) + b*x^3*cos(c + d*x))/d - (6*b*sin
(c + d*x))/d^4
```

3.3 $\int x(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=65

$$\frac{2b\cos(c+dx)}{d^3}-\frac{ax\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}+\frac{2bx\sin(c+dx)}{d^2}$$

```
[Out] 2*b*cos(d*x+c)/d^3-a*x*cos(d*x+c)/d-b*x^2*cos(d*x+c)/d+a*sin(d*x+c)/d^2+2*b
*x*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6874, 3377, 2717, 2718}

$$\frac{a\sin(c+dx)}{d^2}-\frac{ax\cos(c+dx)}{d}+\frac{2b\cos(c+dx)}{d^3}+\frac{2bx\sin(c+dx)}{d^2}-\frac{bx^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (2*b*Cos[c + d*x])/d^3 - (a*x*Cos[c + d*x])/d - (b*x^2*Cos[c + d*x])/d + (a
*Sin[c + d*x])/d^2 + (2*b*x*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int x(a+bx)\sin(c+dx)\,dx &= \int \left(ax\sin(c+dx)+bx^2\sin(c+dx)\right)\,dx \\
&= a\int x\sin(c+dx)\,dx + b\int x^2\sin(c+dx)\,dx \\
&= -\frac{ax\cos(c+dx)}{d} - \frac{bx^2\cos(c+dx)}{d} + \frac{a\int\cos(c+dx)\,dx}{d} + \frac{(2b)\int x\cos(c+dx}{d} \\
&= -\frac{ax\cos(c+dx)}{d} - \frac{bx^2\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} + \frac{2bx\sin(c+dx)}{d^2} - \frac{(2b)\int}{} \\
&= \frac{2b\cos(c+dx)}{d^3} - \frac{ax\cos(c+dx)}{d} - \frac{bx^2\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} + \frac{2bx\sin(}{d^2}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 45, normalized size = 0.69

$$
\frac{-\left((ad^2x+b(-2+d^2x^2))\cos(c+dx)\right)+d(a+2bx)\sin(c+dx)}{d^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (-((a*d^2*x + b*(-2 + d^2*x^2))*Cos[c + d*x]) + d*(a + 2*b*x)*Sin[c + d*x])
/d^3
```

Maple [A]
time = 0.04, size = 121, normalized size = 1.86

method	result
risch	$-\frac{(d^2x^2b+a\,d^2x-2b)\cos(dx+c)}{d^3}+\frac{(2bx+a)\sin(dx+c)}{d^2}$
norman	$\frac{\frac{4b}{d^3}+\frac{ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b\,x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{ax}{d}-\frac{b\,x^2}{d}+\frac{4bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
derivativedivides	$\frac{ac\cos(dx+c)+a(\sin(dx+c)-(dx+c)\cos(dx+c))-\frac{b\,c^2\cos(dx+c)}{d}-\frac{2bc(\sin(dx+c)-(dx+c)\cos(dx+c))}{d}+\frac{b\left(-(dx+c)^2\cos(dx+c)+2\right.}{}}{d^2}$
default	$\frac{ac\cos(dx+c)+a(\sin(dx+c)-(dx+c)\cos(dx+c))-\frac{b\,c^2\cos(dx+c)}{d}-\frac{2bc(\sin(dx+c)-(dx+c)\cos(dx+c))}{d}+\frac{b\left(-(dx+c)^2\cos(dx+c)+2\right.}{}}{d^2}$
meijerg	$\frac{4b\sqrt{\pi}\,\sin(c)\left(\frac{x\left(d^2\right)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\,d^2}-\frac{\left(d^2\right)^{\frac{3}{2}}\left(-\frac{3d^2x^2}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\,d^3}\right)}{d^2\sqrt{d^2}}+\frac{4b\sqrt{\pi}\,\cos(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{d^2x^2}{2}+1\right)\cos(dx)}{2\sqrt{\pi}}+\frac{dx\,\mathrm{s}}{2}\right.}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(a*c*cos(d*x+c)+a*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-1/d*b*c^2*cos(d*x+c
)-2/d*b*c*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+1/d*b*(-(d*x+c)^2*cos(d*x+c)+2*co
s(d*x+c)+2*(d*x+c)*sin(d*x+c)))
```

Maxima [A]
time = 0.31, size = 117, normalized size = 1.80

$$\frac{ac\cos(dx+c) - \frac{bc^2\cos(dx+c)}{d} - ((dx+c)\cos(dx+c) - \sin(dx+c))a + \frac{2((dx+c)\cos(dx+c) - \sin(dx+c))bc}{d} - \frac{\left(\left((dx+c)^2-2\right)\cos(dx+c) - 2(dx+c)\sin(dx+c)\right)b}{d}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c*cos(d*x + c) - b*c^2*cos(d*x + c)/d - ((d*x + c)*cos(d*x + c) - sin(d*
x + c))*a + 2*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c/d - (((d*x + c)^2
 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b/d)/d^2
```

Fricas [A]
time = 0.34, size = 48, normalized size = 0.74

$$-\frac{(bd^2x^2 + ad^2x - 2b)\cos(dx+c) - (2bdx + ad)\sin(dx+c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^2*x^2 + a*d^2*x - 2*b)*cos(d*x + c) - (2*b*d*x + a*d)*sin(d*x + c))/
d^3
```

Sympy [A]
time = 0.14, size = 82, normalized size = 1.26

$$\begin{cases} -\frac{ax\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} - \frac{bx^2\cos(c+dx)}{d} + \frac{2bx\sin(c+dx)}{d^2} + \frac{2b\cos(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^3}{3}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x*cos(c + d*x)/d + a*sin(c + d*x)/d**2 - b*x**2*cos(c + d*x)/
d + 2*b*x*sin(c + d*x)/d**2 + 2*b*cos(c + d*x)/d**3, Ne(d, 0)), ((a*x**2/2
+ b*x**3/3)*sin(c), True))
```

Giac [A]
time = 3.65, size = 49, normalized size = 0.75

$$-\frac{(bd^2x^2 + ad^2x - 2b)\cos(dx+c)}{d^3} + \frac{(2bdx + ad)\sin(dx+c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^2*x^2 + a*d^2*x - 2*b)*cos(d*x + c)/d^3 + (2*b*d*x + a*d)*sin(d*x + c
)/d^3
```

Mupad [B]
time = 4.50, size = 62, normalized size = 0.95

$$\frac{a\,\sin\left(c+d\,x\right)+2\,b\,x\,\sin\left(c+d\,x\right)}{d^2}-\frac{a\,x\,\cos\left(c+d\,x\right)+b\,x^2\,\cos\left(c+d\,x\right)}{d}+\frac{2\,b\,\cos\left(c+d\,x\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(c + d*x)*(a + b*x),x)
```

```
[Out] (a*sin(c + d*x) + 2*b*x*sin(c + d*x))/d^2 - (a*x*cos(c + d*x) + b*x^2*cos(c
 + d*x))/d + (2*b*cos(c + d*x))/d^3
```

3.4 $\int (a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=28

$$-\frac{(a+bx)\cos(c+dx)}{d}+\frac{b\sin(c+dx)}{d^2}$$

```
[Out] -(b*x+a)*cos(d*x+c)/d+b*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3377, 2717}

$$\frac{b\sin(c+dx)}{d^2}-\frac{(a+bx)\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Sin[c + d*x],x]
```

```
[Out] -(((a + b*x)*Cos[c + d*x])/d) + (b*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\sin(c+dx)\,dx &= -\frac{(a+bx)\cos(c+dx)}{d}+\frac{b\int\cos(c+dx)\,dx}{d}\\ &= -\frac{(a+bx)\cos(c+dx)}{d}+\frac{b\sin(c+dx)}{d^2}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 27, normalized size = 0.96

$$\frac{-d(a+bx)\cos(c+dx)+b\sin(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (-(d*(a + b*x)*Cos[c + d*x]) + b*Sin[c + d*x])/d^2
```

Maple [A]
time = 0.03, size = 52, normalized size = 1.86

method	result
risch	$-\frac{(bx+a)\cos(dx+c)}{d}+\frac{b\sin(dx+c)}{d^2}$
derivativedivides	$\frac{-a\cos(dx+c)+\frac{bc\cos(dx+c)}{d}+\frac{b(\sin(dx+c)-(dx+c)\cos(dx+c))}{d}}{d}$
default	$\frac{-a\cos(dx+c)+\frac{bc\cos(dx+c)}{d}+\frac{b(\sin(dx+c)-(dx+c)\cos(dx+c))}{d}}{d}$
norman	$\frac{\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{bx}{d}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
meijerg	$\frac{2b\sqrt{\pi}\,\sin(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b\sqrt{\pi}\,\cos(c)\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{a\sin(c)\sin(dx)}{d}+ a\sqrt{\ }$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a*cos(d*x+c)+1/d*b*c*cos(d*x+c)+1/d*b*(sin(d*x+c)-(d*x+c)*cos(d*x+c))
)
```

Maxima [A]
time = 0.29, size = 53, normalized size = 1.89

$$-\frac{a\cos(dx+c)-\frac{bc\cos(dx+c)}{d}+\frac{((dx+c)\cos(dx+c)-\sin(dx+c))b}{d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*cos(d*x + c) - b*c*cos(d*x + c)/d + ((d*x + c)*cos(d*x + c) - sin(d*x +
 c))*b/d)/d
```

Fricas [A]
time = 0.36, size = 30, normalized size = 1.07

$$-\frac{(bdx+ad)\cos(dx+c)-b\sin(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d*x + a*d)*cos(d*x + c) - b*sin(d*x + c))/d^2
```

Sympy [A]
time = 0.08, size = 46, normalized size = 1.64

$$\begin{cases} -\frac{a\cos(c+dx)}{d} - \frac{bx\cos(c+dx)}{d} + \frac{b\sin(c+dx)}{d^2} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^2}{2}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)/d - b*x*cos(c + d*x)/d + b*sin(c + d*x)/d**2, Ne
(d, 0)), ((a*x + b*x**2/2)*sin(c), True))
```

Giac [A]
time = 4.51, size = 31, normalized size = 1.11

$$-\frac{(bdx + ad)\cos(dx + c)}{d^2} + \frac{b\sin(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d*x + a*d)*cos(d*x + c)/d^2 + b*sin(d*x + c)/d^2
```

Mupad [B]
time = 4.49, size = 35, normalized size = 1.25

$$\frac{b\sin(c + d\,x)}{d^2} - \frac{a\cos(c + d\,x) + b\,x\cos(c + d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*x),x)
```

```
[Out] (b*sin(c + d*x))/d^2 - (a*cos(c + d*x) + b*x*cos(c + d*x))/d
```

3.5 $\int \frac{(a+bx)\sin(c+dx)}{x}\,dx$

Optimal. Leaf size=29

$$-\frac{b\cos(c+dx)}{d}+a\text{Ci}(dx)\sin(c)+a\cos(c)\text{Si}(dx)$$

```
[Out] -b*cos(d*x+c)/d+a*cos(c)*Si(d*x)+a*Ci(d*x)*sin(c)
```

Rubi [A]
time = 0.10, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 2718, 3384, 3380, 3383}

$$a\sin(c)\text{CosIntegral}(dx)+a\cos(c)\text{Si}(dx)-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sin[c + d*x])/x,x]
```

```
[Out] -((b*Cos[c + d*x])/d) + a*CosIntegral[d*x]*Sin[c] + a*Cos[c]*SinIntegral[d*
x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)\sin(c+dx)}{x}\,dx &= \int \left(b\sin(c+dx)+\frac{a\sin(c+dx)}{x}\right)dx\\ &= a\int \frac{\sin(c+dx)}{x}\,dx + b\int \sin(c+dx)\,dx\\ &= -\frac{b\cos(c+dx)}{d} + (a\cos(c))\int \frac{\sin(dx)}{x}\,dx + (a\sin(c))\int \frac{\cos(dx)}{x}\,dx\\ &= -\frac{b\cos(c+dx)}{d} + a\text{Ci}(dx)\sin(c) + a\cos(c)\text{Si}(dx)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.38

$$-\frac{b\cos(c)\cos(dx)}{d} + a\text{Ci}(dx)\sin(c) + \frac{b\sin(c)\sin(dx)}{d} + a\cos(c)\text{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x,x]
```

```
[Out] -((b*Cos[c]*Cos[d*x])/d) + a*CosIntegral[d*x]*Sin[c] + (b*Sin[c]*Sin[d*x])/
d + a*Cos[c]*SinIntegral[d*x]
```

Maple [A]
time = 0.07, size = 31, normalized size = 1.07

method	result
derivativedivides	$a(\text{sinIntegral}(dx)\cos(c) + \text{cosineIntegral}(dx)\sin(c)) - \frac{b\cos(dx+c)}{d}$
default	$a(\text{sinIntegral}(dx)\cos(c) + \text{cosineIntegral}(dx)\sin(c)) - \frac{b\cos(dx+c)}{d}$
risch	$\frac{ia\,\mathrm{e}^{ic}\,\text{expIntegral}(1,-idx)}{2} - \frac{\mathrm{e}^{-ic}\pi\,\text{csgn}(dx)a}{2} + \mathrm{e}^{-ic}\,\text{sinIntegral}(dx)\,a - \frac{i\,\text{expIntegral}(1,-idx)\mathrm{e}^{-ic}a}{2} - b$
meijerg	$\frac{b\sin(c)\sin(dx)}{d} + \frac{b\sqrt{\pi}\,\cos(c)\left(\frac{1}{\sqrt{\pi}} - \frac{\cos(dx)}{\sqrt{\pi}}\right)}{d} + \frac{a\sqrt{\pi}\,\sin(c)\left(\frac{2\gamma+2\ln(x)+\ln(d^2)}{\sqrt{\pi}} - \frac{2\gamma}{\sqrt{\pi}} - \frac{2\ln(2)}{\sqrt{\pi}} - \frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-b*cos(d*x+c)/d
```

Maxima [C] Result contains complex when optimal does not.
time = 0.35, size = 522, normalized size = 18.00

$$-\frac{1}{2}\left(\left(i E_1(i d x)-i E_1(-i d x)\right)\cos(c)+\left(E_1(i d x)+E_1(-i d x)\right)\sin(c)\right)a+\frac{\left(\left(i E_1(i d x)-i E_1(-i d x)\right)\cos(c)+\left(E_1(i d x)+E_1(-i d x)\right)\sin(c)\right)bc}{2d}-\frac{\left(2(dx+c)\left(\cos(c)^2+\sin(c)^2\right)\cos(dx+c)^3+2(dx+c)\left(\cos(c)^2+\sin(c)^2\right)\cos(dx+c)-\left(c\left(E_2(idx)+E_2(\ldots\right.\right.\right.}{\ldots}$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/2*((I*exp_integral_e(1, I*d*x) - I*exp_integral_e(1, -I*d*x))*cos(c) + (
exp_integral_e(1, I*d*x) + exp_integral_e(1, -I*d*x))*sin(c))*a + 1/2*((I*e
xp_integral_e(1, I*d*x) - I*exp_integral_e(1, -I*d*x))*cos(c) + (exp_integr
al_e(1, I*d*x) + exp_integral_e(1, -I*d*x))*sin(c))*b*c/d - 1/4*(2*(d*x + c
)*(cos(c)^2 + sin(c)^2)*cos(d*x + c)^3 + 2*(d*x + c)*(cos(c)^2 + sin(c)^2)*
cos(d*x + c) - (c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*co
s(c)^3 + c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c)*si
n(c)^2 - c*(I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*sin(c
)^3 + c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c) - (c*
(I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c)^2 + c*(I*
exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x)))*sin(c))*cos(d*x +
c)^2 - (c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c)^3 +
 c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c)*sin(c)^2 -
 c*(I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*sin(c)^3 - 2*
(d*x + c)*(cos(c)^2 + sin(c)^2)*cos(d*x + c) + c*(exp_integral_e(2, I*d*x)
+ exp_integral_e(2, -I*d*x))*cos(c) - (c*(I*exp_integral_e(2, I*d*x) - I*ex
p_integral_e(2, -I*d*x))*cos(c)^2 + c*(I*exp_integral_e(2, I*d*x) - I*exp_i
ntegral_e(2, -I*d*x)))*sin(c))*sin(d*x + c)^2)*b/(((d*x + c)*(cos(c)^2 + si
n(c)^2)*d - (c*cos(c)^2 + c*sin(c)^2)*d)*cos(d*x + c)^2 + ((d*x + c)*(cos(c
)^2 + sin(c)^2)*d - (c*cos(c)^2 + c*sin(c)^2)*d)*sin(d*x + c)^2)
```

Fricas [A]
time = 0.34, size = 44, normalized size = 1.52

$$\frac{2\,ad\cos\left(c\right)\operatorname{Si}\left(dx\right)-2\,b\cos\left(dx+c\right)+\left(ad\operatorname{Ci}\left(dx\right)+ad\operatorname{Ci}\left(-dx\right)\right)\sin\left(c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*cos(c)*sin_integral(d*x) - 2*b*cos(d*x + c) + (a*d*cos_integral(
d*x) + a*d*cos_integral(-d*x))*sin(c))/d
```

Sympy [A]
time = 3.12, size = 37, normalized size = 1.28

$$-a(-\sin\left(c\right)\operatorname{Ci}\left(dx\right)-\cos\left(c\right)\operatorname{Si}\left(dx\right))-b\left(\begin{cases}-x\sin\left(c\right) & \text{for}\ d=0\\ \frac{\cos\left(c+dx\right)}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x,x)
```

```
[Out] -a*(-sin(c)*Ci(d*x) - cos(c)*Si(d*x)) - b*Piecewise((-x*sin(c), Eq(d, 0)),
(cos(c + d*x)/d, True))
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.45, size = 339, normalized size = 11.69

$$-\frac{ad\Im\left(\mathrm{Ci}\left(dx\right)\right)\tan\left(\frac{1}{2}dx\right)^{2}\tan\left(\frac{1}{2}c\right)^{2} - ad\Im\left(\mathrm{Ci}\left(-dx\right)\right)\tan\left(\frac{1}{2}dx\right)^{2}\tan\left(\frac{1}{2}c\right)^{2} + 2ad\,\mathrm{Si}\left(dx\right)\tan\left(\frac{1}{2}dx\right)^{2}\tan\left(\frac{1}{2}c\right)^{2} - 2ad\Re\left(\mathrm{Ci}\left(dx\right)\right)\tan\left(\frac{1}{2}dx\right)^{2}\tan\left(\frac{1}{2}c\right) - 2ad\Re\left(\mathrm{Ci}\left(-dx\right)\right)\tan\left(\frac{1}{2}dx\right)^{2}\tan\left(\frac{1}{2}c\right) - ad\Im\left(\mathrm{Ci}\left(dx\right)\right)\tan\left(\frac{1}{2}dx\right)^{2} + ad\Im\left(\mathrm{Ci}\left(-dx\right)\right)\tan\left(\frac{1}{2}dx\right)^{2} - 2ad\,\mathrm{Si}\left(dx\right)\tan\left(\frac{1}{2}dx\right)^{2} + ad\Im\left(\mathrm{Ci}\left(dx\right)\right)\tan\left(\frac{1}{2}c\right)^{2} - ad\Im\left(\mathrm{Ci}\left(-dx\right)\right)\tan\left(\frac{1}{2}c\right)^{2} + 2ad\,\mathrm{Si}\left(dx\right)\tan\left(\frac{1}{2}c\right)^{2} + 2b\tan\left(\frac{1}{2}dx\right)^{2}\tan\left(\frac{1}{2}c\right)^{2} - 2ad\Re\left(\mathrm{Ci}\left(dx\right)\right)\tan\left(\frac{1}{2}c\right) - 2ad\Re\left(\mathrm{Ci}\left(-dx\right)\right)\tan\left(\frac{1}{2}c\right) - ad\Im\left(\mathrm{Ci}\left(dx\right)\right) + ad\Im\left(\mathrm{Ci}\left(-dx\right)\right) - 2ad\,\mathrm{Si}\left(dx\right) - 2b\tan\left(\frac{1}{2}dx\right)^{2} - 8b\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right) - 2b\tan\left(\frac{1}{2}c\right)^{2} + 2b}{2\left(d\tan\left(\frac{1}{2}dx\right)^{2}\tan\left(\frac{1}{2}c\right)^{2} + d\tan\left(\frac{1}{2}dx\right)^{2} + d\tan\left(\frac{1}{2}c\right)^{2} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] -1/2*(a*d*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d*im
ag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d*sin_integra
l(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d*real_part(cos_integral(d*x))*tan
(1/2*d*x)^2*tan(1/2*c) - 2*a*d*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2
*tan(1/2*c) - a*d*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + a*d*imag_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d*sin_integral(d*x)*tan(1/2*d*x
)^2 + a*d*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a*d*imag_part(cos_int
egral(-d*x))*tan(1/2*c)^2 + 2*a*d*sin_integral(d*x)*tan(1/2*c)^2 + 2*b*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d*real_part(cos_integral(d*x))*tan(1/2*c) - 2
*a*d*real_part(cos_integral(-d*x))*tan(1/2*c) - a*d*imag_part(cos_integral(
d*x)) + a*d*imag_part(cos_integral(-d*x)) - 2*a*d*sin_integral(d*x) - 2*b*t
an(1/2*d*x)^2 - 8*b*tan(1/2*d*x)*tan(1/2*c) - 2*b*tan(1/2*c)^2 + 2*b)/(d*ta
n(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$a\,\mathrm{cosint}(d\,x)\,\sin\left(c\right)+a\,\mathrm{sinint}(d\,x)\,\cos\left(c\right)-\frac{b\,\cos\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x))/x,x)
```

```
[Out] a*cosint(d*x)*sin(c) + a*sinint(d*x)*cos(c) - (b*cos(c + d*x))/d
```

3.6 $\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx$

Optimal. Leaf size=48

$$ad\cos(c)\text{Ci}(dx)+b\text{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{x}+b\cos(c)\text{Si}(dx)-ad\sin(c)\text{Si}(dx)$$

```
[Out] a*d*Ci(d*x)*cos(c)+b*cos(c)*Si(d*x)+b*Ci(d*x)*sin(c)-a*d*Si(d*x)*sin(c)-a*s
in(d*x+c)/x
```

Rubi [A]
time = 0.16, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$ad\cos(c)\text{CosIntegral}(dx)-ad\sin(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{x}+b\sin(c)\text{CosIntegral}(dx)+b\cos(c)\text{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sin[c + d*x])/x^2,x]
```

```
[Out] a*d*Cos[c]*CosIntegral[d*x] + b*CosIntegral[d*x]*Sin[c] - (a*Sin[c + d*x])/
x + b*Cos[c]*SinIntegral[d*x] - a*d*Sin[c]*SinIntegral[d*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^2}+\frac{b\sin(c+dx)}{x}\right)dx\\ &= a\int \frac{\sin(c+dx)}{x^2}\,dx+b\int \frac{\sin(c+dx)}{x}\,dx\\ &= -\frac{a\sin(c+dx)}{x}+(ad)\int \frac{\cos(c+dx)}{x}\,dx+(b\cos(c))\int \frac{\sin(dx)}{x}\,dx+(b\sin(c)\\ &= b\text{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{x}+b\cos(c)\text{Si}(dx)+(ad\cos(c))\int \frac{\cos(dx)}{x}\,dx-(\\ &= ad\cos(c)\text{Ci}(dx)+b\text{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{x}+b\cos(c)\text{Si}(dx)-ad\sin(c)\text{S}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 60, normalized size = 1.25

$$-\frac{a\cos(dx)\sin(c)}{x}+b\text{Ci}(dx)\sin(c)-\frac{a\cos(c)\sin(dx)}{x}+b\cos(c)\text{Si}(dx)+ad(\cos(c)\text{Ci}(dx)-\sin(c)\text{Si}(dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x^2,x]
```

```
[Out] -((a*Cos[d*x]*Sin[c])/x) + b*CosIntegral[d*x]*Sin[c] - (a*Cos[c]*Sin[d*x])/
x + b*Cos[c]*SinIntegral[d*x] + a*d*(Cos[c]*CosIntegral[d*x] - Sin[c]*SinIn
tegral[d*x])
```

Maple [A]
time = 0.09, size = 56, normalized size = 1.17

method	result
derivativedivides	$d\left(a\left(-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}(dx)\sin(c)+\text{cosineIntegral}(dx)\cos(c)\right)+\frac{b(\text{sinIntegral}(dx)}{}\right.$
default	$d\left(a\left(-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}(dx)\sin(c)+\text{cosineIntegral}(dx)\cos(c)\right)+\frac{b(\text{sinIntegral}(dx)}{}\right.$
risch	$\frac{ib\,e^{ic}\,\text{expIntegral}(1,-idx)}{2}-\frac{da\,e^{ic}\,\text{expIntegral}(1,-idx)}{2}-\frac{ib\,e^{-ic}\,\text{expIntegral}(1,idx)}{2}-\frac{da\,e^{-ic}\,\text{expIntegral}(1,idx)}{2}$

meijerg	$\frac{b\sqrt{\pi}\,\sin(c)\left(\frac{2\gamma+2\ln(x)+\ln\left(d^2\right)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\,\mathrm{cosineIntegral}(dx)}{\sqrt{\pi}}\right)}{2}+b\cos(c)\,\mathrm{sinIntegral}(dx)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(a*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+1/d*b*(Si(d*x)*cos(c)+
Ci(d*x)*sin(c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.48, size = 108, normalized size = 2.25

$$\frac{\left(\left(a(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos(c)-a(i\,\Gamma(-1,i\,dx)-i\,\Gamma(-1,-i\,dx))\sin(c)\right)d^2-\left(b(-i\,\Gamma(-1,i\,dx)+i\,\Gamma(-1,-i\,dx))\cos(c)-b(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\sin(c)\right)d\right)x-2\,b\cos(dx+c)}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(((a*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) - a*(I*gamma(-1, I*d
*x) - I*gamma(-1, -I*d*x))*sin(c))*d^2 - (b*(-I*gamma(-1, I*d*x) + I*gamma(
-1, -I*d*x))*cos(c) - b*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*sin(c))*d)*x
 - 2*b*cos(d*x + c))/(d*x)
```

Fricas [A]
time = 0.35, size = 75, normalized size = 1.56

$$\frac{\left(adx\,\mathrm{Ci}\,(dx)+adx\,\mathrm{Ci}\,(-dx)+2\,bx\,\mathrm{Si}\,(dx)\right)\cos(c)-2\,a\sin(dx+c)-\left(2\,adx\,\mathrm{Si}\,(dx)-bx\,\mathrm{Ci}\,(dx)-bx\,\mathrm{Ci}\,(-dx)\right)\sin(c)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a*d*x*cos_integral(d*x) + a*d*x*cos_integral(-d*x) + 2*b*x*sin_integr
al(d*x))*cos(c) - 2*a*sin(d*x + c) - (2*a*d*x*sin_integral(d*x) - b*x*cos_i
ntegral(d*x) - b*x*cos_integral(-d*x))*sin(c))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+bx)\sin(c+dx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x)*sin(c + d*x)/x**2, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.79, size = 569, normalized size = 11.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*d*
x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d*x*imag_
part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d*x*imag_part(cos_i
ntegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d*x*sin_integral(d*x)*tan(1/
2*d*x)^2*tan(1/2*c) + b*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1
/2*c)^2 - b*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*b*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d*x*real_part(cos_in
tegral(d*x))*tan(1/2*d*x)^2 - a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d
*x)^2 - 2*b*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b*
x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a*d*x*real_part
(cos_integral(d*x))*tan(1/2*c)^2 + a*d*x*real_part(cos_integral(-d*x))*tan(
1/2*c)^2 - b*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + b*x*imag_part(
cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*b*x*sin_integral(d*x)*tan(1/2*d*x)^2
 + 2*a*d*x*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d*x*imag_part(cos_
integral(-d*x))*tan(1/2*c) + 4*a*d*x*sin_integral(d*x)*tan(1/2*c) + b*x*ima
g_part(cos_integral(d*x))*tan(1/2*c)^2 - b*x*imag_part(cos_integral(-d*x))*
tan(1/2*c)^2 + 2*b*x*sin_integral(d*x)*tan(1/2*c)^2 - a*d*x*real_part(cos_i
ntegral(d*x)) - a*d*x*real_part(cos_integral(-d*x)) - 2*b*x*real_part(cos_i
ntegral(d*x))*tan(1/2*c) - 2*b*x*real_part(cos_integral(-d*x))*tan(1/2*c) -
 4*a*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*tan(1/2*d*x)*tan(1/2*c)^2 - b*x*imag_p
art(cos_integral(d*x)) + b*x*imag_part(cos_integral(-d*x)) - 2*b*x*sin_inte
gral(d*x) + 4*a*tan(1/2*d*x) + 4*a*tan(1/2*c))/(x*tan(1/2*d*x)^2*tan(1/2*c)
^2 + x*tan(1/2*d*x)^2 + x*tan(1/2*c)^2 + x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(c+d\,x)\,\left(a+b\,x\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x))/x^2,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x))/x^2, x)
```

3.7 $\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx$

Optimal. Leaf size=89

$$-\frac{ad\cos(c+dx)}{2x}+bd\cos(c)\mathrm{Ci}(dx)-\frac{1}{2}ad^2\mathrm{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{2x^2}-\frac{b\sin(c+dx)}{x}-\frac{1}{2}ad^2\cos(c)\mathrm{Si}(dx)-bd\,\mathrm{s}$$

```
[Out] b*d*Ci(d*x)*cos(c)-1/2*a*d*cos(d*x+c)/x-1/2*a*d^2*cos(c)*Si(d*x)-1/2*a*d^2*
Ci(d*x)*sin(c)-b*d*Si(d*x)*sin(c)-1/2*a*sin(d*x+c)/x^2-b*sin(d*x+c)/x
```

Rubi [A]
time = 0.19, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$-\frac{1}{2}ad^2\sin(c)\mathrm{CosIntegral}(dx)-\frac{1}{2}ad^2\cos(c)\mathrm{Si}(dx)-\frac{a\sin(c+dx)}{2x^2}-\frac{ad\cos(c+dx)}{2x}+bd\cos(c)\mathrm{CosIntegral}(dx)-bd\sin(c)\mathrm{Si}(dx)-\frac{b\sin(c+dx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sin[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a*d*Cos[c + d*x])/x + b*d*Cos[c]*CosIntegral[d*x] - (a*d^2*CosIntegra
l[d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) - (b*Sin[c + d*x])/x - (a*d^2*C
os[c]*SinIntegral[d*x])/2 - b*d*Sin[c]*SinIntegral[d*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx = \int \left(\frac{a\sin(c+dx)}{x^3}+\frac{b\sin(c+dx)}{x^2}\right)dx$$
$$= a\int \frac{\sin(c+dx)}{x^3}\,dx + b\int \frac{\sin(c+dx)}{x^2}\,dx$$
$$= -\frac{a\sin(c+dx)}{2x^2} - \frac{b\sin(c+dx)}{x} + \frac{1}{2}(ad)\int \frac{\cos(c+dx)}{x^2}\,dx + (bd)\int \frac{\cos(c+d$$
$$= -\frac{ad\cos(c+dx)}{2x} - \frac{a\sin(c+dx)}{2x^2} - \frac{b\sin(c+dx)}{x} - \frac{1}{2}(ad^2)\int \frac{\sin(c+dx)}{x}\,dx$$
$$= -\frac{ad\cos(c+dx)}{2x} + bd\cos(c)\mathrm{Ci}(dx) - \frac{a\sin(c+dx)}{2x^2} - \frac{b\sin(c+dx)}{x} - bd\sin(c$$
$$= -\frac{ad\cos(c+dx)}{2x} + bd\cos(c)\mathrm{Ci}(dx) - \frac{1}{2}ad^2\mathrm{Ci}(dx)\sin(c) - \frac{a\sin(c+dx)}{2x^2} - \frac{b\,\mathrm{si}}{}$$

Mathematica [A]
time = 0.18, size = 76, normalized size = 0.85

$$-\frac{adx\cos(c+dx)+dx^2\mathrm{Ci}(dx)(-2b\cos(c)+ad\sin(c))+a\sin(c+dx)+2bx\sin(c+dx)+dx^2(ad\cos(c)+2b\sin(c))\mathrm{Si}(dx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a*d*x*Cos[c + d*x] + d*x^2*CosIntegral[d*x]*(-2*b*Cos[c] + a*d*Sin[c]
) + a*Sin[c + d*x] + 2*b*x*Sin[c + d*x] + d*x^2*(a*d*Cos[c] + 2*b*Sin[c])*S
inIntegral[d*x])/x^2
```

Maple [A]
time = 0.09, size = 88, normalized size = 0.99

method	result
derivativedivides	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{sinIntegral}(dx)\cos(c)}{2}-\frac{\mathrm{cosineIntegral}(dx)\sin(c)}{2}\right)+\frac{b\left(-\frac{\sin(dx+c)}{dx}-\mathrm{sinI}\right.}{}\right.$

default	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\text{sinIntegral}(dx)\cos(c)}{2}-\frac{\text{cosineIntegral}(dx)\sin(c)}{2}\right)+\frac{b\left(-\frac{\sin(dx+c)}{dx}-\text{sinInte}\right.}{}\right.$
risch	$-\frac{\cos(c)\,\text{expIntegral}(1,idx)bd}{2}-\frac{\cos(c)\,\text{expIntegral}(1,-idx)bd}{2}+\frac{i\cos(c)\,\text{expIntegral}(1,idx)a\,d^2}{4}-\frac{i\cos(c)\,\text{expIntegra}}{4}$
meijerg	$\frac{d^2b\sqrt{\pi}\,\sin(c)\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right)}{x(d^2)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\,\text{sinIntegral}\left(x\sqrt{d^2}\right)}{\sqrt{\pi}}\right)}{4\sqrt{d^2}}+\frac{db\sqrt{\pi}\,\cos(c)\left(\frac{4\gamma-4+4\ln(x)+4\ln(d)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(a*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*C
i(d*x)*sin(c))+1/d*b*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.56, size = 112, normalized size = 1.26

$$-\frac{((a(-i\,\Gamma(-2,i\,dx)+i\,\Gamma(-2,-i\,dx))\cos(c)-a(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin(c))d^3+2\,(b(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\cos(c)-b(i\,\Gamma(-2,i\,dx)-i\,\Gamma(-2,-i\,dx))\sin(c))d^2)x^2+2\,b\cos(dx+c)}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x))*cos(c) - a*(gamma(-2,
 I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^3 + 2*(b*(gamma(-2, I*d*x) + gamma(-
2, -I*d*x))*cos(c) - b*(I*gamma(-2, I*d*x) - I*gamma(-2, -I*d*x))*sin(c))*d
^2)*x^2 + 2*b*cos(d*x + c))/(d*x^2)
```

Fricas [A]
time = 0.34, size = 111, normalized size = 1.25

$$-\frac{2\,adx\cos(dx+c)+2\,(ad^2x^2\,\mathrm{Si}(dx)-bdx^2\,\mathrm{Ci}(dx)-bdx^2\,\mathrm{Ci}(-dx))\cos(c)+2\,(2\,bx+a)\sin(dx+c)+(ad^2x^2\,\mathrm{Ci}(dx)+ad^2x^2\,\mathrm{Ci}(-dx)+4\,bdx^2\,\mathrm{Si}(dx))\sin(c)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*x*cos(d*x + c) + 2*(a*d^2*x^2*sin_integral(d*x) - b*d*x^2*cos_i
ntegral(d*x) - b*d*x^2*cos_integral(-d*x))*cos(c) + 2*(2*b*x + a)*sin(d*x +
 c) + (a*d^2*x^2*cos_integral(d*x) + a*d^2*x^2*cos_integral(-d*x) + 4*b*d*x
^2*sin_integral(d*x))*sin(c))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+bx)\sin(c+dx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x)*sin(c + d*x)/x**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.47, size = 796, normalized size = 8.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a
*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^
2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*x^2*real_part
(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^2*x^2*real_part(cos_i
ntegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b*d*x^2*real_part(cos_integral
(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b*d*x^2*real_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^2*x^2*imag_part(cos_integral(d*x))*tan(
1/2*d*x)^2 + a*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d
^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 4*b*d*x^2*imag_part(cos_integral(
d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*d*x^2*imag_part(cos_integral(-d*x))*t
an(1/2*d*x)^2*tan(1/2*c) - 8*b*d*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1
/2*c) + a*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a*d^2*x^2*ima
g_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^2*x^2*sin_integral(d*x)*tan
(1/2*c)^2 + 2*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 2*b*d*x
^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d^2*x^2*real_part(cos
_integral(d*x))*tan(1/2*c) - 2*a*d^2*x^2*real_part(cos_integral(-d*x))*tan(
1/2*c) - 2*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*c)^2 - 2*b*d*x^2*re
al_part(cos_integral(-d*x))*tan(1/2*c)^2 - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c
)^2 - a*d^2*x^2*imag_part(cos_integral(d*x)) + a*d^2*x^2*imag_part(cos_inte
gral(-d*x)) - 2*a*d^2*x^2*sin_integral(d*x) - 4*b*d*x^2*imag_part(cos_integ
ral(d*x))*tan(1/2*c) + 4*b*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c) -
 8*b*d*x^2*sin_integral(d*x)*tan(1/2*c) + 2*b*d*x^2*real_part(cos_integral(
d*x)) + 2*b*d*x^2*real_part(cos_integral(-d*x)) + 2*a*d*x*tan(1/2*d*x)^2 +
8*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 8*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*d*x
*tan(1/2*c)^2 + 8*b*x*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a*tan(1/2*d*x)^2*tan(1/
2*c) + 4*a*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a*d*x - 8*b*x*tan(1/2*d*x) - 8*b*x
*tan(1/2*c) - 4*a*tan(1/2*d*x) - 4*a*tan(1/2*c))/(x^2*tan(1/2*d*x)^2*tan(1/
2*c)^2 + x^2*tan(1/2*d*x)^2 + x^2*tan(1/2*c)^2 + x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)\,\left(a+b\,x\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x))/x^3,x)
[Out] int((sin(c + d*x)*(a + b*x))/x^3, x)
```

3.8 $\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx$

Optimal. Leaf size=132

$$-\frac{ad\cos(c+dx)}{6x^2}-\frac{bd\cos(c+dx)}{2x}-\frac{1}{6}ad^3\cos(c)\text{Ci}(dx)-\frac{1}{2}bd^2\text{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{2x^2}+\frac{ad}{}$$

```
[Out] -1/6*a*d^3*Ci(d*x)*cos(c)-1/6*a*d*cos(d*x+c)/x^2-1/2*b*d*cos(d*x+c)/x-1/2*b
*d^2*cos(c)*Si(d*x)-1/2*b*d^2*Ci(d*x)*sin(c)+1/6*a*d^3*Si(d*x)*sin(c)-1/3*a
*sin(d*x+c)/x^3-1/2*b*sin(d*x+c)/x^2+1/6*a*d^2*sin(d*x+c)/x
```

Rubi [A]
time = 0.22, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 3378, 3384, 3380, 3383}

$$-\frac{1}{6}ad^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}ad^3\sin(c)\text{Si}(dx)+\frac{ad^2\sin(c+dx)}{6x}-\frac{a\sin(c+dx)}{3x^3}-\frac{ad\cos(c+dx)}{6x^2}-\frac{1}{2}bd^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}bd^2\cos(c)\text{Si}(dx)-\frac{b\sin(c+dx)}{2x^2}-\frac{bd\cos(c+dx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sin[c + d*x])/x^4,x]
```

```
[Out] -1/6*(a*d*Cos[c + d*x])/x^2 - (b*d*Cos[c + d*x])/(2*x) - (a*d^3*Cos[c]*CosI
ntegral[d*x])/6 - (b*d^2*CosIntegral[d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(3*x
^3) - (b*Sin[c + d*x])/(2*x^2) + (a*d^2*Sin[c + d*x])/(6*x) - (b*d^2*Cos[c]
*SinIntegral[d*x])/2 + (a*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^4}+\frac{b\sin(c+dx)}{x^3}\right) dx \\
&= a\int \frac{\sin(c+dx)}{x^4}\,dx + b\int \frac{\sin(c+dx)}{x^3}\,dx \\
&= -\frac{a\sin(c+dx)}{3x^3} - \frac{b\sin(c+dx)}{2x^2} + \frac{1}{3}(ad)\int \frac{\cos(c+dx)}{x^3}\,dx + \frac{1}{2}(bd)\int \frac{\cos(c+d}{x^2} \\
&= -\frac{ad\cos(c+dx)}{6x^2} - \frac{bd\cos(c+dx)}{2x} - \frac{a\sin(c+dx)}{3x^3} - \frac{b\sin(c+dx)}{2x^2} - \frac{1}{6}\left(ad^2\right)\int \\
&= -\frac{ad\cos(c+dx)}{6x^2} - \frac{bd\cos(c+dx)}{2x} - \frac{a\sin(c+dx)}{3x^3} - \frac{b\sin(c+dx)}{2x^2} + \frac{ad^2\sin(c}{6x} \\
&= -\frac{ad\cos(c+dx)}{6x^2} - \frac{bd\cos(c+dx)}{2x} - \frac{1}{2}bd^2\operatorname{Ci}(dx)\sin(c) - \frac{a\sin(c+dx)}{3x^3} - \frac{b\sin(}{2} \\
&= -\frac{ad\cos(c+dx)}{6x^2} - \frac{bd\cos(c+dx)}{2x} - \frac{1}{6}ad^3\cos(c)\operatorname{Ci}(dx) - \frac{1}{2}bd^2\operatorname{Ci}(dx)\sin(c) -
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 110, normalized size = 0.83

$$
-\frac{adx\cos(c+dx)+3bdx^2\cos(c+dx)+d^2x^3\operatorname{Ci}(dx)(ad\cos(c)+3b\sin(c))+2a\sin(c+dx)+3bx\sin(c+dx)-ad^2x^2\sin(c+dx)+d^2x^3(3b\cos(c)-ad\sin(c))\operatorname{Si}(dx)}{6x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x^4,x]
```

```
[Out] -1/6*(a*d*x*Cos[c + d*x] + 3*b*d*x^2*Cos[c + d*x] + d^2*x^3*CosIntegral[d*x
]*(a*d*Cos[c] + 3*b*Sin[c]) + 2*a*Sin[c + d*x] + 3*b*x*Sin[c + d*x] - a*d^2
*x^2*Sin[c + d*x] + d^2*x^3*(3*b*Cos[c] - a*d*Sin[c])*SinIntegral[d*x])/x^3
```

Maple [A]
time = 0.12, size = 117, normalized size = 0.89

method	result
derivativedivides	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{sinIntegral}(dx)\sin(c)}{6}-\frac{\mathrm{cosineIntegral}(dx)\cos(c)}{6}\right)+\frac{b\left(-\right.}{}\right.$
default	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{sinIntegral}(dx)\sin(c)}{6}-\frac{\mathrm{cosineIntegral}(dx)\cos(c)}{6}\right)+\frac{b\left(-\right.}{}\right.$
risch	$\frac{i\,\mathrm{expIntegral}(1,idx)\cos(c)b\,d^2}{4}+\frac{\mathrm{expIntegral}(1,idx)\cos(c)a\,d^3}{12}-\frac{i\,\mathrm{expIntegral}(1,-idx)\cos(c)b\,d^2}{4}+\mathrm{expIntegral}(1,$
meijerg	$\frac{d^2b\sqrt{\pi}\,\sin(c)\left(-\frac{4}{\sqrt{\pi}\,x^2d^2}-\frac{2\left(2\gamma-3+2\ln(x)+\ln\left(d^2\right)\right)}{\sqrt{\pi}}+\frac{-6d^2x^2+4}{\sqrt{\pi}\,x^2d^2}+\frac{4\gamma}{\sqrt{\pi}}+\frac{4\ln(2)}{\sqrt{\pi}}+\frac{4\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}-\frac{4\cos(dx)}{\sqrt{\pi}\,d^2x^2}+\frac{4\sin(c}{\sqrt{\pi}}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(a*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+1
/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+1/d*b*(-1/2*sin(d*x+c)/d^2/x^2-1/2*co
s(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.65, size = 111, normalized size = 0.84

$$-\frac{\left(\left(a(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos(c)+a(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin(c)\right)d^4-3\left(b(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\cos(c)-b(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\sin(c)\right)d^3\right)x^3+2\,b\cos(dx+c)}{2\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a*(-I*gamma(-3, I
*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^4 - 3*(b*(-I*gamma(-3, I*d*x) + I*ga
mma(-3, -I*d*x))*cos(c) - b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*sin(c))*
d^3)*x^3 + 2*b*cos(d*x + c))/(d*x^3)
```

Fricas [A]
time = 0.33, size = 137, normalized size = 1.04

$$-\frac{2\,(3\,bdx^2+adx)\cos(dx+c)+(ad^3x^3\,\mathrm{Ci}(dx)+ad^3x^3\,\mathrm{Ci}(-dx)+6\,bd^2x^3\,\mathrm{Si}(dx))\cos(c)-2\,(ad^2x^2-3\,bx-2\,a)\sin(dx+c)-(2\,ad^3x^3\,\mathrm{Si}(dx)-3\,bd^2x^3\,\mathrm{Ci}(dx)-3\,bd^2x^3\,\mathrm{Ci}(-dx))\sin(c)}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(3*b*d*x^2 + a*d*x)*cos(d*x + c) + (a*d^3*x^3*cos_integral(d*x) +
a*d^3*x^3*cos_integral(-d*x) + 6*b*d^2*x^3*sin_integral(d*x))*cos(c) - 2*(a
*d^2*x^2 - 3*b*x - 2*a)*sin(d*x + c) - (2*a*d^3*x^3*sin_integral(d*x) - 3*b
*d^2*x^3*cos_integral(d*x) - 3*b*d^2*x^3*cos_integral(-d*x))*sin(c))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx)\sin(c + dx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x)*sin(c + d*x)/x**4, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.25, size = 961, normalized size = 7.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d
^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^3
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x^3*sin_
integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 3*b*d^2*x^3*imag_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 3*b*d^2*x^3*imag_part(cos_integral(-d
*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*b*d^2*x^3*sin_integral(d*x)*tan(1/2*d*
x)^2*tan(1/2*c)^2 - a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 -
 a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 6*b*d^2*x^3*real_
part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 6*b*d^2*x^3*real_part(c
os_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a*d^3*x^3*real_part(cos_inte
gral(d*x))*tan(1/2*c)^2 + a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*c
)^2 - 3*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 3*b*d^2*x^3
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 6*b*d^2*x^3*sin_integral(d*
x)*tan(1/2*d*x)^2 + 2*a*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*c) - 2
*a*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a*d^3*x^3*sin_integ
ral(d*x)*tan(1/2*c) + 3*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2
 - 3*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 6*b*d^2*x^3*sin
_integral(d*x)*tan(1/2*c)^2 - a*d^3*x^3*real_part(cos_integral(d*x)) - a*d^
3*x^3*real_part(cos_integral(-d*x)) - 6*b*d^2*x^3*real_part(cos_integral(d*
x))*tan(1/2*c) - 6*b*d^2*x^3*real_part(cos_integral(-d*x))*tan(1/2*c) - 4*a
*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2
- 6*b*d*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 3*b*d^2*x^3*imag_part(cos_integra
l(d*x)) + 3*b*d^2*x^3*imag_part(cos_integral(-d*x)) - 6*b*d^2*x^3*sin_integ
ral(d*x) - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*d^2*x^2*tan(1/2*d*x) +
 6*b*d*x^2*tan(1/2*d*x)^2 + 4*a*d^2*x^2*tan(1/2*c) + 24*b*d*x^2*tan(1/2*d*x
```

```
)*tan(1/2*c) + 6*b*d*x^2*tan(1/2*c)^2 + 2*a*d*x*tan(1/2*d*x)^2 + 8*a*d*x*ta
n(1/2*d*x)*tan(1/2*c) + 12*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*d*x*tan(1/2*
c)^2 + 12*b*x*tan(1/2*d*x)*tan(1/2*c)^2 - 6*b*d*x^2 + 8*a*tan(1/2*d*x)^2*ta
n(1/2*c) + 8*a*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a*d*x - 12*b*x*tan(1/2*d*x) -
12*b*x*tan(1/2*c) - 8*a*tan(1/2*d*x) - 8*a*tan(1/2*c))/(x^3*tan(1/2*d*x)^2*
tan(1/2*c)^2 + x^3*tan(1/2*d*x)^2 + x^3*tan(1/2*c)^2 + x^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)\,\left(a+b\,x\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x))/x^4,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x))/x^4, x)
```

3.9 $\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx$

Optimal. Leaf size=166

$$-\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{6}bd^3\cos(c)\text{Ci}(dx)+\frac{1}{24}ad^4\text{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{4x^4}-$$

```
[Out] -1/6*b*d^3*Ci(d*x)*cos(c)-1/12*a*d*cos(d*x+c)/x^3-1/6*b*d*cos(d*x+c)/x^2+1/
24*a*d^3*cos(d*x+c)/x+1/24*a*d^4*cos(c)*Si(d*x)+1/24*a*d^4*Ci(d*x)*sin(c)+1
/6*b*d^3*Si(d*x)*sin(c)-1/4*a*sin(d*x+c)/x^4-1/3*b*sin(d*x+c)/x^3+1/24*a*d^
2*sin(d*x+c)/x^2+1/6*b*d^2*sin(d*x+c)/x
```

Rubi [A]
time = 0.25, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\frac{1}{24}ad^4\sin(c)\text{CosIntegral}(dx)+\frac{1}{24}ad^4\cos(c)\text{Si}(dx)+\frac{ad^3\cos(c+dx)}{24x}+\frac{ad^2\sin(c+dx)}{24x^2}-\frac{a\sin(c+dx)}{4x^4}-\frac{ad\cos(c+dx)}{12x^3}-\frac{1}{6}bd^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}bd^3\sin(c)\text{Si}(dx)+\frac{bd^2\sin(c+dx)}{6x}-\frac{b\sin(c+dx)}{3x^3}-\frac{bd\cos(c+dx)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sin[c + d*x])/x^5,x]
```

```
[Out] -1/12*(a*d*Cos[c + d*x])/x^3 - (b*d*Cos[c + d*x])/(6*x^2) + (a*d^3*Cos[c +
d*x])/(24*x) - (b*d^3*Cos[c]*CosIntegral[d*x])/6 + (a*d^4*CosIntegral[d*x]*
Sin[c])/24 - (a*Sin[c + d*x])/(4*x^4) - (b*Sin[c + d*x])/(3*x^3) + (a*d^2*S
in[c + d*x])/(24*x^2) + (b*d^2*Sin[c + d*x])/(6*x) + (a*d^4*Cos[c]*SinInteg
ral[d*x])/24 + (b*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^5}+\frac{b\sin(c+dx)}{x^4}\right)dx\\
&= a\int \frac{\sin(c+dx)}{x^5}\,dx+b\int \frac{\sin(c+dx)}{x^4}\,dx\\
&= -\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{3x^3}+\frac{1}{4}(ad)\int \frac{\cos(c+dx)}{x^4}\,dx+\frac{1}{3}(bd)\int \frac{\cos(c+ \cdots}{x^3}\\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}-\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{3x^3}-\frac{1}{12}\left(ad^2\right) \cdots\\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}-\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{3x^3}+\frac{ad^2\sin(\cdots}{24\cdots}\\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}+\frac{ad^3\cos(c+dx)}{24x}-\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(\cdots}{3\cdots}\\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{6}bd^3\cos(c)\text{Ci}(dx)-\cdots\\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{6}bd^3\cos(c)\text{Ci}(dx)+\cdots
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 138, normalized size = 0.83

$$
\frac{-2adx\cos(c+dx)-4bdx^2\cos(c+dx)+ad^3x^3\cos(c+dx)+d^3x^4\text{Ci}(dx)(-4b\cos(c)+ad\sin(c))-6a\sin(c+dx)-8bx\sin(c+dx)+ad^2x^2\sin(c+dx)+4bd^2x^3\sin(c+dx)+d^3x^4(ad\cos(c)+4b\sin(c))\text{Si}(dx)}{24x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x^5,x]
```

```
[Out] (-2*a*d*x*Cos[c + d*x] - 4*b*d*x^2*Cos[c + d*x] + a*d^3*x^3*Cos[c + d*x] +
d^3*x^4*CosIntegral[d*x]*(-4*b*Cos[c] + a*d*Sin[c]) - 6*a*Sin[c + d*x] - 8*
```

```
b*x*Sin[c + d*x] + a*d^2*x^2*Sin[c + d*x] + 4*b*d^2*x^3*Sin[c + d*x] + d^3*
x^4*(a*d*Cos[c] + 4*b*Sin[c])*SinIntegral[d*x])/(24*x^4)
```

Maple [A]
time = 0.12, size = 145, normalized size = 0.87

method	result
derivativedivides	$d^4\left(a\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\text{sinIntegral}(dx)\cos(c)}{24}+\frac{\text{cosineIntegral}(dx)\sin(c}{24}\right.\right.$
default	$d^4\left(a\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\text{sinIntegral}(dx)\cos(c)}{24}+\frac{\text{cosineIntegral}(dx)\sin(c}{24}\right.\right.$
risch	$\frac{\cos(c)\,\text{expIntegral}(1,idx)b\,d^3}{12}+\frac{\cos(c)\,\text{expIntegral}(1,-idx)b\,d^3}{12}-\frac{i\cos(c)\,\text{expIntegral}(1,idx)a\,d^4}{48}+\frac{i\cos(c)\,\text{expIntegr}}{48}$
meijerg	$\frac{d^4b\sqrt{\pi}\,\sin(c)\left(-\frac{8\left(-d^2x^2+2\right)d^2\cos\left(x\sqrt{d^2}\right)}{3x^3\left(d^2\right)^{\frac{5}{2}}\sqrt{\pi}}+\frac{8\sin\left(x\sqrt{d^2}\right)}{3d^2x^2\sqrt{\pi}}+\frac{8\,\text{sinIntegral}\left(x\sqrt{d^2}\right)}{3\sqrt{\pi}}\right)}{16\sqrt{d^2}}+\frac{d^3b\sqrt{\pi}\,\cos(c)\left(-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] d^4*(a*(-1/4*sin(d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/d^2
/x^2+1/24*cos(d*x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c))+1/d*b*(-1
/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+1/6*Si(d*x)
*sin(c)-1/6*Ci(d*x)*cos(c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.57, size = 110, normalized size = 0.66

$$-\frac{\left(\left(a\left(i\,\Gamma\left(-4,i\,dx\right)-i\,\Gamma\left(-4,-i\,dx\right)\right)\cos\left(c\right)+a\left(\Gamma\left(-4,i\,dx\right)+\Gamma\left(-4,-i\,dx\right)\right)\sin\left(c\right)\right)d^5-4\left(b\left(\Gamma\left(-4,i\,dx\right)+\Gamma\left(-4,-i\,dx\right)\right)\cos\left(c\right)+b\left(-i\,\Gamma\left(-4,i\,dx\right)+i\,\Gamma\left(-4,-i\,dx\right)\right)\sin\left(c\right)\right)d^4\right)x^4+2\,b\cos\left(dx+c\right)}{2\,dx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a*(gamma(-4,
I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^5 - 4*(b*(gamma(-4, I*d*x) + gamma(-4
, -I*d*x))*cos(c) + b*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x))*sin(c))*d
^4)*x^4 + 2*b*cos(d*x + c))/(d*x^4)
```

Fricas [A]
time = 0.35, size = 154, normalized size = 0.93

$$\frac{2\left(ad^3x^3-4\,bdx^2-2\,adx\right)\cos\left(dx+c\right)+2\left(ad^4x^4\,\mathrm{Si}\left(dx\right)-2\,bd^3x^4\,\mathrm{Ci}\left(dx\right)-2\,bd^3x^4\,\mathrm{Ci}\left(-dx\right)\right)\cos\left(c\right)+2\left(4\,bd^2x^3+ad^2x^2-8\,bx-6\,a\right)\sin\left(dx+c\right)+\left(ad^4x^4\,\mathrm{Ci}\left(dx\right)+ad^4x^4\,\mathrm{Ci}\left(-dx\right)+8\,bd^3x^4\,\mathrm{Si}\left(dx\right)\right)\sin\left(c\right)}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(a*d^3*x^3 - 4*b*d*x^2 - 2*a*d*x)*cos(d*x + c) + 2*(a*d^4*x^4*sin_i
ntegral(d*x) - 2*b*d^3*x^4*cos_integral(d*x) - 2*b*d^3*x^4*cos_integral(-d*
x))*cos(c) + 2*(4*b*d^2*x^3 + a*d^2*x^2 - 8*b*x - 6*a)*sin(d*x + c) + (a*d^
4*x^4*cos_integral(d*x) + a*d^4*x^4*cos_integral(-d*x) + 8*b*d^3*x^4*sin_in
tegral(d*x))*sin(c))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x)*sin(c + d*x)/x**5, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.89, size = 1108, normalized size = 6.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(a*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*
d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^4*x^4*real_pa
rt(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^4*x^4*real_part(cos
_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*b*d^3*x^4*real_part(cos_inte
gral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b*d^3*x^4*real_part(cos_integral
(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integral(d*x)
)*tan(1/2*d*x)^2 + a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 -
 2*a*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 - 8*b*d^3*x^4*imag_part(cos_i
ntegral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 8*b*d^3*x^4*imag_part(cos_integra
l(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 16*b*d^3*x^4*sin_integral(d*x)*tan(1/2
*d*x)^2*tan(1/2*c) + a*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 -
a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^4*x^4*sin_inte
gral(d*x)*tan(1/2*c)^2 + 4*b*d^3*x^4*real_part(cos_integral(d*x))*tan(1/2*d
*x)^2 + 4*b*d^3*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d^4*
x^4*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^4*x^4*real_part(cos_int
egral(-d*x))*tan(1/2*c) - 4*b*d^3*x^4*real_part(cos_integral(d*x))*tan(1/2*
c)^2 - 4*b*d^3*x^4*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 2*a*d^3*x^3
*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integral(d*x)) + a*d
```

```
^4*x^4*imag_part(cos_integral(-d*x)) - 2*a*d^4*x^4*sin_integral(d*x) - 8*b*
d^3*x^4*imag_part(cos_integral(d*x))*tan(1/2*c) + 8*b*d^3*x^4*imag_part(cos
_integral(-d*x))*tan(1/2*c) - 16*b*d^3*x^4*sin_integral(d*x)*tan(1/2*c) + 4
*b*d^3*x^4*real_part(cos_integral(d*x)) + 4*b*d^3*x^4*real_part(cos_integra
l(-d*x)) + 2*a*d^3*x^3*tan(1/2*d*x)^2 + 8*a*d^3*x^3*tan(1/2*d*x)*tan(1/2*c)
 + 16*b*d^2*x^3*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*d^3*x^3*tan(1/2*c)^2 + 16*b
*d^2*x^3*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c)
+ 4*a*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 8*b*d*x^2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*a*d^3*x^3 - 16*b*d^2*x^3*tan(1/2*d*x) - 16*b*d^2*x^3*tan(1/2*c) +
4*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*d^2*x^2*tan(1/2*d*x) - 8*b*d*x^2*
tan(1/2*d*x)^2 - 4*a*d^2*x^2*tan(1/2*c) - 32*b*d*x^2*tan(1/2*d*x)*tan(1/2*c
) - 8*b*d*x^2*tan(1/2*c)^2 - 4*a*d*x*tan(1/2*d*x)^2 - 16*a*d*x*tan(1/2*d*x)
*tan(1/2*c) - 32*b*x*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*d*x*tan(1/2*c)^2 - 32*
b*x*tan(1/2*d*x)*tan(1/2*c)^2 + 8*b*d*x^2 - 24*a*tan(1/2*d*x)^2*tan(1/2*c)
- 24*a*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a*d*x + 32*b*x*tan(1/2*d*x) + 32*b*x*t
an(1/2*c) + 24*a*tan(1/2*d*x) + 24*a*tan(1/2*c))/(x^4*tan(1/2*d*x)^2*tan(1/
2*c)^2 + x^4*tan(1/2*d*x)^2 + x^4*tan(1/2*c)^2 + x^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)\,\left(a+b\,x\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x))/x^5,x)
[Out] int((sin(c + d*x)*(a + b*x))/x^5, x)
```

3.10 $\int x^2(a+bx)^2\sin(c+dx)\,dx$

Optimal. Leaf size=186

$$-\frac{24b^2\cos(c+dx)}{d^5}+\frac{2a^2\cos(c+dx)}{d^3}+\frac{12abx\cos(c+dx)}{d^3}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^3\cos}{d}$$

```
[Out] -24*b^2*cos(d*x+c)/d^5+2*a^2*cos(d*x+c)/d^3+12*a*b*x*cos(d*x+c)/d^3+12*b^2*
x^2*cos(d*x+c)/d^3-a^2*x^2*cos(d*x+c)/d-2*a*b*x^3*cos(d*x+c)/d-b^2*x^4*cos(
d*x+c)/d-12*a*b*sin(d*x+c)/d^4-24*b^2*x*sin(d*x+c)/d^4+2*a^2*x*sin(d*x+c)/d
^2+6*a*b*x^2*sin(d*x+c)/d^2+4*b^2*x^3*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.21, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.235$, Rules used = {6874, 3377, 2718, 2717}

$$\frac{2a^2\cos(c+dx)}{d^3}+\frac{2a^2x\sin(c+dx)}{d^2}-\frac{a^2x^2\cos(c+dx)}{d}-\frac{12ab\sin(c+dx)}{d^4}+\frac{12abx\cos(c+dx)}{d^3}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{2abx^3\cos(c+dx)}{d}-\frac{24b^2\cos(c+dx)}{d^5}-\frac{24b^2x\sin(c+dx)}{d^4}+\frac{12b^2x^2\cos(c+dx)}{d^3}+\frac{4b^2x^3\sin(c+dx)}{d^2}-\frac{b^2x^4\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (-24*b^2*Cos[c + d*x])/d^5 + (2*a^2*Cos[c + d*x])/d^3 + (12*a*b*x*Cos[c + d
*x])/d^3 + (12*b^2*x^2*Cos[c + d*x])/d^3 - (a^2*x^2*Cos[c + d*x])/d - (2*a*
b*x^3*Cos[c + d*x])/d - (b^2*x^4*Cos[c + d*x])/d - (12*a*b*Sin[c + d*x])/d^
4 - (24*b^2*x*Sin[c + d*x])/d^4 + (2*a^2*x*Sin[c + d*x])/d^2 + (6*a*b*x^2*S
in[c + d*x])/d^2 + (4*b^2*x^3*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2(a+bx)^2\sin(c+dx)\,dx = \int \left(a^2x^2\sin(c+dx)+2abx^3\sin(c+dx)+b^2x^4\sin(c+dx)\right)\,dx$$
$$= a^2\int x^2\sin(c+dx)\,dx+(2ab)\int x^3\sin(c+dx)\,dx+b^2\int x^4\sin(c+dx)\,dx$$
$$= -\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}+\frac{(2a^2)\int x\cos(c}{d}$$
$$= -\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}+\frac{2a^2x\sin(c+d}{d^2}$$
$$= \frac{2a^2\cos(c+dx)}{d^3}+\frac{12abx\cos(c+dx)}{d^3}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx}{d}$$
$$= \frac{2a^2\cos(c+dx)}{d^3}+\frac{12abx\cos(c+dx)}{d^3}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx}{d}$$
$$= -\frac{24b^2\cos(c+dx)}{d^5}+\frac{2a^2\cos(c+dx)}{d^3}+\frac{12abx\cos(c+dx)}{d^3}+\frac{12b^2x^2\cos(c+}{d^3}$$

Mathematica [A]
time = 0.17, size = 101, normalized size = 0.54

$$\frac{-((2abd^2x(-6+d^2x^2)+a^2d^2(-2+d^2x^2)+b^2(24-12d^2x^2+d^4x^4))\cos(c+dx))+2d(a+2bx)\left(ad^2x+b(-6+d^2x^2)\right)\sin(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (-((2*a*b*d^2*x*(-6 + d^2*x^2) + a^2*d^2*(-2 + d^2*x^2) + b^2*(24 - 12*d^2*
x^2 + d^4*x^4))*Cos[c + d*x]) + 2*d*(a + 2*b*x)*(a*d^2*x + b*(-6 + d^2*x^2)
)*Sin[c + d*x])/d^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(186) = 372$.
time = 0.07, size = 468, normalized size = 2.52 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(-a^2*c^2*cos(d*x+c)-2*a^2*c*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+a^2*(-(d
*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+2/d*a*b*c^3*cos(d*x+c
)+6/d*a*b*c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-6/d*a*b*c*(-(d*x+c)^2*cos(d*x
```

```
+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+2/d*a*b*(-(d*x+c)^3*cos(d*x+c)+3*(d*
x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))-1/d^2*b^2*c^4*cos(d*x+
c)-4/d^2*b^2*c^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+6/d^2*b^2*c^2*(-(d*x+c)^2*
cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-4/d^2*b^2*c*(-(d*x+c)^3*cos(d
*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+1/d^2*b^2*(
-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24*cos
(d*x+c)-24*(d*x+c)*sin(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(186) = 372$.
time = 0.36, size = 406, normalized size = 2.18

$$-\frac{a^2c^2\cos\left(dx+c\right)+\frac{b^2c^4\cos(dx+c)}{d^2}-\frac{2\,abc^3\cos(dx+c)}{d}-2\left(\left(dx+c\right)\cos\left(dx+c\right)-\sin\left(dx+c\right)\right)a^2c-\frac{4\left(\left(dx+c\right)\cos(dx+c)-\sin(dx+c)\right)b^2c^3}{d^2}+\frac{6\left(\left(dx+c\right)\cos(dx+c)-\sin(dx+c)\right)abc^2}{d}+\left(\left(\left(dx+c\right)^2-2\right)\cos\left(dx+c\right)-2\left(dx+c\right)\sin\left(dx+c\right)\right)a^2+\frac{6\left(\left(\left(dx+c\right)^2-2\right)\cos(dx+c)-2\left(dx+c\right)\sin(dx+c)\right)b^2c^2}{d^2}-\frac{6\left(\left(\left(dx+c\right)^2-2\right)\cos(dx+c)-2\left(dx+c\right)\sin(dx+c)\right)abc}{d}-\frac{4\left(\left(\left(dx+c\right)^3-6\,dx-6\,c\right)\cos(dx+c)-3\left(\left(dx+c\right)^2-2\right)\sin(dx+c)\right)b^2c}{d^2}+\frac{2\left(\left(\left(dx+c\right)^3-6\,dx-6\,c\right)\cos(dx+c)-3\left(\left(dx+c\right)^2-2\right)\sin(dx+c)\right)ab}{d}+\frac{\left(\left(\left(dx+c\right)^4-12\left(dx+c\right)^2+24\right)\cos(dx+c)-4\left(\left(dx+c\right)^3-6\,dx-6\,c\right)\sin(dx+c)\right)b^2}{d^2}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*c^2*cos(d*x + c) + b^2*c^4*cos(d*x + c)/d^2 - 2*a*b*c^3*cos(d*x + c)/
d - 2*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2*c - 4*((d*x + c)*cos(d*x
+ c) - sin(d*x + c))*b^2*c^3/d^2 + 6*((d*x + c)*cos(d*x + c) - sin(d*x + c)
)*a*b*c^2/d + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a
^2 + 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b^2*c^2/
d^2 - 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*b*c/d
 - 4*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*
x + c))*b^2*c/d^2 + 2*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x +
 c)^2 - 2)*sin(d*x + c))*a*b/d + (((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d
*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b^2/d^2)/d^3
```

Fricas [A]
time = 0.36, size = 126, normalized size = 0.68

$$-\frac{\left(b^2d^4x^4+2\,abd^4x^3-12\,abd^2x-2\,a^2d^2+\left(a^2d^4-12\,b^2d^2\right)x^2+24\,b^2\right)\cos\left(dx+c\right)-2\left(2\,b^2d^3x^3+3\,abd^3x^2-6\,abd+\left(a^2d^3-12\,b^2d\right)x\right)\sin\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^4*x^4 + 2*a*b*d^4*x^3 - 12*a*b*d^2*x - 2*a^2*d^2 + (a^2*d^4 - 12*b
^2*d^2)*x^2 + 24*b^2)*cos(d*x + c) - 2*(2*b^2*d^3*x^3 + 3*a*b*d^3*x^2 - 6*a
*b*d + (a^2*d^3 - 12*b^2*d)*x)*sin(d*x + c))/d^5
```

Sympy [A]
time = 0.32, size = 228, normalized size = 1.23

$$\begin{cases} -\frac{a^2x^2\cos\left(c+dx\right)}{d}+\frac{2a^2x\sin\left(c+dx\right)}{d^2}+\frac{2a^2\cos\left(c+dx\right)}{d^3}-\frac{2abx^3\cos\left(c+dx\right)}{d}+\frac{6abx^2\sin\left(c+dx\right)}{d^2}+\frac{12abx\cos\left(c+dx\right)}{d^3}-\frac{12ab\sin\left(c+dx\right)}{d^4}-\frac{b^2x^4\cos\left(c+dx\right)}{d}+\frac{4b^2x^3\sin\left(c+dx\right)}{d^2}+\frac{12b^2x^2\cos\left(c+dx\right)}{d^3}-\frac{24b^2x\sin\left(c+dx\right)}{d^4}-\frac{24b^2\cos\left(c+dx\right)}{d^5} & \text{for}\: d \neq 0 \\ \left(\frac{a^2x^3}{3}+\frac{abx^4}{2}+\frac{b^2x^5}{5}\right)\sin\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*x**2*cos(c + d*x)/d + 2*a**2*x*sin(c + d*x)/d**2 + 2*a**2*
cos(c + d*x)/d**3 - 2*a*b*x**3*cos(c + d*x)/d + 6*a*b*x**2*sin(c + d*x)/d**
2 + 12*a*b*x*cos(c + d*x)/d**3 - 12*a*b*sin(c + d*x)/d**4 - b**2*x**4*cos(c
 + d*x)/d + 4*b**2*x**3*sin(c + d*x)/d**2 + 12*b**2*x**2*cos(c + d*x)/d**3
- 24*b**2*x*sin(c + d*x)/d**4 - 24*b**2*cos(c + d*x)/d**5, Ne(d, 0)), ((a**
2*x**3/3 + a*b*x**4/2 + b**2*x**5/5)*sin(c), True))
```

Giac [A]
time = 2.74, size = 128, normalized size = 0.69

$$-\frac{(b^2d^4x^4 + 2\,abd^4x^3 + a^2d^4x^2 - 12\,b^2d^2x^2 - 12\,abd^2x - 2\,a^2d^2 + 24\,b^2)\cos(dx+c)}{d^5} + \frac{2\,(2\,b^2d^3x^3 + 3\,abd^3x^2 + a^2d^3x - 12\,b^2dx - 6\,abd)\sin(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^4*x^4 + 2*a*b*d^4*x^3 + a^2*d^4*x^2 - 12*b^2*d^2*x^2 - 12*a*b*d^2*x
 - 2*a^2*d^2 + 24*b^2)*cos(d*x + c)/d^5 + 2*(2*b^2*d^3*x^3 + 3*a*b*d^3*x^2
+ a^2*d^3*x - 12*b^2*d*x - 6*a*b*d)*sin(d*x + c)/d^5
```

Mupad [B]
time = 0.29, size = 172, normalized size = 0.92

$$\frac{4\,b^2\,x^3\,\sin(c+d\,x)}{d^2} - \frac{b^2\,x^4\,\cos(c+d\,x)}{d} - \frac{2\,\cos(c+d\,x)\,(12\,b^2-a^2\,d^2)}{d^5} - \frac{12\,a\,b\,\sin(c+d\,x)}{d^4} - \frac{2\,x\,\sin(c+d\,x)\,(12\,b^2-a^2\,d^2)}{d^4} + \frac{x^2\,\cos(c+d\,x)\,(12\,b^2-a^2\,d^2)}{d^3} - \frac{2\,a\,b\,x^3\,\cos(c+d\,x)}{d} + \frac{6\,a\,b\,x^2\,\sin(c+d\,x)}{d^2} + \frac{12\,a\,b\,x\,\cos(c+d\,x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(c + d*x)*(a + b*x)^2,x)
```

```
[Out] (4*b^2*x^3*sin(c + d*x))/d^2 - (b^2*x^4*cos(c + d*x))/d - (2*cos(c + d*x)*(
12*b^2 - a^2*d^2))/d^5 - (12*a*b*sin(c + d*x))/d^4 - (2*x*sin(c + d*x)*(12*
b^2 - a^2*d^2))/d^4 + (x^2*cos(c + d*x)*(12*b^2 - a^2*d^2))/d^3 - (2*a*b*x^
3*cos(c + d*x))/d + (6*a*b*x^2*sin(c + d*x))/d^2 + (12*a*b*x*cos(c + d*x))/
d^3
```

3.11 $\int x(a+bx)^2 \sin(c+dx)\, dx$

Optimal. Leaf size=135

$$\frac{4ab\cos(c+dx)}{d^3}+\frac{6b^2x\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^3x^3\cos(c+dx)}{d}-\frac{6b^2\sin(c+dx)}{d^4}$$

```
[Out] 4*a*b*cos(d*x+c)/d^3+6*b^2*x*cos(d*x+c)/d^3-a^2*x*cos(d*x+c)/d-2*a*b*x^2*co
s(d*x+c)/d-b^2*x^3*cos(d*x+c)/d-6*b^2*sin(d*x+c)/d^4+a^2*sin(d*x+c)/d^2+4*a
*b*x*sin(d*x+c)/d^2+3*b^2*x^2*sin(d*x+c)/d^2
```

Rubi [A]

time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6874, 3377, 2717, 2718}

$$\frac{a^2\sin(c+dx)}{d^2}-\frac{a^2x\cos(c+dx)}{d}+\frac{4ab\cos(c+dx)}{d^3}+\frac{4abx\sin(c+dx)}{d^2}-\frac{2abx^2\cos(c+dx)}{d}-\frac{6b^2\sin(c+dx)}{d^4}+\frac{6b^2x\cos(c+dx)}{d^3}+\frac{3b^2x^2\sin(c+dx)}{d^2}-\frac{b^2x^3\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (4*a*b*Cos[c + d*x])/d^3 + (6*b^2*x*Cos[c + d*x])/d^3 - (a^2*x*Cos[c + d*x]
)/d - (2*a*b*x^2*Cos[c + d*x])/d - (b^2*x^3*Cos[c + d*x])/d - (6*b^2*Sin[c
+ d*x])/d^4 + (a^2*Sin[c + d*x])/d^2 + (4*a*b*x*Sin[c + d*x])/d^2 + (3*b^2*
x^2*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int x(a+bx)^2\sin(c+dx)\,dx &= \int \left(a^2x\sin(c+dx)+2abx^2\sin(c+dx)+b^2x^3\sin(c+dx)\right)\,dx\\
&= a^2\int x\sin(c+dx)\,dx+(2ab)\int x^2\sin(c+dx)\,dx+b^2\int x^3\sin(c+dx)\,dx\\
&= -\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^3\cos(c+dx)}{d}+\frac{a^2\int\cos(c+dx)\\
&= -\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^3\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d^2}+\\
&= \frac{4ab\cos(c+dx)}{d^3}+\frac{6b^2x\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\\
&= \frac{4ab\cos(c+dx)}{d^3}+\frac{6b^2x\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 87, normalized size = 0.64

$$
\frac{-d(a^2d^2x+b^2x(-6+d^2x^2)+2ab(-2+d^2x^2))\cos(c+dx)+(a^2d^2+4abd^2x+3b^2(-2+d^2x^2))\sin(c+dx)}{d^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (-(d*(a^2*d^2*x + b^2*x*(-6 + d^2*x^2) + 2*a*b*(-2 + d^2*x^2))*Cos[c + d*x]
) + (a^2*d^2 + 4*a*b*d^2*x + 3*b^2*(-2 + d^2*x^2))*Sin[c + d*x])/d^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(135) = 270.
time = 0.07, size = 281, normalized size = 2.08

method	result
risch	$-\frac{(d^2b^2x^3+2ab\,d^2x^2+a^2d^2x-6b^2x-4ab)\cos(dx+c)}{d^3}+\frac{(3d^2x^2b^2+4ab\,d^2x+d^2a^2-6b^2)\sin(dx+c)}{d^4}$
norman	$\frac{\frac{b^2x^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(d^2a^2-6b^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}-\frac{b^2x^3}{d}-\frac{(d^2a^2-6b^2)x}{d^3}+\frac{2(d^2a^2-6b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}-\frac{8ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}+}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
derivativedivides	$\frac{a^2c\cos(dx+c)+a^2(\sin(dx+c)-(dx+c)\cos(dx+c))-\frac{2ab\,c^2\cos(dx+c)}{d}-\frac{4abc(\sin(dx+c)-(dx+c)\cos(dx+c))}{d}+\frac{2ab\left(-(dx+c)^2\cos(\right.}{}}{}$
default	$\frac{a^2c\cos(dx+c)+a^2(\sin(dx+c)-(dx+c)\cos(dx+c))-\frac{2ab\,c^2\cos(dx+c)}{d}-\frac{4abc(\sin(dx+c)-(dx+c)\cos(dx+c))}{d}+\frac{2ab\left(-(dx+c)^2\cos(\right.}{}}{}$

meijerg	$\frac{8b^2\sqrt{\pi}\,\sin(c)\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3d^2x^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}}-\frac{dx\left(-\frac{d^2x^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4}+\frac{8b^2\sqrt{\pi}\,\cos(c)\left(\frac{xd\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx}{20\sqrt{\pi}}\right.}{d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(a^2*c*cos(d*x+c)+a^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-2/d*a*b*c^2*cos
(d*x+c)-4/d*a*b*c*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+2/d*a*b*(-(d*x+c)^2*cos(d
*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+1/d^2*b^2*c^3*cos(d*x+c)+3/d^2*b^2
*c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-3/d^2*b^2*c*(-(d*x+c)^2*cos(d*x+c)+2*c
os(d*x+c)+2*(d*x+c)*sin(d*x+c))+1/d^2*b^2*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^
2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c)))
```

Maxima [A]
time = 0.29, size = 259, normalized size = 1.92

$$\frac{a^2c\cos(dx+c)+\frac{b^2c^3\cos(dx+c)}{d^2}-\frac{2abc^2\cos(dx+c)}{d}-((dx+c)\cos(dx+c)-\sin(dx+c))a^2-\frac{3((dx+c)\cos(dx+c)-\sin(dx+c))b^2c^2}{d^2}+\frac{4((dx+c)\cos(dx+c)-\sin(dx+c))abc}{d}+\frac{3\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)b^2c}{d^2}-\frac{2\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)ab}{d}-\frac{\left(\left((dx+c)^3-6dx-6c\right)\cos(dx+c)-3\left((dx+c)^2-2\right)\sin(dx+c)\right)b^2}{d^2}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a^2*c*cos(d*x + c) + b^2*c^3*cos(d*x + c)/d^2 - 2*a*b*c^2*cos(d*x + c)/d -
 ((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2 - 3*((d*x + c)*cos(d*x + c) -
sin(d*x + c))*b^2*c^2/d^2 + 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a*b*c
/d + 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b^2*c/d^
2 - 2*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*b/d - (
((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c)
)*b^2/d^2)/d^2
```

Fricas [A]
time = 0.34, size = 95, normalized size = 0.70

$$-\frac{\left(b^2d^3x^3+2\,abd^3x^2-4\,abd+\left(a^2d^3-6\,b^2d\right)x\right)\cos(dx+c)-\left(3\,b^2d^2x^2+4\,abd^2x+a^2d^2-6\,b^2\right)\sin(dx+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^3*x^3 + 2*a*b*d^3*x^2 - 4*a*b*d + (a^2*d^3 - 6*b^2*d)*x)*cos(d*x +
 c) - (3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 - 6*b^2)*sin(d*x + c))/d^4
```

Sympy [A]
time = 0.20, size = 172, normalized size = 1.27

$$\begin{cases} -\frac{a^2x\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d^2}-\frac{2abx^2\cos(c+dx)}{d}+\frac{4abx\sin(c+dx)}{d^2}+\frac{4ab\cos(c+dx)}{d^3}-\frac{b^2x^3\cos(c+dx)}{d}+\frac{3b^2x^2\sin(c+dx)}{d^2}+\frac{6b^2x\cos(c+dx)}{d^3}-\frac{6b^2\sin(c+dx)}{d^4} & \text{for } d\neq 0 \\ \left(\frac{a^2x^2}{2}+\frac{2abx^3}{3}+\frac{b^2x^4}{4}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*x*cos(c + d*x)/d + a**2*sin(c + d*x)/d**2 - 2*a*b*x**2*cos
(c + d*x)/d + 4*a*b*x*sin(c + d*x)/d**2 + 4*a*b*cos(c + d*x)/d**3 - b**2*x*
*3*cos(c + d*x)/d + 3*b**2*x**2*sin(c + d*x)/d**2 + 6*b**2*x*cos(c + d*x)/d
**3 - 6*b**2*sin(c + d*x)/d**4, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x**3/3 + b
**2*x**4/4)*sin(c), True))
```

Giac [A]
time = 4.63, size = 95, normalized size = 0.70

$$-\frac{(b^2d^3x^3+2\,abd^3x^2+a^2d^3x-6\,b^2dx-4\,abd)\cos\left(dx+c\right)}{d^4}+\frac{(3\,b^2d^2x^2+4\,abd^2x+a^2d^2-6\,b^2)\sin\left(dx+c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^3*x^3 + 2*a*b*d^3*x^2 + a^2*d^3*x - 6*b^2*d*x - 4*a*b*d)*cos(d*x +
c)/d^4 + (3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 - 6*b^2)*sin(d*x + c)/d^4
```

Mupad [B]
time = 4.76, size = 128, normalized size = 0.95

$$\frac{3\,b^2\,x^2\,\sin\left(c+d\,x\right)}{d^2}-\frac{b^2\,x^3\,\cos\left(c+d\,x\right)}{d}-\frac{\sin\left(c+d\,x\right)\,\left(6\,b^2-a^2\,d^2\right)}{d^4}+\frac{4\,a\,b\,\cos\left(c+d\,x\right)}{d^3}+\frac{x\,\cos\left(c+d\,x\right)\,\left(6\,b^2-a^2\,d^2\right)}{d^3}-\frac{2\,a\,b\,x^2\,\cos\left(c+d\,x\right)}{d}+\frac{4\,a\,b\,x\,\sin\left(c+d\,x\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(c + d*x)*(a + b*x)^2,x)
```

```
[Out] (3*b^2*x^2*sin(c + d*x))/d^2 - (b^2*x^3*cos(c + d*x))/d - (sin(c + d*x)*(6*
b^2 - a^2*d^2))/d^4 + (4*a*b*cos(c + d*x))/d^3 + (x*cos(c + d*x)*(6*b^2 - a
^2*d^2))/d^3 - (2*a*b*x^2*cos(c + d*x))/d + (4*a*b*x*sin(c + d*x))/d^2
```

3.12 $\int (a+bx)^2 \sin(c+dx)\, dx$

Optimal. Leaf size=50

$$\frac{2b^2\cos(c+dx)}{d^3}-\frac{(a+bx)^2\cos(c+dx)}{d}+\frac{2b(a+bx)\sin(c+dx)}{d^2}$$

```
[Out] 2*b^2*cos(d*x+c)/d^3-(b*x+a)^2*cos(d*x+c)/d+2*b*(b*x+a)*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2718}

$$\frac{2b(a+bx)\sin(c+dx)}{d^2}-\frac{(a+bx)^2\cos(c+dx)}{d}+\frac{2b^2\cos(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (2*b^2*Cos[c + d*x])/d^3 - ((a + b*x)^2*Cos[c + d*x])/d + (2*b*(a + b*x)*Si
n[c + d*x])/d^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (a+bx)^2\sin(c+dx)\,dx &= -\frac{(a+bx)^2\cos(c+dx)}{d}+\frac{(2b)\int (a+bx)\cos(c+dx)\,dx}{d}\\
&= -\frac{(a+bx)^2\cos(c+dx)}{d}+\frac{2b(a+bx)\sin(c+dx)}{d^2}-\frac{(2b^2)\int \sin(c+dx)\,dx}{d^2}\\
&= \frac{2b^2\cos(c+dx)}{d^3}-\frac{(a+bx)^2\cos(c+dx)}{d}+\frac{2b(a+bx)\sin(c+dx)}{d^2}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 57, normalized size = 1.14

$$\frac{-((a^2d^2+2abd^2x+b^2(-2+d^2x^2))\cos(c+dx))+2bd(a+bx)\sin(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (-((a^2*d^2 + 2*a*b*d^2*x + b^2*(-2 + d^2*x^2))*Cos[c + d*x]) + 2*b*d*(a +
b*x)*Sin[c + d*x])/d^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(50) = 100$.
time = 0.06, size = 148, normalized size = 2.96

method	result
risch	$-\frac{(d^2x^2b^2+2ab\,d^2x+d^2a^2-2b^2)\cos(dx+c)}{d^3}+\frac{2b(bx+a)\sin(dx+c)}{d^2}$
norman	$\frac{\frac{b^2x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2d^2a^2-4b^2}{d^3}-\frac{b^2x^2}{d}+\frac{4b^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}+\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{2abx}{d}+\frac{2abx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
derivativedivides	$\frac{-a^2\cos(dx+c)+\frac{2abc\cos(dx+c)}{d}+\frac{2ab(\sin(dx+c)-(dx+c)\cos(dx+c))}{d}-\frac{b^2c^2\cos(dx+c)}{d^2}-\frac{2b^2c(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^2}+\frac{b^2(-}{}}{d}$
default	$\frac{-a^2\cos(dx+c)+\frac{2abc\cos(dx+c)}{d}+\frac{2ab(\sin(dx+c)-(dx+c)\cos(dx+c))}{d}-\frac{b^2c^2\cos(dx+c)}{d^2}-\frac{2b^2c(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^2}+\frac{b^2(-}{}}{d}$
meijerg	$\frac{4b^2\sqrt{\pi}\,\sin(c)\left(\frac{x\left(d^2\right)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\,d^2}-\frac{\left(d^2\right)^{\frac{3}{2}}\left(-\frac{3d^2x^2}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\,d^3}\right)}{d^2\sqrt{d^2}}+\frac{4b^2\sqrt{\pi}\,\cos(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{d^2x^2}{2}+1\right)\cos(dx)}{2\sqrt{\pi}}+\frac{d}{}\right.}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*cos(d*x+c)+2/d*a*b*c*cos(d*x+c)+2/d*a*b*(sin(d*x+c)-(d*x+c)*cos(d
*x+c))-1/d^2*b^2*c^2*cos(d*x+c)-2/d^2*b^2*c*(sin(d*x+c)-(d*x+c)*cos(d*x+c))
+1/d^2*b^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(50) = 100$.
time = 0.33, size = 141, normalized size = 2.82

$$-\frac{a^2\cos(dx+c)+\frac{b^2c^2\cos(dx+c)}{d^2}-\frac{2\,abc\cos(dx+c)}{d}-\frac{2\,((dx+c)\cos(dx+c)-\sin(dx+c))b^2c}{d^2}+\frac{2\,((dx+c)\cos(dx+c)-\sin(dx+c))ab}{d}+\frac{\left(\left((dx+c)^2-2\right)\cos(dx+c)-2\,(dx+c)\sin(dx+c)\right)b^2}{d^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*cos(d*x + c) + b^2*c^2*cos(d*x + c)/d^2 - 2*a*b*c*cos(d*x + c)/d - 2*
((d*x + c)*cos(d*x + c) - sin(d*x + c))*b^2*c/d^2 + 2*((d*x + c)*cos(d*x +
c) - sin(d*x + c))*a*b/d + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*si
n(d*x + c))*b^2/d^2)/d
```

Fricas [A]
time = 0.35, size = 63, normalized size = 1.26

$$-\frac{\left(b^{2} d^{2} x^{2}+2 \, a b d^{2} x+a^{2} d^{2}-2 \, b^{2}\right) \cos \left(d x+c\right)-2 \, \left(b^{2} d x+a b d\right) \sin \left(d x+c\right)}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 - 2*b^2)*cos(d*x + c) - 2*(b^2*d*x +
 a*b*d)*sin(d*x + c))/d^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(48) = 96.
time = 0.14, size = 112, normalized size = 2.24

$$\begin{cases} - \frac{a^{2} \cos{\left(c + d x \right)}}{d} - \frac{2 a b x \cos{\left(c + d x \right)}}{d} + \frac{2 a b \sin{\left(c + d x \right)}}{d^{2}} - \frac{b^{2} x^{2} \cos{\left(c + d x \right)}}{d} + \frac{2 b^{2} x \sin{\left(c + d x \right)}}{d^{2}} + \frac{2 b^{2} \cos{\left(c + d x \right)}}{d^{3}} & \text{for}\: d \neq 0 \\\left(a^{2} x + a b x^{2} + \frac{b^{2} x^{3}}{3}\right) \sin{\left(c \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)/d - 2*a*b*x*cos(c + d*x)/d + 2*a*b*sin(c + d*
x)/d**2 - b**2*x**2*cos(c + d*x)/d + 2*b**2*x*sin(c + d*x)/d**2 + 2*b**2*co
s(c + d*x)/d**3, Ne(d, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)*sin(c), True
))
```

Giac [A]
time = 5.15, size = 65, normalized size = 1.30

$$-\frac{\left(b^{2} d^{2} x^{2}+2 \, a b d^{2} x+a^{2} d^{2}-2 \, b^{2}\right) \cos \left(d x+c\right)}{d^{3}}+\frac{2 \, \left(b^{2} d x+a b d\right) \sin \left(d x+c\right)}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 - 2*b^2)*cos(d*x + c)/d^3 + 2*(b^2*d*
x + a*b*d)*sin(d*x + c)/d^3
```

Mupad [B]
time = 4.70, size = 84, normalized size = 1.68

$$\frac{\cos\left(c+d\,x\right)\,\left(2\,b^2-a^2\,d^2\right)}{d^3}-\frac{b^2\,x^2\,\cos\left(c+d\,x\right)}{d}+\frac{2\,a\,b\,\sin\left(c+d\,x\right)}{d^2}+\frac{2\,b^2\,x\,\sin\left(c+d\,x\right)}{d^2}-\frac{2\,a\,b\,x\,\cos\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*x)^2,x)
```

```
[Out] (cos(c + d*x)*(2*b^2 - a^2*d^2))/d^3 - (b^2*x^2*cos(c + d*x))/d + (2*a*b*si
n(c + d*x))/d^2 + (2*b^2*x*sin(c + d*x))/d^2 - (2*a*b*x*cos(c + d*x))/d
```

3.13 $\int \frac{(a+bx)^2 \sin(c+dx)}{x} \, dx$

Optimal. Leaf size=62

$$-\frac{2ab\cos(c+dx)}{d}-\frac{b^2x\cos(c+dx)}{d}+a^2\text{Ci}(dx)\sin(c)+\frac{b^2\sin(c+dx)}{d^2}+a^2\cos(c)\text{Si}(dx)$$

```
[Out] -2*a*b*cos(d*x+c)/d-b^2*x*cos(d*x+c)/d+a^2*cos(c)*Si(d*x)+a^2*Ci(d*x)*sin(c
)+b^2*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.13, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6874, 2718, 3384, 3380, 3383, 3377, 2717}

$$a^2\sin(c)\text{CosIntegral}(dx)+a^2\cos(c)\text{Si}(dx)-\frac{2ab\cos(c+dx)}{d}+\frac{b^2\sin(c+dx)}{d^2}-\frac{b^2x\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x,x]
```

```
[Out] (-2*a*b*Cos[c + d*x])/d - (b^2*x*Cos[c + d*x])/d + a^2*CosIntegral[d*x]*Sin
[c] + (b^2*Sin[c + d*x])/d^2 + a^2*Cos[c]*SinIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2\sin(c+dx)}{x}\,dx &= \int \left(2ab\sin(c+dx)+\frac{a^2\sin(c+dx)}{x}+b^2x\sin(c+dx)\right)dx\\ &= a^2\int \frac{\sin(c+dx)}{x}\,dx+(2ab)\int \sin(c+dx)\,dx+b^2\int x\sin(c+dx)\,dx\\ &= -\frac{2ab\cos(c+dx)}{d}-\frac{b^2x\cos(c+dx)}{d}+\frac{b^2\int\cos(c+dx)\,dx}{d}+\left(a^2\cos(c)\right)\int \frac{\sin}{}\\ &= -\frac{2ab\cos(c+dx)}{d}-\frac{b^2x\cos(c+dx)}{d}+a^2\mathrm{Ci}(dx)\sin(c)+\frac{b^2\sin(c+dx)}{d^2}+a^2\,\mathrm{co}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 51, normalized size = 0.82

$$a^2\mathrm{Ci}(dx)\sin(c)+\frac{b(-d(2a+bx)\cos(c+dx)+b\sin(c+dx))}{d^2}+a^2\cos(c)\mathrm{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x,x]
```

```
[Out] a^2*CosIntegral[d*x]*Sin[c] + (b*(-(d*(2*a + b*x)*Cos[c + d*x]) + b*Sin[c +
 d*x]))/d^2 + a^2*Cos[c]*SinIntegral[d*x]
```

Maple [A]
time = 0.10, size = 79, normalized size = 1.27

method	result
derivativedivides	$a^2(\operatorname{sinIntegral}(dx)\cos(c)+\operatorname{cosineIntegral}(dx)\sin(c))-\frac{2ab\cos(dx+c)}{d}+\frac{2b^2c\cos(dx+c)}{d^2}+$
default	$a^2(\operatorname{sinIntegral}(dx)\cos(c)+\operatorname{cosineIntegral}(dx)\sin(c))-\frac{2ab\cos(dx+c)}{d}+\frac{2b^2c\cos(dx+c)}{d^2}+$
meijerg	$\frac{2b^2\sqrt{\pi}\,\sin(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b^2\sqrt{\pi}\,\cos(c)\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2ab\sin(c)\sin(dx)}{d}$
risch	$-\frac{e^{-ic}\pi\operatorname{csgn}(dx)a^2}{2}+e^{-ic}\operatorname{sinIntegral}(dx)\,a^2-\frac{ie^{-ic}\operatorname{expIntegral}(1,-idx)a^2}{2}+\frac{ia^2e^{ic}\operatorname{expIntegral}(1,-idx}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-2*a*b*cos(d*x+c)/d+2/d^2*b^2*c*cos(d*x+
c)+(c+1)/d^2*b^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 80, normalized size = 1.29

$$\frac{(a^2(-i\operatorname{Ei}(i\,dx)+i\operatorname{Ei}(-i\,dx))\cos(c)+a^2(\operatorname{Ei}(i\,dx)+\operatorname{Ei}(-i\,dx))\sin(c))d^2+2\,b^2\sin(dx+c)-2\,(b^2dx+2\,abd)\cos(dx+c)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a^2*(Ei(I*d*x) + Ei(-I*d*x
))*sin(c))*d^2 + 2*b^2*sin(d*x + c) - 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/d
^2
```

Fricas [A]
time = 0.35, size = 78, normalized size = 1.26

$$\frac{2\,a^2d^2\cos(c)\operatorname{Si}(dx)+2\,b^2\sin(dx+c)-2\,(b^2dx+2\,abd)\cos(dx+c)+(a^2d^2\operatorname{Ci}(dx)+a^2d^2\operatorname{Ci}(-dx))\sin(c)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*d^2*cos(c)*sin_integral(d*x) + 2*b^2*sin(d*x + c) - 2*(b^2*d*x +
 2*a*b*d)*cos(d*x + c) + (a^2*d^2*cos_integral(d*x) + a^2*d^2*cos_integral(
-d*x))*sin(c))/d^2
```

Sympy [A]
time = 2.24, size = 92, normalized size = 1.48

$$a^2\sin(c)\operatorname{Ci}(dx)+a^2\cos(c)\operatorname{Si}(dx)+2ab\left(\begin{cases}x\sin(c) & \text{for } d=0\\ -\frac{\cos(c+dx)}{d} & \text{otherwise}\end{cases}\right)+b^2x\left(\begin{cases}x\sin(c) & \text{for } d=0\\ -\frac{\cos(c+dx)}{d} & \text{otherwise}\end{cases}\right)-b^2\left(\begin{cases}\frac{x^2\sin(c)}{2} & \text{for } d=0\\ -\frac{\begin{cases}\frac{\sin(c+dx)}{d} & \text{for } d\neq 0\\ x\cos(c) & \text{otherwise}\end{cases}}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*sin(d*x+c)/x,x)
```

```
[Out] a**2*sin(c)*Ci(d*x) + a**2*cos(c)*Si(d*x) + 2*a*b*Piecewise((x*sin(c), Eq(d
, 0)), (-cos(c + d*x)/d, True)) + b**2*x*Piecewise((x*sin(c), Eq(d, 0)), (-
cos(c + d*x)/d, True)) - b**2*Piecewise((x**2*sin(c)/2, Eq(d, 0)), (-Piecew
ise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))/d, True))
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.82, size = 551, normalized size = 8.89

$$-\frac{a^2d^2\Im(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\tan\left(\frac{1}{2}c\right)^2 - a^2d^2\Im(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\tan\left(\frac{1}{2}c\right)^2 + 2a^2d^2\,\mathrm{Si}(dx)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\tan\left(\frac{1}{2}c\right)^2 - 2a^2d^2\Re(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\tan\left(\frac{1}{2}c\right) - 2a^2d^2\Re(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\tan\left(\frac{1}{2}c\right) - 2b^2dx\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\tan\left(\frac{1}{2}c\right)^2 - a^2d^2\Im(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + a^2d^2\Im(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 2a^2d^2\,\mathrm{Si}(dx)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + a^2d^2\Im(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}c\right)^2 - a^2d^2\Im(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}c\right)^2 + 2a^2d^2\,\mathrm{Si}(dx)\tan\left(\frac{1}{2}c\right)^2 - 4abd\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\tan\left(\frac{1}{2}c\right)^2 - 2b^2dx\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 2a^2d^2\Re(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}c\right) - 2a^2d^2\Re(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}c\right) + 2b^2dx\tan\left(\frac{1}{2}c\right)^2 - a^2d^2\Im(\mathrm{Ci}(dx)) + a^2d^2\Im(\mathrm{Ci}(-dx)) - 2a^2d^2\,\mathrm{Si}(dx) - 4abd\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 4abd\tan\left(\frac{1}{2}c\right)^2 - 4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\tan\left(\frac{1}{2}c\right)^2 + 2b^2dx + 4abd - 4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{2\left(d^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\tan\left(\frac{1}{2}c\right)^2 + d^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + d^2\tan\left(\frac{1}{2}c\right)^2 + d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*d^2*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c
)^2 - a^2*d^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
c)^2 + 2*a^2*d^2*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*
a^2*d^2*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*
a^2*d^2*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2
*b^2*d*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - a^2*d^2*imag_part(cos_integr
al(d*x))*tan(1/2*d*x + 1/2*c)^2 + a^2*d^2*imag_part(cos_integral(-d*x))*tan
(1/2*d*x + 1/2*c)^2 - 2*a^2*d^2*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2 +
a^2*d^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^2*imag_part(cos_i
ntegral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^2*sin_integral(d*x)*tan(1/2*c)^2 - 4*
a*b*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*b^2*d*x*tan(1/2*d*x + 1/2*c)^
2 - 2*a^2*d^2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^2*real_part
(cos_integral(-d*x))*tan(1/2*c) + 2*b^2*d*x*tan(1/2*c)^2 - a^2*d^2*imag_par
t(cos_integral(d*x)) + a^2*d^2*imag_part(cos_integral(-d*x)) - 2*a^2*d^2*si
n_integral(d*x) - 4*a*b*d*tan(1/2*d*x + 1/2*c)^2 + 4*a*b*d*tan(1/2*c)^2 - 4
*b^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 + 2*b^2*d*x + 4*a*b*d - 4*b^2*tan(1/
2*d*x + 1/2*c))/(d^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + d^2*tan(1/2*d*x
+ 1/2*c)^2 + d^2*tan(1/2*c)^2 + d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$b^2\cos(c)\left(\frac{\sin(d\,x)}{d^2} - \frac{x\cos(d\,x)}{d}\right) + b^2\sin(c)\left(\frac{\cos(d\,x)}{d^2} + \frac{x\sin(d\,x)}{d}\right) + a^2\,\mathrm{cosint}(d\,x)\sin(c) + a^2\,\mathrm{sinint}(d\,x)\cos(c) - \frac{2\,a\,b\cos(d\,x)\cos(c)}{d} + \frac{2\,a\,b\sin(d\,x)\sin(c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x)^2)/x,x)
```

```
[Out] b^2*cos(c)*(sin(d*x)/d^2 - (x*cos(d*x))/d) + b^2*sin(c)*(cos(d*x)/d^2 + (x*
sin(d*x))/d) + a^2*cosint(d*x)*sin(c) + a^2*sinint(d*x)*cos(c) - (2*a*b*cos
(d*x)*cos(c))/d + (2*a*b*sin(d*x)*sin(c))/d
```

3.14 $\int \frac{(a+bx)^2 \sin(c+dx)}{x^2} dx$

Optimal. Leaf size=72

$$-\frac{b^2 \cos(c+dx)}{d}+a^2 d \cos(c)\text{Ci}(dx)+2ab\text{Ci}(dx)\sin(c)-\frac{a^2 \sin(c+dx)}{x}+2ab\cos(c)\text{Si}(dx)-a^2 d \sin(c)\text{Si}(dx)$$

```
[Out] a^2*d*Ci(d*x)*cos(c)-b^2*cos(d*x+c)/d+2*a*b*cos(c)*Si(d*x)+2*a*b*Ci(d*x)*si
n(c)-a^2*d*Si(d*x)*sin(c)-a^2*sin(d*x+c)/x
```

Rubi [A]
time = 0.17, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6874, 2718, 3378, 3384, 3380, 3383}

$$a^2 d \cos(c)\text{CosIntegral}(dx) - a^2 d \sin(c)\text{Si}(dx) - \frac{a^2 \sin(c+dx)}{x} + 2ab\sin(c)\text{CosIntegral}(dx) + 2ab\cos(c)\text{Si}(dx) - \frac{b^2 \cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b^2*Cos[c + d*x])/d) + a^2*d*Cos[c]*CosIntegral[d*x] + 2*a*b*CosIntegral
[d*x]*Sin[c] - (a^2*Sin[c + d*x])/x + 2*a*b*Cos[c]*SinIntegral[d*x] - a^2*d
*Sin[c]*SinIntegral[d*x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2\sin(c+dx)}{x^2}\,dx &= \int \left(b^2\sin(c+dx)+\frac{a^2\sin(c+dx)}{x^2}+\frac{2ab\sin(c+dx)}{x}\right)dx\\ &= a^2\int \frac{\sin(c+dx)}{x^2}\,dx+(2ab)\int \frac{\sin(c+dx)}{x}\,dx+b^2\int \sin(c+dx)\,dx\\ &= -\frac{b^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{x}+\left(a^2d\right)\int \frac{\cos(c+dx)}{x}\,dx+(2ab\cos(c))\int \\ &= -\frac{b^2\cos(c+dx)}{d}+2ab\text{Ci}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{x}+2ab\cos(c)\text{Si}(dx)+\left(a^2\right.\\ &= -\frac{b^2\cos(c+dx)}{d}+a^2d\cos(c)\text{Ci}(dx)+2ab\text{Ci}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{x}+2ab\end{aligned}$$

Mathematica [A]
time = 0.16, size = 64, normalized size = 0.89

$$-\frac{b^2\cos(c+dx)}{d}+a\text{Ci}(dx)(ad\cos(c)+2b\sin(c))-\frac{a^2\sin(c+dx)}{x}-a(-2b\cos(c)+ad\sin(c))\text{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b^2*Cos[c + d*x])/d) + a*CosIntegral[d*x]*(a*d*Cos[c] + 2*b*Sin[c]) - (a
^2*Sin[c + d*x])/x - a*(-2*b*Cos[c] + a*d*Sin[c])*SinIntegral[d*x]
```

Maple [A]
time = 0.09, size = 74, normalized size = 1.03

method	result

derivativedivides	$d\left(a^2\left(-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}\left(dx\right)\sin\left(c\right)+\text{cosineIntegral}\left(dx\right)\cos\left(c\right)\right)+\frac{2ab(\text{sinIntegral}($
default	$d\left(a^2\left(-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}\left(dx\right)\sin\left(c\right)+\text{cosineIntegral}\left(dx\right)\cos\left(c\right)\right)+\frac{2ab(\text{sinIntegral}($
risch	$-i\cos\left(c\right)\text{expIntegral}\left(1,idx\right)ab-\frac{d\cos(c)a^2\,\text{expIntegral}(1,idx)}{2}+i\cos\left(c\right)\text{expIntegral}\left(1,-id\right.$
meijerg	$\frac{b^2\sin(c)\sin(dx)}{d}+\frac{b^2\sqrt{\pi}\,\cos(c)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}+ab\sqrt{\pi}\,\sin\left(c\right)\left(\frac{2\gamma+2\ln(x)+\ln(d^2)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln}{\sqrt{}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(a^2*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+2/d*a*b*(Si(d*x)*cos
(c)+Ci(d*x)*sin(c))-1/d^2*b^2*cos(d*x+c))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.90, size = 122, normalized size = 1.69

$$\frac{\left(\left(a^2(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos\left(c\right)-a^2(i\,\Gamma(-1,i\,dx)-i\,\Gamma(-1,-i\,dx))\sin\left(c\right)\right)d^2+2\left(ab(i\,\Gamma(-1,i\,dx)-i\,\Gamma(-1,-i\,dx))\cos\left(c\right)+ab(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\sin\left(c\right)\right)d\right)x-2\left(b^2x+2\,ab\right)\cos\left(dx+c\right)}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(((a^2*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) - a^2*(I*gamma(-1,
 I*d*x) - I*gamma(-1, -I*d*x))*sin(c))*d^2 + 2*(a*b*(I*gamma(-1, I*d*x) - I
*gamma(-1, -I*d*x))*cos(c) + a*b*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*sin
(c))*d)*x - 2*(b^2*x + 2*a*b)*cos(d*x + c))/(d*x)
```

Fricas [A]
time = 0.35, size = 111, normalized size = 1.54

$$-\frac{2\,b^2x\cos\left(dx+c\right)+2\,a^2d\sin\left(dx+c\right)-\left(a^2d^2x\,\mathrm{Ci}\left(dx\right)+a^2d^2x\,\mathrm{Ci}\left(-dx\right)+4\,abdx\,\mathrm{Si}\left(dx\right)\right)\cos\left(c\right)+2\left(a^2d^2x\,\mathrm{Si}\left(dx\right)-abdx\,\mathrm{Ci}\left(dx\right)-abdx\,\mathrm{Ci}\left(-dx\right)\right)\sin\left(c\right)}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x*cos(d*x + c) + 2*a^2*d*sin(d*x + c) - (a^2*d^2*x*cos_integral
(d*x) + a^2*d^2*x*cos_integral(-d*x) + 4*a*b*d*x*sin_integral(d*x))*cos(c)
+ 2*(a^2*d^2*x*sin_integral(d*x) - a*b*d*x*cos_integral(d*x) - a*b*d*x*cos_
integral(-d*x))*sin(c))/(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx\right)^2\sin\left(c+dx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x)**2*sin(c + d*x)/x**2, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.91, size = 743, normalized size = 10.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*d^2*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
a^2*d^2*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2
*d^2*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^2*x
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^2*x*sin_
integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*b*d*x*imag_part(cos_integral(
d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*b*d*x*imag_part(cos_integral(-d*x))
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b*d*x*sin_integral(d*x)*tan(1/2*d*x)^2*t
an(1/2*c)^2 - a^2*d^2*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 - a^2*d
^2*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 4*a*b*d*x*real_part(cos
_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b*d*x*real_part(cos_integra
l(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^2*x*real_part(cos_integral(d*x))
*tan(1/2*c)^2 + a^2*d^2*x*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 2*a*
b*d*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 2*a*b*d*x*imag_part(cos
_integral(-d*x))*tan(1/2*d*x)^2 - 4*a*b*d*x*sin_integral(d*x)*tan(1/2*d*x)^
2 + 2*a^2*d^2*x*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^2*x*imag_
part(cos_integral(-d*x))*tan(1/2*c) + 4*a^2*d^2*x*sin_integral(d*x)*tan(1/2
*c) + 2*a*b*d*x*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - 2*a*b*d*x*imag_
part(cos_integral(-d*x))*tan(1/2*c)^2 + 4*a*b*d*x*sin_integral(d*x)*tan(1/2
*c)^2 + 2*b^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^2*x*real_part(cos_integ
ral(d*x)) - a^2*d^2*x*real_part(cos_integral(-d*x)) - 4*a*b*d*x*real_part(c
os_integral(d*x))*tan(1/2*c) - 4*a*b*d*x*real_part(cos_integral(-d*x))*tan(
1/2*c) - 4*a^2*d*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*d*tan(1/2*d*x)*tan(1/2*c
)^2 - 2*a*b*d*x*imag_part(cos_integral(d*x)) + 2*a*b*d*x*imag_part(cos_inte
gral(-d*x)) - 4*a*b*d*x*sin_integral(d*x) - 2*b^2*x*tan(1/2*d*x)^2 - 8*b^2*
x*tan(1/2*d*x)*tan(1/2*c) - 2*b^2*x*tan(1/2*c)^2 + 4*a^2*d*tan(1/2*d*x) + 4
*a^2*d*tan(1/2*c) + 2*b^2*x)/(d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*x*tan(1/2
*d*x)^2 + d*x*tan(1/2*c)^2 + d*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)\,{\left(a+b\,x\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x)^2)/x^2,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x)^2)/x^2, x)
```

3.15 $\int \frac{(a+bx)^2 \sin(c+dx)}{x^3} \, dx$

Optimal. Leaf size=121

$$-\frac{a^2 d \cos(c+dx)}{2x}+2abd\cos(c)\text{Ci}(dx)+b^2\text{Ci}(dx)\sin(c)-\frac{1}{2}a^2d^2\text{Ci}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{2x^2}-\frac{2ab\sin(c+dx)}{x}$$

```
[Out] 2*a*b*d*Ci(d*x)*cos(c)-1/2*a^2*d*cos(d*x+c)/x+b^2*cos(c)*Si(d*x)-1/2*a^2*d^
2*cos(c)*Si(d*x)+b^2*Ci(d*x)*sin(c)-1/2*a^2*d^2*Ci(d*x)*sin(c)-2*a*b*d*Si(d
*x)*sin(c)-1/2*a^2*sin(d*x+c)/x^2-2*a*b*sin(d*x+c)/x
```

Rubi [A]
time = 0.24, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$-\frac{1}{2}a^2d^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)-\frac{a^2\sin(c+dx)}{2x^2}-\frac{a^2d\cos(c+dx)}{2x}+2abd\cos(c)\text{CosIntegral}(dx)-2abd\sin(c)\text{Si}(dx)-\frac{2ab\sin(c+dx)}{x}+b^2\sin(c)\text{CosIntegral}(dx)+b^2\cos(c)\text{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a^2*d*Cos[c + d*x])/x + 2*a*b*d*Cos[c]*CosIntegral[d*x] + b^2*CosInte
gral[d*x]*Sin[c] - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 - (a^2*Sin[c + d*x])
/(2*x^2) - (2*a*b*Sin[c + d*x])/x + b^2*Cos[c]*SinIntegral[d*x] - (a^2*d^2*
Cos[c]*SinIntegral[d*x])/2 - 2*a*b*d*Sin[c]*SinIntegral[d*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2\sin(c+dx)}{x^3}\,dx &= \int \left(\frac{a^2\sin(c+dx)}{x^3}+\frac{2ab\sin(c+dx)}{x^2}+\frac{b^2\sin(c+dx)}{x}\right)dx\\
&= a^2\int \frac{\sin(c+dx)}{x^3}\,dx+(2ab)\int \frac{\sin(c+dx)}{x^2}\,dx+b^2\int \frac{\sin(c+dx)}{x}\,dx\\
&= -\frac{a^2\sin(c+dx)}{2x^2}-\frac{2ab\sin(c+dx)}{x}+\frac{1}{2}\left(a^2d\right)\int \frac{\cos(c+dx)}{x^2}\,dx+(2abd)\int \frac{\mathrm{c}}{}\\
&= -\frac{a^2d\cos(c+dx)}{2x}+b^2\mathrm{Ci}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{2x^2}-\frac{2ab\sin(c+dx)}{x}+b^2\,\mathrm{c}\\
&= -\frac{a^2d\cos(c+dx)}{2x}+2abd\cos(c)\mathrm{Ci}(dx)+b^2\mathrm{Ci}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{2x^2}-\\
&= -\frac{a^2d\cos(c+dx)}{2x}+2abd\cos(c)\mathrm{Ci}(dx)+b^2\mathrm{Ci}(dx)\sin(c)-\frac{1}{2}a^2d^2\mathrm{Ci}(dx)\sin(c
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 95, normalized size = 0.79

$$\frac{1}{2}\left(\mathrm{Ci}(dx)\left(4abd\cos(c)+\left(2b^2-a^2d^2\right)\sin(c)\right)-\frac{a(adx\cos(c+dx)+(a+4bx)\sin(c+dx))}{x^2}+\left(\left(2b^2-a^2d^2\right)\cos(c)-4abd\sin(c)\right)\mathrm{Si}(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] (CosIntegral[d*x]*(4*a*b*d*Cos[c] + (2*b^2 - a^2*d^2)*Sin[c]) - (a*(a*d*x*C
os[c + d*x] + (a + 4*b*x)*Sin[c + d*x]))/x^2 + ((2*b^2 - a^2*d^2)*Cos[c] -
4*a*b*d*Sin[c])*SinIntegral[d*x])/2
```

Maple [A]
time = 0.10, size = 114, normalized size = 0.94

method	result

derivativedivides	$d^2\left(a^2\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\text{sinIntegral}(dx)\cos(c)}{2}-\frac{\text{cosineIntegral}(dx)\sin(c)}{2}\right)+\frac{2ab\left(-\frac{\sin(dx+c)}{dx}-\sin\right.}{}\right.$
default	$d^2\left(a^2\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\text{sinIntegral}(dx)\cos(c)}{2}-\frac{\text{cosineIntegral}(dx)\sin(c)}{2}\right)+\frac{2ab\left(-\frac{\sin(dx+c)}{dx}-\sin\right.}{}\right.$
risch	$-\cos(c)\,\text{expIntegral}(1,idx)\,abd-\cos(c)\,\text{expIntegral}(1,-idx)\,abd+\frac{i\cos(c)\,\text{expIntegral}(1,id}{4}$
meijerg	$\frac{b^2\sqrt{\pi}\,\sin(c)\left(\frac{2\gamma+2\ln(x)+\ln(d^2)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\,\text{cosineIntegral}(dx)}{\sqrt{\pi}}\right)}{2}+b^2\cos(c)\,\text{sinIntegral}(dx$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(a^2*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2
*Ci(d*x)*sin(c))+2/d*a*b*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+1/
d^2*b^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 1.40, size = 189, normalized size = 1.56

$$-\frac{\left(\left(a^2(-i\,\Gamma(-2,i\,dx)+i\,\Gamma(-2,-i\,dx))\cos(c)-a^2(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin(c)\right)d^4+4\left(ab(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\cos(c)-ab(i\,\Gamma(-2,i\,dx)-i\,\Gamma(-2,-i\,dx))\sin(c)\right)d^3-2\left(b^2(-i\,\Gamma(-2,i\,dx)+i\,\Gamma(-2,-i\,dx))\cos(c)-b^2(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin(c)\right)d^2\right)x^2+2\,b^2\sin(dx+c)+2\,(b^2dx+2\,abd)\cos(dx+c)}{2\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x))*cos(c) - a^2*(gamma
(-2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^4 + 4*(a*b*(gamma(-2, I*d*x) + g
amma(-2, -I*d*x))*cos(c) - a*b*(I*gamma(-2, I*d*x) - I*gamma(-2, -I*d*x))*s
in(c))*d^3 - 2*(b^2*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x))*cos(c) - b^
2*(gamma(-2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^2)*x^2 + 2*b^2*sin(d*x +
 c) + 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/(d^2*x^2)
```

Fricas [A]
time = 0.37, size = 147, normalized size = 1.21

$$-\frac{2\,a^2dx\cos(dx+c)-2\left(2\,abdx^2\,\mathrm{Ci}(dx)+2\,abdx^2\,\mathrm{Ci}(-dx)-(a^2d^2-2\,b^2)x^2\,\mathrm{Si}(dx)\right)\cos(c)+2\,(4\,abx+a^2)\sin(dx+c)+\left(8\,abdx^2\,\mathrm{Si}(dx)+(a^2d^2-2\,b^2)x^2\,\mathrm{Ci}(dx)+(a^2d^2-2\,b^2)x^2\,\mathrm{Ci}(-dx)\right)\sin(c)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*d*x*cos(d*x + c) - 2*(2*a*b*d*x^2*cos_integral(d*x) + 2*a*b*d*x
^2*cos_integral(-d*x) - (a^2*d^2 - 2*b^2)*x^2*sin_integral(d*x))*cos(c) + 2
*(4*a*b*x + a^2)*sin(d*x + c) + (8*a*b*d*x^2*sin_integral(d*x) + (a^2*d^2 -
```

```
2*b^2)*x^2*cos_integral(d*x) + (a^2*d^2 - 2*b^2)*x^2*cos_integral(-d*x))*s
in(c))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x)**2*sin(c + d*x)/x**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.33, size = 1182, normalized size = 9.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^2*x^2*r
eal_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^2*x^2*real_
part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b*d*x^2*real_part(
cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*d*x^2*real_part(cos_
integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^2*x^2*imag_part(cos_int
egral(d*x))*tan(1/2*d*x)^2 + a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(
1/2*d*x)^2 - 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 8*a*b*d*x^2*i
mag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*b*d*x^2*imag_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 16*a*b*d*x^2*sin_integra
l(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^2*x^2*imag_part(cos_integral(d*x))
*tan(1/2*c)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*
a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 - 2*b^2*x^2*imag_part(cos_integr
al(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_integral(-d*
x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 4*a*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 +
4*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a^2*d^2*x^2*re
al_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^2*x^2*real_part(cos_integra
l(-d*x))*tan(1/2*c) + 4*b^2*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2
*tan(1/2*c) + 4*b^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/
2*c) - 4*a*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*c)^2 - 4*a*b*d*x^2*
real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 2*a^2*d*x*tan(1/2*d*x)^2*tan(1
/2*c)^2 - a^2*d^2*x^2*imag_part(cos_integral(d*x)) + a^2*d^2*x^2*imag_part(
```

```
cos_integral(-d*x)) - 2*a^2*d^2*x^2*sin_integral(d*x) + 2*b^2*x^2*imag_part
(cos_integral(d*x))*tan(1/2*d*x)^2 - 2*b^2*x^2*imag_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2 + 4*b^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 8*a*b*d*x^2
*imag_part(cos_integral(d*x))*tan(1/2*c) + 8*a*b*d*x^2*imag_part(cos_integr
al(-d*x))*tan(1/2*c) - 16*a*b*d*x^2*sin_integral(d*x)*tan(1/2*c) - 2*b^2*x^
2*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_integ
ral(-d*x))*tan(1/2*c)^2 - 4*b^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 + 4*a*b*
d*x^2*real_part(cos_integral(d*x)) + 4*a*b*d*x^2*real_part(cos_integral(-d*
x)) + 2*a^2*d*x*tan(1/2*d*x)^2 + 4*b^2*x^2*real_part(cos_integral(d*x))*tan
(1/2*c) + 4*b^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a^2*d*x*ta
n(1/2*d*x)*tan(1/2*c) + 16*a*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d*x*tan(
1/2*c)^2 + 16*a*b*x*tan(1/2*d*x)*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_int
egral(d*x)) - 2*b^2*x^2*imag_part(cos_integral(-d*x)) + 4*b^2*x^2*sin_integ
ral(d*x) + 4*a^2*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*tan(1/2*d*x)*tan(1/2*c)^
2 - 2*a^2*d*x - 16*a*b*x*tan(1/2*d*x) - 16*a*b*x*tan(1/2*c) - 4*a^2*tan(1/2
*d*x) - 4*a^2*tan(1/2*c))/(x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + x^2*tan(1/2*d*
x)^2 + x^2*tan(1/2*c)^2 + x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin \left(c+d\,x\right)\,{\left(a+b\,x\right)}^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x)^2)/x^3,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x)^2)/x^3, x)
```

3.16 $\int \frac{(a+bx)^2 \sin(c+dx)}{x^4} \, dx$

Optimal. Leaf size=175

$$-\frac{a^2 d \cos(c+dx)}{6x^2} - \frac{abd \cos(c+dx)}{x} + b^2 d \cos(c) \text{Ci}(dx) - \frac{1}{6} a^2 d^3 \cos(c) \text{Ci}(dx) - abd^2 \text{Ci}(dx) \sin(c) - \frac{a^2 \sin(c+d}{3x^3}$$

```
[Out] b^2*d*Ci(d*x)*cos(c)-1/6*a^2*d^3*Ci(d*x)*cos(c)-1/6*a^2*d*cos(d*x+c)/x^2-a*
b*d*cos(d*x+c)/x-a*b*d^2*cos(c)*Si(d*x)-a*b*d^2*Ci(d*x)*sin(c)-b^2*d*Si(d*x
)*sin(c)+1/6*a^2*d^3*Si(d*x)*sin(c)-1/3*a^2*sin(d*x+c)/x^3-a*b*sin(d*x+c)/x
^2-b^2*sin(d*x+c)/x+1/6*a^2*d^2*sin(d*x+c)/x
```

Rubi [A]
time = 0.27, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$-\frac{1}{6} a^2 d^3 \cos(c) \text{CosIntegral}(dx) + \frac{1}{6} a^2 d^3 \sin(c) \text{Si}(dx) + \frac{a^2 d^2 \sin(c+dx)}{6x} - \frac{a^2 \sin(c+dx)}{3x^3} - \frac{a^2 d \cos(c+dx)}{6x^2} - abd^2 \sin(c) \text{CosIntegral}(dx) - abd^2 \cos(c) \text{Si}(dx) - \frac{ab \sin(c+dx)}{x^2} - \frac{abd \cos(c+dx)}{x} + b^2 d \cos(c) \text{CosIntegral}(dx) - b^2 d \sin(c) \text{Si}(dx) - \frac{b^2 \sin(c+dx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] -1/6*(a^2*d*Cos[c + d*x])/x^2 - (a*b*d*Cos[c + d*x])/x + b^2*d*Cos[c]*CosIn
tegral[d*x] - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 - a*b*d^2*CosIntegral[d*x
]*Sin[c] - (a^2*Sin[c + d*x])/(3*x^3) - (a*b*Sin[c + d*x])/x^2 - (b^2*Sin[c
 + d*x])/x + (a^2*d^2*Sin[c + d*x])/(6*x) - a*b*d^2*Cos[c]*SinIntegral[d*x]
 - b^2*d*Sin[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2\sin(c+dx)}{x^4}\,dx &= \int \left(\frac{a^2\sin(c+dx)}{x^4}+\frac{2ab\sin(c+dx)}{x^3}+\frac{b^2\sin(c+dx)}{x^2}\right)dx \\
&= a^2\int \frac{\sin(c+dx)}{x^4}\,dx+(2ab)\int \frac{\sin(c+dx)}{x^3}\,dx+b^2\int \frac{\sin(c+dx)}{x^2}\,dx \\
&= -\frac{a^2\sin(c+dx)}{3x^3}-\frac{ab\sin(c+dx)}{x^2}-\frac{b^2\sin(c+dx)}{x}+\frac{1}{3}\left(a^2d\right)\int \frac{\cos(c+dx)}{x^3}\,dx \\
&= -\frac{a^2d\cos(c+dx)}{6x^2}-\frac{abd\cos(c+dx)}{x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{ab\sin(c+dx)}{x^2}-\frac{b^2\sin \\
&= -\frac{a^2d\cos(c+dx)}{6x^2}-\frac{abd\cos(c+dx)}{x}+b^2d\cos(c)\mathrm{Ci}(dx)-\frac{a^2\sin(c+dx)}{3x^3}-\frac{ab \\
&= -\frac{a^2d\cos(c+dx)}{6x^2}-\frac{abd\cos(c+dx)}{x}+b^2d\cos(c)\mathrm{Ci}(dx)-abd^2\mathrm{Ci}(dx)\sin(c)- \\
&= -\frac{a^2d\cos(c+dx)}{6x^2}-\frac{abd\cos(c+dx)}{x}+b^2d\cos(c)\mathrm{Ci}(dx)-\frac{1}{6}a^2d^3\cos(c)\mathrm{Ci}(dx)
\end{aligned}$$

Mathematica [A]
time = 0.32, size = 154, normalized size = 0.88

$$-\frac{a^2dx\cos(c+dx)+6abdx^2\cos(c+dx)+dx^3\mathrm{Ci}(dx)\left(\left(-6b^2+a^2d^2\right)\cos(c)+6abd\sin(c)\right)+2a^2\sin(c+dx)+6abx\sin(c+dx)+6b^2x^2\sin(c+dx)-a^2d^2x^2\sin(c+dx)+dx^3\left(6abd\cos(c)+6b^2\sin(c)-a^2d^2\sin(c)\right)\mathrm{Si}(dx)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] -1/6*(a^2*d*x*Cos[c + d*x] + 6*a*b*d*x^2*Cos[c + d*x] + d*x^3*CosIntegral[d
*x]*((-6*b^2 + a^2*d^2)*Cos[c] + 6*a*b*d*Sin[c]) + 2*a^2*Sin[c + d*x] + 6*a
*b*x*Sin[c + d*x] + 6*b^2*x^2*Sin[c + d*x] - a^2*d^2*x^2*Sin[c + d*x] + d*x
^3*(6*a*b*d*Cos[c] + 6*b^2*Sin[c] - a^2*d^2*Sin[c])*SinIntegral[d*x])/x^3
```

Maple [A]
time = 0.12, size = 158, normalized size = 0.90 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(a^2*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x
+1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+2/d*a*b*(-1/2*sin(d*x+c)/d^2/x^2-1/
2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c))+1/d^2*b^2*(-sin(d*x
+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 1.42, size = 187, normalized size = 1.07

$$-\frac{\left(\left(a^{2}(\Gamma(-3, i\,dx)+\Gamma(-3,-i\,dx))\cos(c)+a^{2}(-i\,\Gamma(-3, i\,dx)+i\,\Gamma(-3,-i\,dx))\sin(c)\right)d^{5}-6\left(ab(-i\,\Gamma(-3, i\,dx)+i\,\Gamma(-3,-i\,dx))\cos(c)-ab(\Gamma(-3, i\,dx)+\Gamma(-3,-i\,dx))\sin(c)\right)d^{4}-6\left(b^{2}(\Gamma(-3, i\,dx)+\Gamma(-3,-i\,dx))\cos(c)+b^{2}(-i\,\Gamma(-3, i\,dx)+i\,\Gamma(-3,-i\,dx))\sin(c)\right)d^{3}\right)x^{3}+4\,b^{2}\sin(dx+c)+2\left(b^{2}dx+2\,abd\right)\cos(dx+c)}{2\,d^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a^2*(-I*gamma(-
3, I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^5 - 6*(a*b*(-I*gamma(-3, I*d*x)
+ I*gamma(-3, -I*d*x))*cos(c) - a*b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*
sin(c))*d^4 - 6*(b^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + b^2*(-
I*gamma(-3, I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^3)*x^3 + 4*b^2*sin(d*x
+ c) + 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/(d^2*x^3)
```

Fricas [A]
time = 0.36, size = 186, normalized size = 1.06

$$-\frac{2\left(6\,abdx^{2}+a^{2}dx\right)\cos(dx+c)+\left(12\,abd^{2}x^{3}\operatorname{Si}(dx)+(a^{2}d^{3}-6\,b^{2}d)x^{3}\operatorname{Ci}(dx)+(a^{2}d^{3}-6\,b^{2}d)x^{3}\operatorname{Ci}(-dx)\right)\cos(c)+2\left(6\,abx-(a^{2}d^{2}-6\,b^{2})x^{2}+2\,a^{2}\right)\sin(dx+c)+2\left(3\,abd^{2}x^{3}\operatorname{Ci}(dx)+3\,abd^{2}x^{3}\operatorname{Ci}(-dx)-(a^{2}d^{3}-6\,b^{2}d)x^{3}\operatorname{Si}(dx)\right)\sin(c)}{12\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(6*a*b*d*x^2 + a^2*d*x)*cos(d*x + c) + (12*a*b*d^2*x^3*sin_integra
l(d*x) + (a^2*d^3 - 6*b^2*d)*x^3*cos_integral(d*x) + (a^2*d^3 - 6*b^2*d)*x^
3*cos_integral(-d*x))*cos(c) + 2*(6*a*b*x - (a^2*d^2 - 6*b^2)*x^2 + 2*a^2)*
sin(d*x + c) + 2*(3*a*b*d^2*x^3*cos_integral(d*x) + 3*a*b*d^2*x^3*cos_integ
ral(-d*x) - (a^2*d^3 - 6*b^2*d)*x^3*sin_integral(d*x))*sin(c))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{2}\sin(c+dx)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x)**2*sin(c + d*x)/x**4, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 6.54, size = 1400, normalized size = 8.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a^2*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ a^2*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*a^2*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2
*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^
3*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 6*a*b*d^2*x^3*imag_part
(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a*b*d^2*x^3*imag_part(c
os_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a*b*d^2*x^3*sin_integra
l(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^3*x^3*real_part(cos_integral(d*x
))*tan(1/2*d*x)^2 - a^2*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^
2 - 12*a*b*d^2*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) -
 12*a*b*d^2*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a
^2*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a^2*d^3*x^3*real_par
t(cos_integral(-d*x))*tan(1/2*c)^2 - 6*b^2*d*x^3*real_part(cos_integral(d*x
))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*b^2*d*x^3*real_part(cos_integral(-d*x))*
tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a*b*d^2*x^3*imag_part(cos_integral(d*x))*ta
n(1/2*d*x)^2 + 6*a*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 -
 12*a*b*d^2*x^3*sin_integral(d*x)*tan(1/2*d*x)^2 + 2*a^2*d^3*x^3*imag_part(
cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^3*x^3*imag_part(cos_integral(-d*x))
*tan(1/2*c) + 4*a^2*d^3*x^3*sin_integral(d*x)*tan(1/2*c) - 12*b^2*d*x^3*ima
g_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 12*b^2*d*x^3*imag_par
t(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 24*b^2*d*x^3*sin_integral
(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 6*a*b*d^2*x^3*imag_part(cos_integral(d*x)
)*tan(1/2*c)^2 - 6*a*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 +
 12*a*b*d^2*x^3*sin_integral(d*x)*tan(1/2*c)^2 - a^2*d^3*x^3*real_part(cos_
integral(d*x)) - a^2*d^3*x^3*real_part(cos_integral(-d*x)) + 6*b^2*d*x^3*re
al_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 6*b^2*d*x^3*real_part(cos_integ
ral(-d*x))*tan(1/2*d*x)^2 - 12*a*b*d^2*x^3*real_part(cos_integral(d*x))*tan
(1/2*c) - 12*a*b*d^2*x^3*real_part(cos_integral(-d*x))*tan(1/2*c) - 4*a^2*d
^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) - 6*b^2*d*x^3*real_part(cos_integral(d*x))
*tan(1/2*c)^2 - 6*b^2*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*
a^2*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2 - 12*a*b*d*x^2*tan(1/2*d*x)^2*tan(1/2
*c)^2 - 6*a*b*d^2*x^3*imag_part(cos_integral(d*x)) + 6*a*b*d^2*x^3*imag_par
t(cos_integral(-d*x)) - 12*a*b*d^2*x^3*sin_integral(d*x) - 12*b^2*d*x^3*ima
g_part(cos_integral(d*x))*tan(1/2*c) + 12*b^2*d*x^3*imag_part(cos_integral(
```

```
-d*x))*tan(1/2*c) - 24*b^2*d*x^3*sin_integral(d*x)*tan(1/2*c) - 2*a^2*d*x*t
an(1/2*d*x)^2*tan(1/2*c)^2 + 6*b^2*d*x^3*real_part(cos_integral(d*x)) + 6*b
^2*d*x^3*real_part(cos_integral(-d*x)) + 4*a^2*d^2*x^2*tan(1/2*d*x) + 12*a*
b*d*x^2*tan(1/2*d*x)^2 + 4*a^2*d^2*x^2*tan(1/2*c) + 48*a*b*d*x^2*tan(1/2*d*
x)*tan(1/2*c) + 24*b^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 12*a*b*d*x^2*tan(1/2
*c)^2 + 24*b^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a^2*d*x*tan(1/2*d*x)^2 + 8
*a^2*d*x*tan(1/2*d*x)*tan(1/2*c) + 24*a*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 2*a
^2*d*x*tan(1/2*c)^2 + 24*a*b*x*tan(1/2*d*x)*tan(1/2*c)^2 - 12*a*b*d*x^2 - 2
4*b^2*x^2*tan(1/2*d*x) - 24*b^2*x^2*tan(1/2*c) + 8*a^2*tan(1/2*d*x)^2*tan(1
/2*c) + 8*a^2*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a^2*d*x - 24*a*b*x*tan(1/2*d*x)
 - 24*a*b*x*tan(1/2*c) - 8*a^2*tan(1/2*d*x) - 8*a^2*tan(1/2*c))/(x^3*tan(1/
2*d*x)^2*tan(1/2*c)^2 + x^3*tan(1/2*d*x)^2 + x^3*tan(1/2*c)^2 + x^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (c+d\,x)\ (a+b\,x)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x)^2)/x^4,x)
[Out] int((sin(c + d*x)*(a + b*x)^2)/x^4, x)
```

3.17 $\int \frac{(a+bx)^2 \sin(c+dx)}{x^5} \, dx$

Optimal. Leaf size=248

$$-\frac{a^2 d \cos(c+dx)}{12x^3} - \frac{abd \cos(c+dx)}{3x^2} - \frac{b^2 d \cos(c+dx)}{2x} + \frac{a^2 d^3 \cos(c+dx)}{24x} - \frac{1}{3} abd^3 \cos(c) \text{Ci}(dx) - \frac{1}{2} b^2 d^2 \text{Ci}(dx) \text{si}$$

```
[Out] -1/3*a*b*d^3*Ci(d*x)*cos(c)-1/12*a^2*d*cos(d*x+c)/x^3-1/3*a*b*d*cos(d*x+c)/
x^2-1/2*b^2*d*cos(d*x+c)/x+1/24*a^2*d^3*cos(d*x+c)/x-1/2*b^2*d^2*cos(c)*Si(
d*x)+1/24*a^2*d^4*cos(c)*Si(d*x)-1/2*b^2*d^2*Ci(d*x)*sin(c)+1/24*a^2*d^4*Ci
(d*x)*sin(c)+1/3*a*b*d^3*Si(d*x)*sin(c)-1/4*a^2*sin(d*x+c)/x^4-2/3*a*b*sin(
d*x+c)/x^3-1/2*b^2*sin(d*x+c)/x^2+1/24*a^2*d^2*sin(d*x+c)/x^2+1/3*a*b*d^2*s
in(d*x+c)/x
```

Rubi [A]
time = 0.32, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\frac{1}{24} a^2 d^4 \sin(c) \text{CosIntegral}(dx) + \frac{1}{24} a^2 d^4 \cos(c) \text{Si}(dx) + \frac{a^2 d^3 \cos(c+dx)}{24x} + \frac{a^2 d^2 \sin(c+dx)}{24x^2} - \frac{a^2 \sin(c+dx)}{4x^4} - \frac{a^2 d \cos(c+dx)}{12x^3} - \frac{1}{3} abd^3 \cos(c) \text{CosIntegral}(dx) + \frac{1}{3} abd^3 \sin(c) \text{Si}(dx) + \frac{abd^2 \sin(c+dx)}{3x} - \frac{2ab \sin(c+dx)}{3x^3} - \frac{abd \cos(c+dx)}{3x^2} - \frac{1}{2} b^2 d^2 \sin(c) \text{CosIntegral}(dx) - \frac{1}{2} b^2 d^2 \cos(c) \text{Si}(dx) - \frac{b^2 \sin(c+dx)}{2x^2} - \frac{b^2 d \cos(c+dx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] -1/12*(a^2*d*Cos[c + d*x])/x^3 - (a*b*d*Cos[c + d*x])/(3*x^2) - (b^2*d*Cos[
c + d*x])/(2*x) + (a^2*d^3*Cos[c + d*x])/(24*x) - (a*b*d^3*Cos[c]*CosIntegr
al[d*x])/3 - (b^2*d^2*CosIntegral[d*x]*Sin[c])/2 + (a^2*d^4*CosIntegral[d*x
]*Sin[c])/24 - (a^2*Sin[c + d*x])/(4*x^4) - (2*a*b*Sin[c + d*x])/(3*x^3) -
(b^2*Sin[c + d*x])/(2*x^2) + (a^2*d^2*Sin[c + d*x])/(24*x^2) + (a*b*d^2*Sin
[c + d*x])/(3*x) - (b^2*d^2*Cos[c]*SinIntegral[d*x])/2 + (a^2*d^4*Cos[c]*Si
nIntegral[d*x])/24 + (a*b*d^3*Sin[c]*SinIntegral[d*x])/3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx &= \int \left(\frac{a^2\sin(c+dx)}{x^5}+\frac{2ab\sin(c+dx)}{x^4}+\frac{b^2\sin(c+dx)}{x^3}\right)dx \\
&= a^2\int \frac{\sin(c+dx)}{x^5}\,dx+(2ab)\int \frac{\sin(c+dx)}{x^4}\,dx+b^2\int \frac{\sin(c+dx)}{x^3}\,dx \\
&= -\frac{a^2\sin(c+dx)}{4x^4}-\frac{2ab\sin(c+dx)}{3x^3}-\frac{b^2\sin(c+dx)}{2x^2}+\frac{1}{4}\left(a^2d\right)\int \frac{\cos(c+dx)}{x^4} \\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{3x^2}-\frac{b^2d\cos(c+dx)}{2x}-\frac{a^2\sin(c+dx)}{4x^4}-2a \\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{3x^2}-\frac{b^2d\cos(c+dx)}{2x}-\frac{a^2\sin(c+dx)}{4x^4}-2a \\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{3x^2}-\frac{b^2d\cos(c+dx)}{2x}+\frac{a^2d^3\cos(c+dx)}{24x}- \\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{3x^2}-\frac{b^2d\cos(c+dx)}{2x}+\frac{a^2d^3\cos(c+dx)}{24x}- \\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{3x^2}-\frac{b^2d\cos(c+dx)}{2x}+\frac{a^2d^3\cos(c+dx)}{24x}-
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 204, normalized size = 0.82

$$\frac{-2a^2dx\cos(c+dx)-8abdx^2\cos(c+dx)-12b^2dx^3\cos(c+dx)+a^2d^3x^3\cos(c+dx)+d^2x^4\operatorname{Ci}(dx)\left(-8abd\cos(c)+\left(-12b^2+a^2d^2\right)\sin(c)\right)-6a^2\sin(c+dx)-16abx\sin(c+dx)-12b^2x^2\sin(c+dx)+a^2d^2x^2\sin(c+dx)+8abd^2x^3\sin(c+dx)+d^2x^4\left(-12b^2\cos(c)+a^2d^2\cos(c)+8abd\sin(c)\right)\operatorname{Si}(dx)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] (-2*a^2*d*x*Cos[c + d*x] - 8*a*b*d*x^2*Cos[c + d*x] - 12*b^2*d*x^3*Cos[c +
d*x] + a^2*d^3*x^3*Cos[c + d*x] + d^2*x^4*CosIntegral[d*x]*(-8*a*b*d*Cos[c]
 + (-12*b^2 + a^2*d^2)*Sin[c]) - 6*a^2*Sin[c + d*x] - 16*a*b*x*Sin[c + d*x]
 - 12*b^2*x^2*Sin[c + d*x] + a^2*d^2*x^2*Sin[c + d*x] + 8*a*b*d^2*x^3*Sin[c
 + d*x] + d^2*x^4*(-12*b^2*Cos[c] + a^2*d^2*Cos[c] + 8*a*b*d*Sin[c])*SinInt
egral[d*x])/(24*x^4)
```

Maple [A]
time = 0.14, size = 201, normalized size = 0.81 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] d^4*(a^2*(-1/4*sin(d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/d
^2/x^2+1/24*cos(d*x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c))+2/d*a*b
*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+1/6*Si(
d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+1/d^2*b^2*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(
d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 1.58, size = 186, normalized size = 0.75

$$-\frac{\left(\left(a^{2}\left(i\,\Gamma\left(-4,i\,dx\right)-i\,\Gamma\left(-4,-i\,dx\right)\right)\cos\left(c\right)+a^{2}\left(\Gamma\left(-4,i\,dx\right)+\Gamma\left(-4,-i\,dx\right)\right)\sin\left(c\right)\right)d^{6}-8\left(ab\left(\Gamma\left(-4,i\,dx\right)+\Gamma\left(-4,-i\,dx\right)\right)\cos\left(c\right)+ab\left(-i\,\Gamma\left(-4,i\,dx\right)+i\,\Gamma\left(-4,-i\,dx\right)\right)\sin\left(c\right)\right)d^{5}-12\left(b^{2}\left(i\,\Gamma\left(-4,i\,dx\right)-i\,\Gamma\left(-4,-i\,dx\right)\right)\cos\left(c\right)+b^{2}\left(\Gamma\left(-4,i\,dx\right)+\Gamma\left(-4,-i\,dx\right)\right)\sin\left(c\right)\right)d^{4}\right)x^{4}+6\,b^{2}\sin\left(dx+c\right)+2\left(b^{2}dx+2\,abd\right)\cos\left(dx+c\right)}{2\,d^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a^2*(gamma(
-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^6 - 8*(a*b*(gamma(-4, I*d*x) + ga
mma(-4, -I*d*x))*cos(c) + a*b*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x))*s
in(c))*d^5 - 12*(b^2*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + b^
2*(gamma(-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^4)*x^4 + 6*b^2*sin(d*x +
 c) + 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/(d^2*x^4)
```

Fricas [A]
time = 0.36, size = 222, normalized size = 0.90

$$-\frac{2\left(8\,abdx^{2}+2\,a^{2}dx-\left(a^{2}d^{3}-12\,b^{2}d\right)x^{3}\right)\cos\left(dx+c\right)+2\left(4\,abd^{3}x^{4}\operatorname{Ci}\left(dx\right)+4\,abd^{3}x^{4}\operatorname{Ci}\left(-dx\right)-\left(a^{2}d^{4}-12\,b^{2}d^{2}\right)x^{4}\operatorname{Si}\left(dx\right)\right)\cos\left(c\right)-2\left(8\,abd^{2}x^{3}-16\,abx+\left(a^{2}d^{2}-12\,b^{2}\right)x^{2}-6\,a^{2}\right)\sin\left(dx+c\right)-\left(16\,abd^{3}x^{4}\operatorname{Si}\left(dx\right)+\left(a^{2}d^{4}-12\,b^{2}d^{2}\right)x^{4}\operatorname{Ci}\left(dx\right)+\left(a^{2}d^{4}-12\,b^{2}d^{2}\right)x^{4}\operatorname{Ci}\left(-dx\right)\right)\sin\left(c\right)}{48\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*(8*a*b*d*x^2 + 2*a^2*d*x - (a^2*d^3 - 12*b^2*d)*x^3)*cos(d*x + c)
+ 2*(4*a*b*d^3*x^4*cos_integral(d*x) + 4*a*b*d^3*x^4*cos_integral(-d*x) - (
a^2*d^4 - 12*b^2*d^2)*x^4*sin_integral(d*x))*cos(c) - 2*(8*a*b*d^2*x^3 - 16
```

```
*a*b*x + (a^2*d^2 - 12*b^2)*x^2 - 6*a^2)*sin(d*x + c) - (16*a*b*d^3*x^4*sin
_integral(d*x) + (a^2*d^4 - 12*b^2*d^2)*x^4*cos_integral(d*x) + (a^2*d^4 -
12*b^2*d^2)*x^4*cos_integral(-d*x))*sin(c))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*sin(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x)**2*sin(c + d*x)/x**5, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.04, size = 1712, normalized size = 6.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(a^2*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
 - a^2*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^4*x^4
*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x^4*rea
l_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 8*a*b*d^3*x^4*real_p
art(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 8*a*b*d^3*x^4*real_par
t(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x^4*imag_part(c
os_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^4*x^4*imag_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2 - 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 - 16*a*b*
d^3*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 16*a*b*d^3
*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 32*a*b*d^3*x
^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^4*x^4*imag_part(cos_
integral(d*x))*tan(1/2*c)^2 - a^2*d^4*x^4*imag_part(cos_integral(-d*x))*tan
(1/2*c)^2 + 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*c)^2 - 12*b^2*d^2*x^4*i
mag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*b^2*d^2*x^4*im
ag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*b^2*d^2*x^4*si
n_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a*b*d^3*x^4*real_part(cos_i
ntegral(d*x))*tan(1/2*d*x)^2 + 8*a*b*d^3*x^4*real_part(cos_integral(-d*x))*
tan(1/2*d*x)^2 - 2*a^2*d^4*x^4*real_part(cos_integral(d*x))*tan(1/2*c) - 2*
a^2*d^4*x^4*real_part(cos_integral(-d*x))*tan(1/2*c) + 24*b^2*d^2*x^4*real_
part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 24*b^2*d^2*x^4*real_par
t(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 8*a*b*d^3*x^4*real_part(c
os_integral(d*x))*tan(1/2*c)^2 - 8*a*b*d^3*x^4*real_part(cos_integral(-d*x)
```

```
)*tan(1/2*c)^2 - 2*a^2*d^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x^4*im
ag_part(cos_integral(d*x)) + a^2*d^4*x^4*imag_part(cos_integral(-d*x)) - 2*
a^2*d^4*x^4*sin_integral(d*x) + 12*b^2*d^2*x^4*imag_part(cos_integral(d*x))
*tan(1/2*d*x)^2 - 12*b^2*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)
^2 + 24*b^2*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 - 16*a*b*d^3*x^4*imag_
part(cos_integral(d*x))*tan(1/2*c) + 16*a*b*d^3*x^4*imag_part(cos_integral(
-d*x))*tan(1/2*c) - 32*a*b*d^3*x^4*sin_integral(d*x)*tan(1/2*c) - 12*b^2*d^
2*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 12*b^2*d^2*x^4*imag_part(
cos_integral(-d*x))*tan(1/2*c)^2 - 24*b^2*d^2*x^4*sin_integral(d*x)*tan(1/2
*c)^2 + 8*a*b*d^3*x^4*real_part(cos_integral(d*x)) + 8*a*b*d^3*x^4*real_par
t(cos_integral(-d*x)) + 2*a^2*d^3*x^3*tan(1/2*d*x)^2 + 24*b^2*d^2*x^4*real_
part(cos_integral(d*x))*tan(1/2*c) + 24*b^2*d^2*x^4*real_part(cos_integral(
-d*x))*tan(1/2*c) + 8*a^2*d^3*x^3*tan(1/2*d*x)*tan(1/2*c) + 32*a*b*d^2*x^3*
tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d^3*x^3*tan(1/2*c)^2 + 32*a*b*d^2*x^3*tan
(1/2*d*x)*tan(1/2*c)^2 + 24*b^2*d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*b^2*
d^2*x^4*imag_part(cos_integral(d*x)) - 12*b^2*d^2*x^4*imag_part(cos_integra
l(-d*x)) + 24*b^2*d^2*x^4*sin_integral(d*x) + 4*a^2*d^2*x^2*tan(1/2*d*x)^2*
tan(1/2*c) + 4*a^2*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 16*a*b*d*x^2*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^3*x^3 - 32*a*b*d^2*x^3*tan(1/2*d*x) - 24*b^2
*d*x^3*tan(1/2*d*x)^2 - 32*a*b*d^2*x^3*tan(1/2*c) - 96*b^2*d*x^3*tan(1/2*d*
x)*tan(1/2*c) - 24*b^2*d*x^3*tan(1/2*c)^2 + 4*a^2*d*x*tan(1/2*d*x)^2*tan(1/
2*c)^2 - 4*a^2*d^2*x^2*tan(1/2*d*x) - 16*a*b*d*x^2*tan(1/2*d*x)^2 - 4*a^2*d
^2*x^2*tan(1/2*c) - 64*a*b*d*x^2*tan(1/2*d*x)*tan(1/2*c) - 48*b^2*x^2*tan(1
/2*d*x)^2*tan(1/2*c) - 16*a*b*d*x^2*tan(1/2*c)^2 - 48*b^2*x^2*tan(1/2*d*x)*
tan(1/2*c)^2 + 24*b^2*d*x^3 - 4*a^2*d*x*tan(1/2*d*x)^2 - 16*a^2*d*x*tan(1/2
*d*x)*tan(1/2*c) - 64*a*b*x*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*d*x*tan(1/2*c
)^2 - 64*a*b*x*tan(1/2*d*x)*tan(1/2*c)^2 + 16*a*b*d*x^2 + 48*b^2*x^2*tan(1/
2*d*x) + 48*b^2*x^2*tan(1/2*c) - 24*a^2*tan(1/2*d*x)^2*tan(1/2*c) - 24*a^2*
tan(1/2*d*x)*tan(1/2*c)^2 + 4*a^2*d*x + 64*a*b*x*tan(1/2*d*x) + 64*a*b*x*ta
n(1/2*c) + 24*a^2*tan(1/2*d*x) + 24*a^2*tan(1/2*c))/(x^4*tan(1/2*d*x)^2*tan
(1/2*c)^2 + x^4*tan(1/2*d*x)^2 + x^4*tan(1/2*c)^2 + x^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin \left(c+d\,x\right)\,{\left(a+b\,x\right)}^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x)^2)/x^5,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x)^2)/x^5, x)
```

3.18 $\int \frac{x^4 \sin(c+dx)}{a+bx} \, dx$

Optimal. Leaf size=218

$$-\frac{2a\cos(c+dx)}{b^2d^3}+\frac{a^3\cos(c+dx)}{b^4d}+\frac{6x\cos(c+dx)}{bd^3}-\frac{a^2x\cos(c+dx)}{b^3d}+\frac{ax^2\cos(c+dx)}{b^2d}-\frac{x^3\cos(c+dx)}{bd}+\frac{a^4\mathrm{C}$$

```
[Out] -2*a*cos(d*x+c)/b^2/d^3+a^3*cos(d*x+c)/b^4/d+6*x*cos(d*x+c)/b/d^3-a^2*x*cos
(d*x+c)/b^3/d+a*x^2*cos(d*x+c)/b^2/d-x^3*cos(d*x+c)/b/d+a^4*cos(-c+a*d/b)*S
i(a*d/b+d*x)/b^5-a^4*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^5-6*sin(d*x+c)/b/d^4+a^2
*sin(d*x+c)/b^3/d^2-2*a*x*sin(d*x+c)/b^2/d^2+3*x^2*sin(d*x+c)/b/d^2
```

Rubi [A]
time = 0.32, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6874, 2718, 3377, 2717, 3384, 3380, 3383}

$$\frac{a^4\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5}+\frac{a^4\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{a^3\cos(c+dx)}{b^4d}+\frac{a^2\sin(c+dx)}{b^3d^2}-\frac{a^2x\cos(c+dx)}{b^3d}-\frac{2a\cos(c+dx)}{b^2d^3}-\frac{2ax\sin(c+dx)}{b^2d^2}+\frac{ax^2\cos(c+dx)}{b^2d}-\frac{6\sin(c+dx)}{bd^4}+\frac{6x\cos(c+dx)}{bd^3}+\frac{3x^2\sin(c+dx)}{bd^2}-\frac{x^3\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] (-2*a*Cos[c + d*x])/(b^2*d^3) + (a^3*Cos[c + d*x])/(b^4*d) + (6*x*Cos[c + d
*x])/(b*d^3) - (a^2*x*Cos[c + d*x])/(b^3*d) + (a*x^2*Cos[c + d*x])/(b^2*d)
- (x^3*Cos[c + d*x])/(b*d) + (a^4*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/
b])/b^5 - (6*Sin[c + d*x])/(b*d^4) + (a^2*Sin[c + d*x])/(b^3*d^2) - (2*a*x*
Sin[c + d*x])/(b^2*d^2) + (3*x^2*Sin[c + d*x])/(b*d^2) + (a^4*Cos[c - (a*d)
/b]*SinIntegral[(a*d)/b + d*x])/b^5
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^4\sin(c+dx)}{a+bx}\,dx = \int \left(-\frac{a^3\sin(c+dx)}{b^4}+\frac{a^2x\sin(c+dx)}{b^3}-\frac{ax^2\sin(c+dx)}{b^2}+\frac{x^3\sin(c+dx)}{b}+\frac{a^4\sin(c}{b^4(a-}\right.$$
$$= -\frac{a^3\int\sin(c+dx)\,dx}{b^4}+\frac{a^4\int\frac{\sin(c+dx)}{a+bx}\,dx}{b^4}+\frac{a^2\int x\sin(c+dx)\,dx}{b^3}-\frac{a\int x^2\sin(c+dx)\,c}{b^2}$$
$$= \frac{a^3\cos(c+dx)}{b^4d}-\frac{a^2x\cos(c+dx)}{b^3d}+\frac{ax^2\cos(c+dx)}{b^2d}-\frac{x^3\cos(c+dx)}{bd}+\frac{a^2\int\cos(c+}{b^3d}$$
$$= \frac{a^3\cos(c+dx)}{b^4d}-\frac{a^2x\cos(c+dx)}{b^3d}+\frac{ax^2\cos(c+dx)}{b^2d}-\frac{x^3\cos(c+dx)}{bd}+\frac{a^4\text{Ci}\left(\frac{ad}{b}+dx\right.}{}$$
$$= -\frac{2a\cos(c+dx)}{b^2d^3}+\frac{a^3\cos(c+dx)}{b^4d}+\frac{6x\cos(c+dx)}{bd^3}-\frac{a^2x\cos(c+dx)}{b^3d}+\frac{ax^2\cos(c+}{b^2d}$$
$$= -\frac{2a\cos(c+dx)}{b^2d^3}+\frac{a^3\cos(c+dx)}{b^4d}+\frac{6x\cos(c+dx)}{bd^3}-\frac{a^2x\cos(c+dx)}{b^3d}+\frac{ax^2\cos(c+}{b^2d}$$

Mathematica [A]
time = 0.42, size = 158, normalized size = 0.72

$$\frac{a^4d^4\text{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right)+b\left(d\left(a^3d^2-a^2bd^2x+b^3x\left(6-d^2x^2\right)+ab^2\left(-2+d^2x^2\right)\right)\cos(c+dx)+b\left(a^2d^2-2abd^2x+3b^2\left(-2+d^2x^2\right)\right)\sin(c+dx)\right)+a^4d^4\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{b^5d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] (a^4*d^4*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + b*(d*(a^3*d^2 - a^2*b*
d^2*x + b^3*x*(6 - d^2*x^2) + a*b^2*(-2 + d^2*x^2))*Cos[c + d*x] + b*(a^2*d
^2 - 2*a*b*d^2*x + 3*b^2*(-2 + d^2*x^2))*Sin[c + d*x]) + a^4*d^4*Cos[c - (a
*d)/b]*SinIntegral[d*(a/b + x)])/(b^5*d^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 783 vs. $2(221) = 442$.
time = 0.23, size = 784, normalized size = 3.60 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^5*(d*c^4*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b
)*sin((a*d-b*c)/b)/b)+4*(a*d-b*c)*d*c^3/b*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b
*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+4*d*c^3/b*cos(d*x+c)+6*(
a^2*d^2-2*a*b*c*d+b^2*c^2)*d*c^2/b^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b
)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-6*d*c^2*(a*d-b*c-b)/b^2*(sin(
d*x+c)-(d*x+c)*cos(d*x+c))+4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*
d*c/b^3*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin
((a*d-b*c)/b)/b)-4*d*c*(a^2*d^2-2*a*b*c*d+b^2*c^2-a*b*d+b^2*c+b^2)/b^3*(-(d
*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+(a^4*d^4-4*a^3*b*c*d^
3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*d/b^4*(Si(d*x+c+(a*d-b*c)/b)*cos
((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-d*(a^3*d^3-3*a^2*
b*c*d^2+3*a*b^2*c^2*d-b^3*c^3-a^2*b*d^2+2*a*b^2*c*d-b^3*c^2+a*b^2*d-b^3*c-b
^3)/b^4*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c
)*cos(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(((6*a*b^2*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_
e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + 6*a*b^2*(I*exp_integral_e(2, (I*b*d*
x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2 - (a^3*
(-I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x
+ I*a*d)/b))*cos(c)^2 + a^3*(-I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) + I*
exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d^2 - 4*(a^2*b*(exp_inte
gral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*c
os(c)^2 + a^2*b*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2,
 -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) - (6*a*b^2*(exp_in
tegral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))
```

```
*cos(c)^2 + 6*a*b^2*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_
e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2 + (a^3*(exp_integral_e(2, (I*b*d*x + I
*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^3*(exp_int
egral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*
sin(c)^2)*d^2 - 4*(a^2*b*(-I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) + I*exp
_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^2*b*(-I*exp_integral_e(2
, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^
2)*d)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((6*a*b^2*(I*exp_integral_e(2,
(I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2
+ 6*a*b^2*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -
(I*b*d*x + I*a*d)/b))*sin(c)^2 - (a^3*(-I*exp_integral_e(2, (I*b*d*x + I*a*
d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^3*(-I*exp_i
ntegral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/
b))*sin(c)^2)*d^2 - 4*(a^2*b*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_
integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^2*b*(exp_integral_e(2, (I
*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*
cos(-(b*c - a*d)/b) - (6*a*b^2*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + ex
p_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + 6*a*b^2*(exp_integral_e(2
, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2
+ (a^3*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*
x + I*a*d)/b))*cos(c)^2 + a^3*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp
_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d^2 - 4*(a^2*b*(-I*exp_inte
gral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))
*cos(c)^2 + a^2*b*(-I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integr
al_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*sin(-(b*c - a*d)/b))*sin(d*x +
c)^2 + ((b^3*d^3*x^4*cos(c) + 3*b^3*d^2*x^3*sin(c) + (a*b^2*d^2*sin(c) - 6*
b^3*d*cos(c))*x^2 - (a^2*b*d^2*sin(c) + 4*a*b^2*d*cos(c) + 6*b^3*sin(c))*x)
*cos(d*x + c)^2 + (b^3*d^3*x^4*cos(c) + 3*b^3*d^2*x^3*sin(c) + (a*b^2*d^2*s
in(c) - 6*b^3*d*cos(c))*x^2 - (a^2*b*d^2*sin(c) + 4*a*b^2*d*cos(c) + 6*b^3*
sin(c))*x)*sin(d*x + c)^2)*cos(d*x + 2*c) + ((b^3*cos(c)^2 + b^3*sin(c)^2)*
d^3*x^4 - 6*(b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^2 - 4*(a*b^2*cos(c)^2 + a*b^2
*sin(c)^2)*d*x)*cos(d*x + c) - 2*(((a^4*b^3*cos(c)^2 + a^4*b^3*sin(c)^2)*d^
7 - 6*(a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^5 + ((a^3*b^4*cos(c)^2 + a^3*
b^4*sin(c)^2)*d^7 - 6*(a*b^6*cos(c)^2 + a*b^6*sin(c)^2)*d^5)*x)*cos(d*x + c
)^2 + ((a^4*b^3*cos(c)^2 + a^4*b^3*sin(c)^2)*d^7 - 6*(a^2*b^5*cos(c)^2 + a^
2*b^5*sin(c)^2)*d^5 + ((a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^7 - 6*(a*b^6
*cos(c)^2 + a*b^6*sin(c)^2)*d^5)*x)*sin(d*x + c)^2)*integrate(1/2*x*cos(d*x
 + c)/(b^4*d^4*x^2 + 2*a*b^3*d^4*x + a^2*b^2*d^4), x) - 2*(((a^4*b^3*cos(c)
^2 + a^4*b^3*sin(c)^2)*d^7 - 6*(a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^5 +
((a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^7 - 6*(a*b^6*cos(c)^2 + a*b^6*sin(
c)^2)*d^5)*x)*cos(d*x + c)^2 + ((a^4*b^3*cos(c)^2 + a^4*b^3*sin(c)^2)*d^7 -
 6*(a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^5 + ((a^3*b^4*cos(c)^2 + a^3*b^4
*sin(c)^2)*d^7 - 6*(a*b^6*cos(c)^2 + a*b^6*sin(c)^2)*d^5)*x)*sin(d*x + c)^2
)*integrate(1/2*x*cos(d*x + c)/((b^4*d^4*x^2 + 2*a*b^3*d^4*x + a^2*b^2*d^4)
*cos(d*x + c)^2 + (b^4*d^4*x^2 + 2*a*b^3*d^4*x + a^2*b^2*d^4)*sin(d*x + c)^
```

```
2), x) - 4*(((a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^6*x + (a^3*b^4*cos(c)^
2 + a^3*b^4*sin(c)^2)*d^6)*cos(d*x + c)^2 + ((a^2*b^5*cos(c)^2 + a^2*b^5*si
n(c)^2)*d^6*x + (a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^6)*sin(d*x + c)^2)*
integrate(1/2*x*sin(d*x + c)/(b^4*d^4*x^2 + 2*a*b^3*d^4*x + a^2*b^2*d^4), x
) - 4*(((a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^6*x + (a^3*b^4*cos(c)^2 + a
^3*b^4*sin(c)^2)*d^6)*cos(d*x + c)^2 + ((a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^
2)*d^6*x + (a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^6)*sin(d*x + c)^2)*integ
rate(1/2*x*sin(d*x + c)/((b^4*d^4*x^2 + 2*a*b^3*d^4*x + a^2*b^2*d^4)*cos(d*
x + c)^2 + (b^4*d^4*x^2 + 2*a*b^3*d^4*x + a^2*b^2*d^4)*sin(d*x + c)^2), x)
+ ((b^3*d^3*x^4*sin(c) - 3*b^3*d^2*x^3*cos(c) - (a*b^2*d^2*cos(c) + 6*b^3*d
*sin(c))*x^2 + (a^2*b*d^2*cos(c) - 4*a*b^2*d*sin(c) + 6*b^3*cos(c))*x)*cos(
d*x + c)^2 + (b^3*d^3*x^4*sin(c) - 3*b^3*d^2*x^...
```

Fricas [**A**]
time = 0.36, size = 213, normalized size = 0.98

$$\frac{2\,a^4 d^4 \cos\left(-\frac{bc-ad}{b}\right) \operatorname{Si}\left(\frac{bdx+ad}{b}\right) - 2\,(b^4d^3x^3 - ab^3d^3x^2 - a^3bd^3 + 2\,ab^3d + (a^2b^2d^3 - 6\,b^4d)x)\cos\left(dx+c\right) + 2\,(3\,b^4d^2x^2 - 2\,ab^3d^2x + a^2b^2d^2 - 6\,b^4)\sin\left(dx+c\right) - \left(a^4d^4 \operatorname{Ci}\left(\frac{bdx+ad}{b}\right) + a^4d^4 \operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\,b^5d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^4*d^4*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) - 2*(b^4*d
^3*x^3 - a*b^3*d^3*x^2 - a^3*b*d^3 + 2*a*b^3*d + (a^2*b^2*d^3 - 6*b^4*d)*x)
*cos(d*x + c) + 2*(3*b^4*d^2*x^2 - 2*a*b^3*d^2*x + a^2*b^2*d^2 - 6*b^4)*sin
(d*x + c) - (a^4*d^4*cos_integral((b*d*x + a*d)/b) + a^4*d^4*cos_integral(-
(b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^5*d^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sin\left(c+dx\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sin(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**4*sin(c + d*x)/(a + b*x), x)
```

Giac [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.54, size = 3337, normalized size = 15.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

[Out] 1/2*(2*b^4*d^3*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2
*a*b^3*d^3*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d
^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 - a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*d^4*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^4*d^3
*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 2*a^4*d^4*real_part(cos_integral
(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*d
^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^
2*tan(1/2*a*d/b) + 2*b^4*d^3*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 -
2*a^4*d^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*d^4*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^4*d^3*x^3*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^3*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*t
an(1/2*a*d/b)^2 - 2*a*b^3*d^3*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - a^4
*d^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)
^2 + a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*t
an(1/2*c)^2 - 2*a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^
2*tan(1/2*c)^2 + 4*a^4*d^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
 + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^4*d^4*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^4*d^4*s
in_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/
b) - 2*a*b^3*d^3*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 - a^4*d^4*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 +
a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*a*d/b)^2 - 2*a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*a*d/b)^2 + 2*a*b^3*d^3*x^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d^4
*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^4*d
^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*
a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 12*b^
4*d^2*x^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^3*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^4*d^3*x^3*tan(1/
2*d*x + 1/2*c)^2 + 2*a^4*d^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*c) + 2*a^4*d^4*real_part(cos_integral(-d*x - a*d/b))*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*b^4*d^3*x^3*tan(1/2*c)^2 + 2*a^2*b^2*
d^3*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^4*d^4*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b) - 2*a^4*d^4*real_part
(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*
d^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^
4*d^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2
*b^4*d^3*x^3*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^3*x*tan(1/2*d*x + 1/2*c)^2*tan(
1/2*a*d/b)^2 - 2*a^4*d^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*ta
n(1/2*a*d/b)^2 - 2*a^4*d^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)
*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^3*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a*b^3
*d^2*x*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*b^4*d*x*tan(

```
1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b^3*d^3*x^2*tan(1/2*
d*x + 1/2*c)^2 + a^4*d^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x +
 1/2*c)^2 - a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2
*c)^2 + 2*a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2 + 2*
a*b^3*d^3*x^2*tan(1/2*c)^2 - a^4*d^4*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*c)^2 + a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 -
2*a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 12*b^4*d^2*x^2*tan(1
/2*d*x + 1/2*c)*tan(1/2*c)^2 - 2*a^3*b*d^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c
)^2 + 4*a^4*d^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d
/b) - 4*a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*
d/b) + 8*a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) +
2*a*b^3*d^3*x^2*tan(1/2*a*d/b)^2 - a^4*d^4*imag_part(cos_integral(d*x + a*d
/b))*tan(1/2*a*d/b)^2 + a^4*d^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*a*d/b)^2 - 2*a^4*d^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 12
*b^4*d^2*x^2*tan(1/2*d*x + 1/2*c)*tan(1/2*a*d/b)^2 - 2*a^3*b*d^3*tan(1/2*d*
x + 1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d^3*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 4*a^2*b^2*d^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b^3
*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^4*d^3*x^3 + 2
*a^2*b^2*d^3*x*tan(1/2*d*x + 1/2*c)^2 + 2*a^4*d^4*real_part(cos_integral(d*
x + a*d/b))*tan(1/2*c) + 2*a^4*d^4*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*c) - 2*a^2*b^2*d^3*x*tan(1/2*c)^2 - 8*a*b^3*d^2*x*tan(1/2*d*x + 1/2*c
)*tan(1/2*c)^2 - 12*b^4*d*x*tan(1/2*d*x + 1/2*c...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sin\left(c+d\,x\right)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*sin(c + d*x))/(a + b*x),x)
```

```
[Out] int((x^4*sin(c + d*x))/(a + b*x), x)
```

3.19 $\int \frac{x^3 \sin(c+dx)}{a+bx}\,dx$

Optimal. Leaf size=152

$$\frac{2\cos(c+dx)}{bd^3}-\frac{a^2\cos(c+dx)}{b^3d}+\frac{ax\cos(c+dx)}{b^2d}-\frac{x^2\cos(c+dx)}{bd}-\frac{a^3\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4}-\frac{a\sin(c+dx)}{b^2d^2}+$$

```
[Out] 2*cos(d*x+c)/b/d^3-a^2*cos(d*x+c)/b^3/d+a*x*cos(d*x+c)/b^2/d-x^2*cos(d*x+c)
/b/d-a^3*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^4+a^3*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^
4-a*sin(d*x+c)/b^2/d^2+2*x*sin(d*x+c)/b/d^2
```

Rubi [A]
time = 0.22, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6874, 2718, 3377, 2717, 3384, 3380, 3383}

$$-\frac{a^3\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{a^3\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^2\cos(c+dx)}{b^3d}-\frac{a\sin(c+dx)}{b^2d^2}+\frac{ax\cos(c+dx)}{b^2d}+\frac{2\cos(c+dx)}{bd^3}+\frac{2x\sin(c+dx)}{bd^2}-\frac{x^2\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] (2*Cos[c + d*x])/(b*d^3) - (a^2*Cos[c + d*x])/(b^3*d) + (a*x*Cos[c + d*x])/
(b^2*d) - (x^2*Cos[c + d*x])/(b*d) - (a^3*CosIntegral[(a*d)/b + d*x]*Sin[c
- (a*d)/b])/b^4 - (a*Sin[c + d*x])/(b^2*d^2) + (2*x*Sin[c + d*x])/(b*d^2) -
 (a^3*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3\sin(c+dx)}{a+bx}\,dx = \int \left(\frac{a^2\sin(c+dx)}{b^3} - \frac{ax\sin(c+dx)}{b^2} + \frac{x^2\sin(c+dx)}{b} - \frac{a^3\sin(c+dx)}{b^3(a+bx)}\right)dx$$

$$= \frac{a^2\int\sin(c+dx)\,dx}{b^3} - \frac{a^3\int\frac{\sin(c+dx)}{a+bx}\,dx}{b^3} - \frac{a\int x\sin(c+dx)\,dx}{b^2} + \frac{\int x^2\sin(c+dx)\,dx}{b}$$

$$= -\frac{a^2\cos(c+dx)}{b^3d} + \frac{ax\cos(c+dx)}{b^2d} - \frac{x^2\cos(c+dx)}{bd} - \frac{a\int\cos(c+dx)\,dx}{b^2d} + \frac{2\int x\,\mathrm{co}}{}$$

$$= -\frac{a^2\cos(c+dx)}{b^3d} + \frac{ax\cos(c+dx)}{b^2d} - \frac{x^2\cos(c+dx)}{bd} - \frac{a^3\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} -$$

$$= \frac{2\cos(c+dx)}{bd^3} - \frac{a^2\cos(c+dx)}{b^3d} + \frac{ax\cos(c+dx)}{b^2d} - \frac{x^2\cos(c+dx)}{bd} - \frac{a^3\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{b^4}$$

Mathematica [A]
time = 0.35, size = 117, normalized size = 0.77

$$-\frac{a^3d^3\mathrm{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right)+b\left(\left(a^2d^2-abd^2x+b^2(-2+d^2x^2)\right)\cos(c+dx)+bd(a-2bx)\sin(c+dx)\right)+a^3d^3\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{b^4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] -((a^3*d^3*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + b*((a^2*d^2 - a*b*d^
2*x + b^2*(-2 + d^2*x^2))*Cos[c + d*x] + b*d*(a - 2*b*x)*Sin[c + d*x]) + a^
3*d^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(b^4*d^3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(153) = 306$.
time = 0.07, size = 516, normalized size = 3.39

method	result
derivativedivides	$\frac{-d\,c^3\left(\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)-\frac{3(da-cb)d\,c^2\left(\frac{\mathrm{sinIntegral}\left(dx+c+\frac{d}{}\right.}{b}\right.}{}}{}$
default	$\frac{-d\,c^3\left(\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)-\frac{3(da-cb)d\,c^2\left(\frac{\mathrm{sinIntegral}\left(dx+c+\frac{d}{}\right.}{b}\right.}{}}{}$
risch	$\frac{i\left(2ib^4d^4x^4-4ia\,b^3d^4x^3+6ib^4c\,d^3x^3+6ia^2b^2d^4x^2-12ia\,b^3c\,d^3x^2+6ib^4c^2d^2x^2-4ia^3b\,d^4x+12ia^2b^2c\,d^3x-6ia\,b^3c^2d^2x+2ia\right.}{2d^3\left(d^2x^2b^2-ab\,d^2x+3b^2cdx+d^2a^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-d*c^3*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/
b)*sin((a*d-b*c)/b)/b)-3*(a*d-b*c)*d*c^2/b*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-
b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-3*d*c^2/b*cos(d*x+c)-3*
(a^2*d^2-2*a*b*c*d+b^2*c^2)*d*c/b^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)
/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+3*(a*d-b*c-b)*d*c/b^2*(sin(d*x
+c)-(d*x+c)*cos(d*x+c))-(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*d/b^3
*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b
*c)/b)/b)+(a^2*d^2-2*a*b*c*d+b^2*c^2-a*b*d+b^2*c+b^2)*d/b^3*(-(d*x+c)^2*cos
(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(((2*a*b*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(
I*b*d*x + I*a*d)/b))*cos(c)^2 + 2*a*b*(exp_integral_e(2, (I*b*d*x + I*a*d)/
b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2 - (a^2*(-I*exp_integ
ral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*
cos(c)^2 + a^2*(-I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_
e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) + (2*a*b*(I*ex
p_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*
d)/b))*cos(c)^2 + 2*a*b*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_i
ntegral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2 - (a^2*(exp_integral_e(2, (I*b
```

```
*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^2*
(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a
*d)/b))*sin(c)^2)*d)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((2*a*b*(exp_int
egral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*
cos(c)^2 + 2*a*b*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2
, -(I*b*d*x + I*a*d)/b))*sin(c)^2 - (a^2*(-I*exp_integral_e(2, (I*b*d*x + I
*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^2*(-I*ex
p_integral_e(2, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(2, -(I*b*d*x + I*a*
d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) + (2*a*b*(I*exp_integral_e(2, (I*b*
d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + 2*a
*b*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*
x + I*a*d)/b))*sin(c)^2 - (a^2*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + ex
p_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^2*(exp_integral_e(2, (I
*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*
sin(-(b*c - a*d)/b))*sin(d*x + c)^2 - ((b^2*d^2*x^3*cos(c) + 2*b^2*d*x^2*si
n(c) + (a*b*d*sin(c) - 2*b^2*cos(c))*x)*cos(d*x + c)^2 + (b^2*d^2*x^3*cos(c
) + 2*b^2*d*x^2*sin(c) + (a*b*d*sin(c) - 2*b^2*cos(c))*x)*sin(d*x + c)^2)*c
os(d*x + 2*c) - ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^3 - 2*(b^2*cos(c)^2 +
b^2*sin(c)^2)*x)*cos(d*x + c) - 2*(((a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d
^5*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^5)*cos(d*x + c)^2 + ((a^2*b^
3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^5*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2
)*d^5)*sin(d*x + c)^2)*integrate(1/2*x*cos(d*x + c)/(b^3*d^3*x^2 + 2*a*b^2*
d^3*x + a^2*b*d^3), x) - 2*(((a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^5*x +
(a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^5)*cos(d*x + c)^2 + ((a^2*b^3*cos(c
)^2 + a^2*b^3*sin(c)^2)*d^5*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^5)*
sin(d*x + c)^2)*integrate(1/2*x*cos(d*x + c)/((b^3*d^3*x^2 + 2*a*b^2*d^3*x
+ a^2*b*d^3)*cos(d*x + c)^2 + (b^3*d^3*x^2 + 2*a*b^2*d^3*x + a^2*b*d^3)*sin
(d*x + c)^2), x) - 4*(((a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^4*x + (a^2*b^3*c
os(c)^2 + a^2*b^3*sin(c)^2)*d^4)*cos(d*x + c)^2 + ((a*b^4*cos(c)^2 + a*b^4*
sin(c)^2)*d^4*x + (a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^4)*sin(d*x + c)^2
)*integrate(1/2*x*sin(d*x + c)/(b^3*d^3*x^2 + 2*a*b^2*d^3*x + a^2*b*d^3), x
) - 4*(((a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^4*x + (a^2*b^3*cos(c)^2 + a^2*b
^3*sin(c)^2)*d^4)*cos(d*x + c)^2 + ((a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^4*x
 + (a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^4)*sin(d*x + c)^2)*integrate(1/2
*x*sin(d*x + c)/((b^3*d^3*x^2 + 2*a*b^2*d^3*x + a^2*b*d^3)*cos(d*x + c)^2 +
 (b^3*d^3*x^2 + 2*a*b^2*d^3*x + a^2*b*d^3)*sin(d*x + c)^2), x) - ((b^2*d^2*
x^3*sin(c) - 2*b^2*d*x^2*cos(c) - (a*b*d*cos(c) + 2*b^2*sin(c))*x)*cos(d*x
+ c)^2 + (b^2*d^2*x^3*sin(c) - 2*b^2*d*x^2*cos(c) - (a*b*d*cos(c) + 2*b^2*s
in(c))*x)*sin(d*x + c)^2)*sin(d*x + 2*c) + (2*(b^2*cos(c)^2 + b^2*sin(c)^2)
*d*x^2 + (a*b*cos(c)^2 + a*b*sin(c)^2)*d*x)*sin(d*x + c))/(((b^3*cos(c)^2 +
 b^3*sin(c)^2)*d^3*x + (a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3)*cos(d*x + c)^
2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x + (a*b^2*cos(c)^2 + a*b^2*sin(c)^2
)*d^3)*sin(d*x + c)^2)
```

Fricas [A]

time = 0.35, size = 167, normalized size = 1.10

$$-\frac{2\,a^3d^3\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)+2\left(b^3d^2x^2-ab^2d^2x+a^2bd^2-2\,b^3\right)\cos\left(dx+c\right)-2\left(2\,b^3dx-ab^2d\right)\sin\left(dx+c\right)-\left(a^3d^3\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+a^3d^3\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\,b^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*d^3*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + 2*(b^3*
d^2*x^2 - a*b^2*d^2*x + a^2*b*d^2 - 2*b^3)*cos(d*x + c) - 2*(2*b^3*d*x - a*
b^2*d)*sin(d*x + c) - (a^3*d^3*cos_integral((b*d*x + a*d)/b) + a^3*d^3*cos_
integral(-(b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^4*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sin\left(c+dx\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.55, size = 2709, normalized size = 17.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*d^2*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a
^3*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*d^3*sin_integral((b
*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3
*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)
^2*tan(1/2*a*d/b) - 2*a^3*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*d^3*real_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^
3*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*
c)*tan(1/2*a*d/b)^2 - 2*a*b^2*d^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan
(1/2*a*d/b)^2 + 2*b^3*d^2*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + a^3*d^3
*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 -
 a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*c)^2 + 2*a^3*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*ta
```

n(1/2*c)^2 - 4*a^3*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^3*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) + 2*b^3*d^2*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 - a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^3*d^2*x^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^3*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b*d^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*a^3*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*a*b^2*d^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 2*a^3*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b) - 2*a^3*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^3*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a*b^2*d^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a*b^2*d^2*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 8*b^3*d*x*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^3*d^2*x^2*tan(1/2*d*x + 1/2*c)^2 - a^3*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2 + a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2 - 2*a^3*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2 - 2*b^3*d^2*x^2*tan(1/2*c)^2 + a^3*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 2*a^3*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 2*a^2*b*d^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 4*a^3*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^3*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 2*b^3*d^2*x^2*tan(1/2*a*d/b)^2 + a^3*d^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - a^3*d^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + 2*a^3*d^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 2*a^2*b*d^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*b*d^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^2*d*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*b^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b^2*d^2*x*tan(1/2*d*x + 1/2*c)^2 - 2*a^3*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) - 2*a^3*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 2*a*b^2*d^2*x*tan(1/2*c)^2 + 8*b^3*d*x*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 + 2*a^3*d^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 2*a^3*d^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 2*a*b^2*d^2*x*tan(1/2*a*d/

```
b)^2 + 8*b^3*d*x*tan(1/2*d*x + 1/2*c)*tan(1/2*a*d/b)^2 - 2*b^3*d^2*x^2 - a^
3*d^3*imag_part(cos_integral(d*x + a*d/b)) + a^3*d^3*imag_part(cos_integral
(-d*x - a*d/b)) - 2*a^3*d^3*sin_integral((b*d*x + a*d)/b) + 2*a^2*b*d^2*tan
(1/2*d*x + 1/2*c)^2 - 2*a^2*b*d^2*tan(1/2*c)^2 - 4*a*b^2*d*tan(1/2*d*x + 1/
2*c)*tan(1/2*c)^2 - 4*b^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^2*b*d^2
*tan(1/2*a*d/b)^2 - 4*a*b^2*d*tan(1/2*d*x + 1/2*c)*tan(1/2*a*d/b)^2 - 4*b^3
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + 4*b^...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sin\left(c+d\,x\right)}{a+b\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sin(c + d*x))/(a + b*x),x)
```

```
[Out] int((x^3*sin(c + d*x))/(a + b*x), x)
```

3.20 $\int \frac{x^2 \sin(c+dx)}{a+bx} \, dx$

Optimal. Leaf size=99

$$\frac{a\cos(c+dx)}{b^2d}-\frac{x\cos(c+dx)}{bd}+\frac{a^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3}+\frac{\sin(c+dx)}{bd^2}+\frac{a^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^3}$$

```
[Out] a*cos(d*x+c)/b^2/d-x*cos(d*x+c)/b/d+a^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^3-a^2
*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^3+sin(d*x+c)/b/d^2
```

Rubi [A]
time = 0.18, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6874, 2718, 3377, 2717, 3384, 3380, 3383}

$$\frac{a^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{a^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{a\cos(c+dx)}{b^2d}+\frac{\sin(c+dx)}{bd^2}-\frac{x\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] (a*Cos[c + d*x])/(b^2*d) - (x*Cos[c + d*x])/(b*d) + (a^2*CosIntegral[(a*d)/
b + d*x]*Sin[c - (a*d)/b])/b^3 + Sin[c + d*x]/(b*d^2) + (a^2*Cos[c - (a*d)/
b]*SinIntegral[(a*d)/b + d*x])/b^3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2\sin(c+dx)}{a+bx}\,dx = \int \left(-\frac{a\sin(c+dx)}{b^2}+\frac{x\sin(c+dx)}{b}+\frac{a^2\sin(c+dx)}{b^2(a+bx)}\right)dx$$
$$= -\frac{a\int\sin(c+dx)\,dx}{b^2}+\frac{a^2\int\frac{\sin(c+dx)}{a+bx}\,dx}{b^2}+\frac{\int x\sin(c+dx)\,dx}{b}$$
$$= \frac{a\cos(c+dx)}{b^2d}-\frac{x\cos(c+dx)}{bd}+\frac{\int\cos(c+dx)\,dx}{bd}+\frac{\left(a^2\cos\left(c-\frac{ad}{b}\right)\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2}$$
$$= \frac{a\cos(c+dx)}{b^2d}-\frac{x\cos(c+dx)}{bd}+\frac{a^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3}+\frac{\sin(c+dx)}{bd^2}+\frac{a^2\cos(}{}$$

Mathematica [A]
time = 0.19, size = 87, normalized size = 0.88

$$\frac{a^2d^2\text{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right)+b(d(a-bx)\cos(c+dx)+b\sin(c+dx))+a^2d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{b^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] (a^2*d^2*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + b*(d*(a - b*x)*Cos[c +
 d*x] + b*Sin[c + d*x]) + a^2*d^2*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)]
)/(b^3*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(102) = 204$.
time = 0.06, size = 318, normalized size = 3.21

method	result
risch	$-\frac{(dxb-da)\cos(dx+c)}{d^2b^2}+\frac{\sin(dx+c)}{b\,d^2}-\frac{ia^2\cos\left(\frac{da-cb}{b}\right)\mathrm{expIntegral}\left(1,\frac{id(bx+a)}{b}\right)}{2b^3}+\frac{ia^2\cos\left(\frac{da-cb}{b}\right)\mathrm{expIntegral}\left(1\right.}{2b^3}$
derivativedivides	$\frac{d\,c^2\left(\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)+\frac{2(da-cb)dc\left(\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da}{}\right.}{b}\right.}{}}{}$
default	$\frac{d\,c^2\left(\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)+\frac{2(da-cb)dc\left(\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da}{}\right.}{b}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(d*c^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b
)*sin((a*d-b*c)/b)/b)+2*(a*d-b*c)*d*c/b*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c
)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+2*d*c/b*cos(d*x+c)+(a^2*d^
2-2*a*b*c*d+b^2*c^2)*d/b^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x
+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-d*(a*d-b*c-b)/b^2*(sin(d*x+c)-(d*x+c)*c
os(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*((b*cos(c)^2 + b*sin(c)^2)*d*x^2*cos(d*x + c) - ((a*(I*exp_integral_e(
2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)
^2 + a*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*
b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) - (a*(exp_integral_e(2, (I
*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*
(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a
*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 - (b*cos(c)^2 + b*sin
(c)^2)*x*sin(d*x + c) - ((a*(I*exp_integral_e(2, (I*b*d*x + I*a*d)/b) - I*e
xp_integral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*exp_integral_e(2, (
I*b*d*x + I*a*d)/b) - I*exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*
cos(-(b*c - a*d)/b) - (a*(exp_integral_e(2, (I*b*d*x + I*a*d)/b) + exp_inte
gral_e(2, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(exp_integral_e(2, (I*b*d*x +
 I*a*d)/b) + exp_integral_e(2, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c -
 a*d)/b))*sin(d*x + c)^2 + ((b*d*x^2*cos(c) + b*x*sin(c))*cos(d*x + c)^2 +
(b*d*x^2*cos(c) + b*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) - 2*(((a*b^2*c
```

```
os(c)^2 + a*b^2*sin(c)^2)*d^3*x + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3)*co
s(d*x + c)^2 + ((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x + (a^2*b*cos(c)^2 +
 a^2*b*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(1/2*x*cos(d*x + c)/(b^2*d^2
*x^2 + 2*a*b*d^2*x + a^2*d^2), x) - 2*(((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d
^3*x + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((a*b^2*cos(
c)^2 + a*b^2*sin(c)^2)*d^3*x + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3)*sin(d
*x + c)^2)*integrate(1/2*x*cos(d*x + c)/((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d
^2)*cos(d*x + c)^2 + (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sin(d*x + c)^2),
 x) + ((b*d*x^2*sin(c) - b*x*cos(c))*cos(d*x + c)^2 + (b*d*x^2*sin(c) - b*x
*cos(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^
2*x + (a*b*cos(c)^2 + a*b*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c)^2 +
b^2*sin(c)^2)*d^2*x + (a*b*cos(c)^2 + a*b*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Fricas [A]
time = 0.35, size = 133, normalized size = 1.34

$$\frac{2\,a^2d^2\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)+2\,b^2\sin\left(dx+c\right)-2\left(b^2dx-abd\right)\cos\left(dx+c\right)-\left(a^2d^2\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+a^2d^2\,\mathrm{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\,b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*d^2*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + 2*b^2*si
n(d*x + c) - 2*(b^2*d*x - a*b*d)*cos(d*x + c) - (a^2*d^2*cos_integral((b*d*
x + a*d)/b) + a^2*d^2*cos_integral(-(b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/
(b^3*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\sin\left(c+dx\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.30, size = 2205, normalized size = 22.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))
```

*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 2*a^2*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(
1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*d^2*real_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b)^2
- 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 + 2*b^2*d*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2*t
an(1/2*a*d/b)^2 - a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
+ 1/2*c)^2*tan(1/2*c)^2 + a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^2*d^2*sin_integral((b*d*x + a*d)/b)*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 4*a^2*d^2*imag_part(cos_integral(d*x
+ a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*d^2*imag
_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)*tan(1/2
*a*d/b) + 8*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*c)*tan(1/2*a*d/b) - a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(
1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*d^2*imag_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*d^2*sin_integral((
b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*d^2*imag_part
(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*d^2*imag_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*si
n_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b*d*tan(1/2
*d*x + 1/2*c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*real_part(cos_int
egral(d*x + a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) + 2*a^2*d^2*real_part
(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) + 2*b^2*d*x*
tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^2*d^2*real_part(cos_integral(d*x
+ a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*d^2*real_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*d^2*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*d^2*
real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*b^2*d
*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*d^2*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d^2*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2*d*x*tan(1/2*c)^2*ta
n(1/2*a*d/b)^2 + a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x +
 1/2*c)^2 - a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x + 1/2
*c)^2 + 2*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x + 1/2*c)^2 - a^
2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + a^2*d^2*imag_part
(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*a^2*d^2*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*c)^2 - 2*a*b*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 4*a^2
*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2
*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^
2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - a^2*d^2*ima
g_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + a^2*d^2*imag_part(cos_
integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*a^2*d^2*sin_integral((b*d*x +
a*d)/b)*tan(1/2*a*d/b)^2 - 2*a*b*d*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2

```
+ 2*a*b*d*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*b^2*tan(1/2*d*x + 1/2*c)*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d*x*tan(1/2*d*x + 1/2*c)^2 + 2*a^2*d^2*real
_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a^2*d^2*real_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*c) - 2*b^2*d*x*tan(1/2*c)^2 - 2*a^2*d^2*real_par
t(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^2*d^2*real_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*a*d/b) - 2*b^2*d*x*tan(1/2*a*d/b)^2 + a^2*d^2*im
ag_part(cos_integral(d*x + a*d/b)) - a^2*d^2*imag_part(cos_integral(-d*x -
a*d/b)) + 2*a^2*d^2*sin_integral((b*d*x + a*d)/b) - 2*a*b*d*tan(1/2*d*x + 1
/2*c)^2 + 2*a*b*d*tan(1/2*c)^2 + 4*b^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 +
2*a*b*d*tan(1/2*a*d/b)^2 + 4*b^2*tan(1/2*d*x + 1/2*c)*tan(1/2*a*d/b)^2 - 2*
b^2*d*x + 2*a*b*d + 4*b^2*tan(1/2*d*x + 1/2*c))/(b^3*d^2*tan(1/2*d*x + 1/2*
c)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^3*d^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2
*c)^2 + b^3*d^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*a*d/b)^2 + b^3*d^2*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + b^3*d^2*tan(1/2*d*x + 1/2*c)^2 + b^3*d^2*tan(1/2*c)^
2 + b^3*d^2*tan(1/2*a*d/b)^2 + b^3*d^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \sin(c+dx)}{a+bx} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sin(c + d*x))/(a + b*x),x)
[Out] int((x^2*sin(c + d*x))/(a + b*x), x)
```

3.21 $\int \frac{x\sin(c+dx)}{a+bx}\,dx$

Optimal. Leaf size=69

$$-\frac{\cos(c+dx)}{bd}-\frac{a\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^2}-\frac{a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^2}$$

```
[Out] -cos(d*x+c)/b/d-a*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^2+a*Ci(a*d/b+d*x)*sin(-c+a*
d/b)/b^2
```

Rubi [A]
time = 0.11, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6874, 2718, 3384, 3380, 3383}

$$-\frac{a\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^2}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] -(Cos[c + d*x]/(b*d)) - (a*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b^2
 - (a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sin(c+dx)}{a+bx}\,dx &= \int \left(\frac{\sin(c+dx)}{b}-\frac{a\sin(c+dx)}{b(a+bx)}\right)dx\\
&= \frac{\int \sin(c+dx)\,dx}{b}-\frac{a\int \frac{\sin(c+dx)}{a+bx}\,dx}{b}\\
&= -\frac{\cos(c+dx)}{bd}-\frac{\left(a\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}-\frac{\left(a\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}\\
&= -\frac{\cos(c+dx)}{bd}-\frac{a\operatorname{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^2}-\frac{a\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 63, normalized size = 0.91

$$
-\frac{b\cos(c+dx)+ad\operatorname{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right)+ad\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] -((b*Cos[c + d*x] + a*d*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + a*d*Cos
[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(b^2*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(70) = 140.
time = 0.06, size = 180, normalized size = 2.61

method	result
risch	$-\frac{\cos(dx+c)}{bd}+\frac{ia\cos\left(\frac{da-cb}{b}\right)\mathrm{expIntegral}\left(1,\frac{id(bx+a)}{b}\right)}{2b^2}-\frac{ia\cos\left(\frac{da-cb}{b}\right)\mathrm{expIntegral}\left(1,-\frac{id(bx+a)}{b}\right)}{2b^2}-\frac{a\sin\left(\frac{da-cb}{b}\right.}{}$
derivativedivides	$\frac{-\frac{(da-cb)d\left(\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)}{b}-\frac{d\cos(dx+c)}{b}-dc\left(\frac{\mathrm{sinIntegral}(}{}\right.}{d^2}$
default	$\frac{-\frac{(da-cb)d\left(\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}\right)}{b}-\frac{d\cos(dx+c)}{b}-dc\left(\frac{\mathrm{sinIntegral}(}{}\right.}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(-(a*d-b*c)*d/b*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a
*d-b*c)/b)*sin((a*d-b*c)/b)/b)-d/b*cos(d*x+c)-d*c*(Si(d*x+c+(a*d-b*c)/b)*co
s((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.42, size = 776, normalized size = 11.25

$$-\frac{\frac{\left(d\left(-i\,E_1\left(\frac{i(dx+c)b-ibc+iad}{b}\right)+i\,E_1\left(-\frac{i(dx+c)b-ibc+iad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+d\left(E_1\left(\frac{i(dx+c)b-ibc+iad}{b}\right)+E_1\left(-\frac{i(dx+c)b-ibc+iad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)\right)c}{b}+\frac{(dx+c)bd\cos(dx+c)^3+(dx+c)bd\cos(dx+c)-\left(\left(bcd\left(E_2\left(\frac{i(dx+c)b-ibc+iad}{b}\right)+E_2\left(-\frac{i(dx+c)b-ibc+iad}{b}\right)\right)-ad^2\left(E_2\left(\frac{i(dx+c)b-ibc+iad}{b}\right)+E_2\left(-\frac{i(dx+c)b-ibc+iad}{b}\right)\right)\right)\cos\left(-\frac{bc-ad}{b}\right)-\left(ad^2\left(i\,E_2\left(\frac{i(dx+c)b-ibc+iad}{b}\right)-i\,E_2\left(-\frac{i(dx+c)b-ibc+iad}{b}\right)\right)+bcd\left(-i\,E_2\left(\frac{i(dx+c)b-ibc+iad}{b}\right)+i\,E_2\left(-\frac{i(dx+c)b-ibc+iad}{b}\right)\right)\right)\sin\left(-\frac{bc-ad}{b}\right)\right)\cos(dx+c)^2+\left((dx+c)bd\cos(dx+c)-\left(bcd\left(E_2\left(\frac{i(dx+c)b-ibc+iad}{b}\right)+E_2\left(-\frac{i(dx+c)b-ibc+iad}{b}\right)\right)-ad^2\left(E_2\left(\frac{i(dx+c)b-ibc+iad}{b}\right)+E_2\left(-\frac{i(dx+c)b-ibc+iad}{b}\right)\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+\left(ad^2\left(i\,E_2\left(\frac{i(dx+c)b-ibc+iad}{b}\right)-i\,E_2\left(-\frac{i(dx+c)b-ibc+iad}{b}\right)\right)+bcd\left(-i\,E_2\left(\frac{i(dx+c)b-ibc+iad}{b}\right)+i\,E_2\left(-\frac{i(dx+c)b-ibc+iad}{b}\right)\right)\right)\sin\left(-\frac{bc-ad}{b}\right)\right)\sin(dx+c)^2}{\left((dx+c)b^2-b^2c+abd\right)\cos(dx+c)^2+\left((dx+c)b^2-b^2c+abd\right)\sin(dx+c)^2}}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*((d*(-I*exp_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_i
ntegral_e(1, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d*(
exp_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(1, -(
I*(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))*c/b + ((d*x + c)*b*
d*cos(d*x + c)^3 + (d*x + c)*b*d*cos(d*x + c) - ((b*c*d*(exp_integral_e(2,
(I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(2, -(I*(d*x + c)*b - I*
b*c + I*a*d)/b)) - a*d^2*(exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)
/b) + exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)))*cos(-(b*c - a
*d)/b) - (a*d^2*(I*exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) - I
*exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)) + b*c*d*(-I*exp_int
egral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_integral_e(2, -(I*(d*
x + c)*b - I*b*c + I*a*d)/b)))*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((d*x
+ c)*b*d*cos(d*x + c) - (b*c*d*(exp_integral_e(2, (I*(d*x + c)*b - I*b*c +
I*a*d)/b) + exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)) - a*d^2*
(exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(2, -
(I*(d*x + c)*b - I*b*c + I*a*d)/b)))*cos(-(b*c - a*d)/b) + (a*d^2*(I*exp_in
tegral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) - I*exp_integral_e(2, -(I*(d
*x + c)*b - I*b*c + I*a*d)/b)) + b*c*d*(-I*exp_integral_e(2, (I*(d*x + c)*b
 - I*b*c + I*a*d)/b) + I*exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)
/b)))*sin(-(b*c - a*d)/b))*sin(d*x + c)^2)/((((d*x + c)*b^2 - b^2*c + a*b*d)
*cos(d*x + c)^2 + ((d*x + c)*b^2 - b^2*c + a*b*d)*sin(d*x + c)^2))/d^2
```

Fricas [A]
time = 0.35, size = 99, normalized size = 1.43

$$-\frac{2\,ad\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)+2\,b\cos\left(dx+c\right)-\left(ad\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+ad\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + 2*b*cos(d*x
 + c) - (a*d*cos_integral((b*d*x + a*d)/b) + a*d*cos_integral(-(b*d*x + a*d
)/b))*sin(-(b*c - a*d)/b))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(c + dx)}{a + bx} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.39, size = 1647, normalized size = 23.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 - a*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*d*real_part(cos_integral(d*x +
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a*d*real_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a*d*r
eal_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b
)^2 - 2*a*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
*tan(1/2*a*d/b)^2 - a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)^2 + a*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^
2 + 4*a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*ta
n(1/2*a*d/b) - 4*a*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)*tan(1/2*a*d/b) + 8*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)
^2*tan(1/2*c)*tan(1/2*a*d/b) - a*d*imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a*d*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2
*d*x)^2*tan(1/2*a*d/b)^2 + a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2
*c)^2*tan(1/2*a*d/b)^2 - a*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + 2*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*ta
n(1/2*a*d/b)^2 + 2*b*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*d*r
```

```
eal_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*d*real_
part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d*real_par
t(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a*d*real_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 2*a*d*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a*d*real_part
(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a*d*real_part(
cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*d*real_part(co
s_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + a*d*imag_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*d*x)^2 - a*d*imag_part(cos_integral(-d*x - a*
d/b))*tan(1/2*d*x)^2 + 2*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 -
 a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + a*d*imag_part(cos_
integral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*a*d*sin_integral((b*d*x + a*d)/b)*
tan(1/2*c)^2 + 2*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*d*imag_part(cos_integr
al(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a*d*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a*d*sin_integral((b*d*x + a*d)/
b)*tan(1/2*c)*tan(1/2*a*d/b) - a*d*imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*a*d/b)^2 + a*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2
- 2*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 2*b*tan(1/2*d*x)^2
*tan(1/2*a*d/b)^2 - 8*b*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b*tan(
1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*d*real_part(cos_integral(d*x + a*d/b))*tan(
1/2*c) + 2*a*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 2*a*d*rea
l_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a*d*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*a*d/b) + a*d*imag_part(cos_integral(d*x + a*d/b
)) - a*d*imag_part(cos_integral(-d*x - a*d/b)) + 2*a*d*sin_integral((b*d*x
+ a*d)/b) - 2*b*tan(1/2*d*x)^2 - 8*b*tan(1/2*d*x)*tan(1/2*c) - 2*b*tan(1/2*
c)^2 + 2*b*tan(1/2*a*d/b)^2 + 2*b)/(b^2*d*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 + b^2*d*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*d*tan(1/2*d*x)^2*tan(
1/2*a*d/b)^2 + b^2*d*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^2*d*tan(1/2*d*x)^2 +
 b^2*d*tan(1/2*c)^2 + b^2*d*tan(1/2*a*d/b)^2 + b^2*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin\left(c+d\,x\right)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(c + d*x))/(a + b*x),x)
```

```
[Out] int((x*sin(c + d*x))/(a + b*x), x)
```

3.22 $\int \frac{\sin(c+dx)}{a+bx}\,dx$

Optimal. Leaf size=51

$$\frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b}$$

```
[Out] cos(-c+a*d/b)*Si(a*d/b+d*x)/b-Ci(a*d/b+d*x)*sin(-c+a*d/b)/b
```

Rubi [A]

time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3384, 3380, 3383}

$$\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x),x]
```

```
[Out] (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b + (Cos[c - (a*d)/b]*SinInte
gral[(a*d)/b + d*x])/b
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a+bx}\,dx &= \cos\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx+\sin\left(c-\frac{ad}{b}\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx\\ &= \frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 0.96

$$\frac{\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)+\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(\frac{ad}{b}+dx\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*x),x]
```

```
[Out] (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b] + Cos[c - (a*d)/b]*SinIntegral
[(a*d)/b + d*x])/b
```

Maple [A]
time = 0.05, size = 73, normalized size = 1.43

method	result	size
derivativedivides	$\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}$	73
default	$\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}$	73
risch	$\frac{i\,{\mathrm e}^{-\frac{i(da-cb)}{b}}\,\mathrm{expIntegral}\left(1,-idx-ic-\frac{iad-ibc}{b}\right)}{2b}-\frac{i\,{\mathrm e}^{\frac{i(da-cb)}{b}}\,\mathrm{expIntegral}\left(1,idx+ic+\frac{i(da-cb)}{b}\right)}{2b}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c
)/b)/b
```

Maxima [C] Result contains complex when optimal does not.
time = 0.39, size = 141, normalized size = 2.76

$$\frac{d\left(-i\,E_{1}\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+i\,E_{1}\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+d\left(E_{1}\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+E_{1}\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(d*(-I*exp_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_int
egral_e(1, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d*(ex
p_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(1, -(I*
(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))/(b*d)
```

Fricas [A]
time = 0.35, size = 78, normalized size = 1.53

$$-\frac{\left(\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+\mathrm{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)-2\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*((cos_integral((b*d*x + a*d)/b) + cos_integral(-(b*d*x + a*d)/b))*sin(
-(b*c - a*d)/b) - 2*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c + dx)}{a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.47, size = 597, normalized size = 11.71

$$\frac{\Im\left(\operatorname{Ci}\left(dx + \frac{ad}{b}\right)\right)\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{ad}{2b}\right)^2 - \Im\left(\operatorname{Ci}\left(-dx - \frac{ad}{b}\right)\right)\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{ad}{2b}\right)^2 + 2\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{ad}{2b}\right)^2 + 2\Re\left(\operatorname{Ci}\left(dx + \frac{ad}{b}\right)\right)\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{ad}{2b}\right) + 2\Re\left(\operatorname{Ci}\left(-dx - \frac{ad}{b}\right)\right)\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{ad}{2b}\right) - 2\Re\left(\operatorname{Ci}\left(dx + \frac{ad}{b}\right)\right)\tan\left(\frac{1}{2}c\right)\tan\left(\frac{ad}{2b}\right)^2 - 2\Re\left(\operatorname{Ci}\left(-dx - \frac{ad}{b}\right)\right)\tan\left(\frac{1}{2}c\right)\tan\left(\frac{ad}{2b}\right)^2 - \Im\left(\operatorname{Ci}\left(dx + \frac{ad}{b}\right)\right)\tan\left(\frac{1}{2}c\right)^2 + \Im\left(\operatorname{Ci}\left(-dx - \frac{ad}{b}\right)\right)\tan\left(\frac{1}{2}c\right)^2 - 2\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\tan\left(\frac{1}{2}c\right)^2 + 4\Im\left(\operatorname{Ci}\left(dx + \frac{ad}{b}\right)\right)\tan\left(\frac{1}{2}c\right)\tan\left(\frac{ad}{2b}\right) - 4\Im\left(\operatorname{Ci}\left(-dx - \frac{ad}{b}\right)\right)\tan\left(\frac{1}{2}c\right)\tan\left(\frac{ad}{2b}\right) + 8\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\tan\left(\frac{1}{2}c\right)\tan\left(\frac{ad}{2b}\right) - \Im\left(\operatorname{Ci}\left(dx + \frac{ad}{b}\right)\right)\tan\left(\frac{ad}{2b}\right)^2 + \Im\left(\operatorname{Ci}\left(-dx - \frac{ad}{b}\right)\right)\tan\left(\frac{ad}{2b}\right)^2 - 2\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\tan\left(\frac{ad}{2b}\right)^2 + 2\Re\left(\operatorname{Ci}\left(dx + \frac{ad}{b}\right)\right)\tan\left(\frac{1}{2}c\right) + 2\Re\left(\operatorname{Ci}\left(-dx - \frac{ad}{b}\right)\right)\tan\left(\frac{1}{2}c\right) - 2\Re\left(\operatorname{Ci}\left(dx + \frac{ad}{b}\right)\right)\tan\left(\frac{ad}{2b}\right) - 2\Re\left(\operatorname{Ci}\left(-dx - \frac{ad}{b}\right)\right)\tan\left(\frac{ad}{2b}\right) + \Im\left(\operatorname{Ci}\left(dx + \frac{ad}{b}\right)\right) - \Im\left(\operatorname{Ci}\left(-dx - \frac{ad}{b}\right)\right) + 2\operatorname{Si}\left(\frac{bdx+ad}{b}\right)}{2\left(b\tan\left(\frac{1}{2}c\right)^2\tan\left(\frac{ad}{2b}\right)^2 + b\tan\left(\frac{1}{2}c\right)^2 + b\tan\left(\frac{ad}{2b}\right)^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*sin_
integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*real_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*real_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*real_part(cos_integral(d*x
 + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - imag_part(cos_integral(d*x + a*d/b))*ta
n(1/2*c)^2 + imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*sin_int
egral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 4*imag_part(cos_integral(d*x + a*d/b)
)*tan(1/2*c)*tan(1/2*a*d/b) - 4*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*c)*tan(1/2*a*d/b) + 8*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a
*d/b) - imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*sin_integral((b*d*x + a*d)/
b)*tan(1/2*a*d/b)^2 + 2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2
*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 2*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*a*d/b) - 2*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*a*d/b) + imag_part(cos_integral(d*x + a*d/b)) - imag_part(cos_integra
l(-d*x - a*d/b)) + 2*sin_integral((b*d*x + a*d)/b))/(b*tan(1/2*c)^2*tan(1/2
*a*d/b)^2 + b*tan(1/2*c)^2 + b*tan(1/2*a*d/b)^2 + b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(c+d\,x)}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*x),x)
```

```
[Out] int(sin(c + d*x)/(a + b*x), x)
```

3.23 $\int \frac{\sin(c+dx)}{x(a+bx)}\,dx$

Optimal. Leaf size=73

$$\frac{\text{Ci}(dx)\sin(c)}{a}-\frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a}+\frac{\cos(c)\text{Si}(dx)}{a}-\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a}$$

[Out] cos(c)*Si(d*x)/a-cos(-c+a*d/b)*Si(a*d/b+d*x)/a+Ci(d*x)*sin(c)/a+Ci(a*d/b+d*x)*sin(-c+a*d/b)/a

Rubi [A]
time = 0.17, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.235$, Rules used = {6874, 3384, 3380, 3383}

$$-\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a}-\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a}+\frac{\sin(c)\text{CosIntegral}(dx)}{a}+\frac{\cos(c)\text{Si}(dx)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]/(x*(a + b*x)),x]

[Out] (CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a + (Cos[c]*SinIntegral[d*x])/a - (Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx = \int \left(\frac{\sin(c+dx)}{ax} - \frac{b\sin(c+dx)}{a(a+bx)}\right)dx$$
$$= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{a+bx}\,dx}{a}$$
$$= \frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a} - \frac{\left(b\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a} + \frac{\sin(c)\int \frac{\cos(dx)}{x}\,dx}{a} - \frac{\left(b\sin\left(c-\right.\right.}{}$$
$$= \frac{\operatorname{Ci}(dx)\sin(c)}{a} - \frac{\operatorname{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a} + \frac{\cos(c)\operatorname{Si}(dx)}{a} - \frac{\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{a}$$

Mathematica [A]
time = 0.09, size = 63, normalized size = 0.86

$$\frac{\operatorname{Ci}(dx)\sin(c) - \operatorname{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right) + \cos(c)\operatorname{Si}(dx) - \cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x)),x]
```

```
[Out] (CosIntegral[d*x]*Sin[c] - CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + Cos[
c]*SinIntegral[d*x] - Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/a
```

Maple [A]
time = 0.07, size = 99, normalized size = 1.36

method	result
derivativedivides	$\frac{\text{sinIntegral}(dx)\cos(c)+\text{cosineIntegral}(dx)\sin(c)}{a} - \frac{b\left(\frac{\text{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b} - \frac{\text{cosineIntegral}\left(dx+c+\frac{da-c}{b}\right.}{b}\right.}{a}$
default	$\frac{\text{sinIntegral}(dx)\cos(c)+\text{cosineIntegral}(dx)\sin(c)}{a} - \frac{b\left(\frac{\text{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b} - \frac{\text{cosineIntegral}\left(dx+c+\frac{da-c}{b}\right.}{b}\right.}{a}$
risch	$-\frac{i\,e^{-\frac{i(da-cb)}{b}}\,\text{expIntegral}\left(1,-idx-ic-\frac{iad-ibc}{b}\right)}{2a} + \frac{i\,e^{ic}\,\text{expIntegral}(1,-idx)}{2a} + \frac{i\,e^{\frac{i(da-cb)}{b}}\,\text{expIntegral}\left(1,idx+ic+\right.}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-1/a*b*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b
*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)*x), x)
```

Fricas [A]
time = 0.39, size = 99, normalized size = 1.36

$$\frac{\left(\operatorname{Ci}\left(dx\right)+\operatorname{Ci}\left(-dx\right)\right)\sin\left(c\right)+\left(\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)+2\,\cos\left(c\right)\operatorname{Si}\left(dx\right)-2\,\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*((cos_integral(d*x) + cos_integral(-d*x))*sin(c) + (cos_integral((b*d*x
 + a*d)/b) + cos_integral(-(b*d*x + a*d)/b))*sin(-(b*c - a*d)/b) + 2*cos(c)
*sin_integral(d*x) - 2*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(c+dx\right)}{x\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x*(a + b*x)), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.45, size = 838, normalized size = 11.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
imag_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - imag_part(cos_
integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - imag_part(cos_integ
ral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*sin_integral(d*x)*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 + 2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*
```

```
d/b)^2 + 2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)
 + 2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*
real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_p
art(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_part(cos_integral(-d
*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*c)^2 + imag_part(cos_integral(d*x))*tan(1/2*c)^2 + imag_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*c)^2 - imag_part(cos_integral(-d*x))*tan(1/2*c)^
2 + 2*sin_integral(d*x)*tan(1/2*c)^2 - 2*sin_integral((b*d*x + a*d)/b)*tan(
1/2*c)^2 + 4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)
 - 4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*si
n_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - imag_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - imag_part(cos_integral(d*x))*tan(1/2*a
*d/b)^2 + imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + imag_par
t(cos_integral(-d*x))*tan(1/2*a*d/b)^2 - 2*sin_integral(d*x)*tan(1/2*a*d/b)
^2 - 2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 2*real_part(cos_int
egral(d*x + a*d/b))*tan(1/2*c) - 2*real_part(cos_integral(d*x))*tan(1/2*c)
+ 2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 2*real_part(cos_inte
gral(-d*x))*tan(1/2*c) - 2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d
/b) - 2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + imag_part(co
s_integral(d*x + a*d/b)) - imag_part(cos_integral(d*x)) - imag_part(cos_int
egral(-d*x - a*d/b)) + imag_part(cos_integral(-d*x)) - 2*sin_integral(d*x)
+ 2*sin_integral((b*d*x + a*d)/b))/(a*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*tan
(1/2*c)^2 + a*tan(1/2*a*d/b)^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)}{x\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x*(a + b*x)),x)
```

```
[Out] int(sin(c + d*x)/(x*(a + b*x)), x)
```

3.24 $\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx$

Optimal. Leaf size=114

$$\frac{d\cos(c)\operatorname{Ci}(dx)}{a}-\frac{b\operatorname{Ci}(dx)\sin(c)}{a^2}+\frac{b\operatorname{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2}-\frac{\sin(c+dx)}{ax}-\frac{b\cos(c)\operatorname{Si}(dx)}{a^2}-\frac{d\sin(c)\operatorname{Si}(dx)}{a}+\frac{b\,c}{}$$

```
[Out] d*Ci(d*x)*cos(c)/a-b*cos(c)*Si(d*x)/a^2+b*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^2-b
*Ci(d*x)*sin(c)/a^2-d*Si(d*x)*sin(c)/a-b*Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^2-si
n(d*x+c)/a/x
```

Rubi [A]
time = 0.24, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$-\frac{b\sin(c)\text{CosIntegral}(dx)}{a^2}+\frac{b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2}-\frac{b\cos(c)\text{Si}(dx)}{a^2}+\frac{b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{d\cos(c)\text{CosIntegral}(dx)}{a}-\frac{d\sin(c)\text{Si}(dx)}{a}-\frac{\sin(c+dx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x)),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a - (b*CosIntegral[d*x]*Sin[c])/a^2 + (b*CosInt
egral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^2 - Sin[c + d*x]/(a*x) - (b*Cos[c]
*SinIntegral[d*x])/a^2 - (d*Sin[c]*SinIntegral[d*x])/a + (b*Cos[c - (a*d)/b
]*SinIntegral[(a*d)/b + d*x])/a^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx &= \int \left(\frac{\sin(c+dx)}{ax^2}-\frac{b\sin(c+dx)}{a^2x}+\frac{b^2\sin(c+dx)}{a^2(a+bx)}\right)dx\\ &= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a}-\frac{b\int \frac{\sin(c+dx)}{x}\,dx}{a^2}+\frac{b^2\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^2}\\ &= -\frac{\sin(c+dx)}{ax}+\frac{d\int \frac{\cos(c+dx)}{x}\,dx}{a}-\frac{(b\cos(c))\int \frac{\sin(dx)}{x}\,dx}{a^2}+\frac{\left(b^2\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\\ &= -\frac{b\text{Ci}(dx)\sin(c)}{a^2}+\frac{b\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2}-\frac{\sin(c+dx)}{ax}-\frac{b\cos(c)\text{Si}(dx)}{a^2}+\frac{b\cos\left(c\right.}{}\\ &= \frac{d\cos(c)\text{Ci}(dx)}{a}-\frac{b\text{Ci}(dx)\sin(c)}{a^2}+\frac{b\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2}-\frac{\sin(c+dx)}{ax}-\frac{b\cos(c)\text{S}}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 101, normalized size = 0.89

$$\frac{x\text{Ci}(dx)(ad\cos(c)-b\sin(c))+bx\text{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right)-a\sin(c+dx)-bx\cos(c)\text{Si}(dx)-adx\sin(c)\text{Si}(dx)+bx\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x)),x]
```

```
[Out] (x*CosIntegral[d*x]*(a*d*Cos[c] - b*Sin[c]) + b*x*CosIntegral[d*(a/b + x)]*
Sin[c - (a*d)/b] - a*Sin[c + d*x] - b*x*Cos[c]*SinIntegral[d*x] - a*d*x*Sin
[c]*SinIntegral[d*x] + b*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(a^2*
x)
```

Maple [A]
time = 0.10, size = 144, normalized size = 1.26

method	result

derivativedivides	$d\left(\frac{-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}(dx)\sin(c)+\text{cosineIntegral}(dx)\cos(c)}{a}-\frac{b(\text{sinIntegral}(dx)\cos(c)+\text{cosineIntegral}(dx)\sin(c))}{a^2d}\right.$
default	$d\left(\frac{-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}(dx)\sin(c)+\text{cosineIntegral}(dx)\cos(c)}{a}-\frac{b(\text{sinIntegral}(dx)\cos(c)+\text{cosineIntegral}(dx)\sin(c))}{a^2d}\right.$
risch	$-\frac{d\,e^{ic}\,\text{expIntegral}(1,-idx)}{2a}+\frac{ib\,e^{-\frac{i(da-cb)}{b}}\,\text{expIntegral}\left(1,-idx-ic-\frac{iad-ibc}{b}\right)}{2a^2}-\frac{ib\,e^{ic}\,\text{expIntegral}(1,-idx)}{2a^2}-\frac{ib\,e^{i}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] d*(1/a*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))-1/a^2*b/d*(Si(d*x)*c
os(c)+Ci(d*x)*sin(c))+b^2/a^2/d*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-C
i(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)*x^2), x)
```

Fricas [A]
time = 0.35, size = 157, normalized size = 1.38

$$\frac{2\,bx\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)+\left(adx\operatorname{Ci}\left(dx\right)+adx\operatorname{Ci}\left(-dx\right)-2\,bx\operatorname{Si}\left(dx\right)\right)\cos\left(c\right)-2\,a\sin\left(dx+c\right)-\left(2\,adx\operatorname{Si}\left(dx\right)+bx\operatorname{Ci}\left(dx\right)+bx\operatorname{Ci}\left(-dx\right)\right)\sin\left(c\right)-\left(bx\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+bx\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\,a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + (a*d*x*cos_i
ntegral(d*x) + a*d*x*cos_integral(-d*x) - 2*b*x*sin_integral(d*x))*cos(c) -
 2*a*sin(d*x + c) - (2*a*d*x*sin_integral(d*x) + b*x*cos_integral(d*x) + b*
x*cos_integral(-d*x))*sin(c) - (b*x*cos_integral((b*d*x + a*d)/b) + b*x*cos
_integral(-(b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(c+dx\right)}{x^{2}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x**2*(a + b*x)), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.83, size = 2897, normalized size = 25.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(a*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
2*a*d/b)^2 + a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^
2*tan(1/2*a*d/b)^2 + 2*a*d*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*d*x*imag_part(cos_integral(-d*x))*tan(1/2*d
*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*d*x*sin_integral(d*x)*tan(1/2*d*x)^
2*tan(1/2*c)*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x + a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b*x*imag_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b*x*imag_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b*x
*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b*x*sin
_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a
*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*d*x*real_
part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b*x*real_part(cos_
integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*b*x*r
eal_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*
d/b) - a*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 -
 a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*b*
x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*
d/b)^2 + 2*b*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1
/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(-d*x))*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a*d*x*real_part(cos_integral(d*x))*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + a*d*x*real_part(cos_integral(-d*x))*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 + 2*a*d*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(
1/2*c) - 2*a*d*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) +
4*a*d*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + b*x*imag_part(cos_int
egral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*x*imag_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*x*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + b*x*imag_part(cos_integral(-d*x))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*b*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*b*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b*x*
```

```
imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/
b) + 4*b*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*
tan(1/2*a*d/b) - 8*b*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2
*c)*tan(1/2*a*d/b) + b*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^
2*tan(1/2*a*d/b)^2 + b*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/
2*a*d/b)^2 - b*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*a*d/b)^2 - b*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/
b)^2 + 2*b*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*b*x*sin_
integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*d*x*imag_pa
rt(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*d*x*imag_part(cos_i
ntegral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*d*x*sin_integral(d*x)*tan(
1/2*c)*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 + b*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/
2*a*d/b)^2 + b*x*imag_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^
2 - 2*b*x*sin_integral(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b*x*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a*d*x*real_part(cos_in
tegral(d*x))*tan(1/2*d*x)^2 - a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d
*x)^2 - 2*b*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
) + 2*b*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b*x*re
al_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*b*x*real_
part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a*d*x*real_part(cos_in
tegral(d*x))*tan(1/2*c)^2 + a*d*x*real_part(cos_integral(-d*x))*tan(1/2*c)^
2 + 2*b*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b
) + 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/
b) - 2*b*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)
 - 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)
- a*d*x*real_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 - a*d*x*real_part(cos
_integral(-d*x))*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(d*x + a*d/
b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(d*x))*tan(1/
2*c)*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2
*c)*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(-d*x))*tan(1/2*c)*tan(1
/2*a*d/b)^2 - 4*a*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a*tan(1/2*
d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2 + b*x*imag_part(cos_integral(...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (c+d\,x)}{x^2\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x^2*(a + b*x)),x)
[Out] int(sin(c + d*x)/(x^2*(a + b*x)), x)
```

3.25 $\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx$

Optimal. Leaf size=189

$$-\frac{d\cos(c+dx)}{2ax}-\frac{bd\cos(c)\text{Ci}(dx)}{a^2}+\frac{b^2\text{Ci}(dx)\sin(c)}{a^3}-\frac{d^2\text{Ci}(dx)\sin(c)}{2a}-\frac{b^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3}-\frac{\sin(c+d}{2ax^2}$$

```
[Out] -b*d*Ci(d*x)*cos(c)/a^2-1/2*d*cos(d*x+c)/a/x+b^2*cos(c)*Si(d*x)/a^3-1/2*d^2
*cos(c)*Si(d*x)/a-b^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^3+b^2*Ci(d*x)*sin(c)/a^
3-1/2*d^2*Ci(d*x)*sin(c)/a+b*d*Si(d*x)*sin(c)/a^2+b^2*Ci(a*d/b+d*x)*sin(-c+
a*d/b)/a^3-1/2*sin(d*x+c)/a/x^2+b*sin(d*x+c)/a^2/x
```

Rubi [A]
time = 0.34, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\frac{b^2\sin(c)\text{CosIntegral}(dx)}{a^3}-\frac{b^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^3}+\frac{b^2\cos(c)\text{Si}(dx)}{a^3}-\frac{b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{bd\cos(c)\text{CosIntegral}(dx)}{a^2}+\frac{bd\sin(c)\text{Si}(dx)}{a^2}+\frac{b\sin(c+dx)}{a^2x}-\frac{d^2\sin(c)\text{CosIntegral}(dx)}{2a}-\frac{d^2\cos(c)\text{Si}(dx)}{2a}-\frac{\sin(c+dx)}{2ax^2}-\frac{d\cos(c+dx)}{2ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x)),x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(a*x) - (b*d*Cos[c]*CosIntegral[d*x])/a^2 + (b^2*CosI
ntegral[d*x]*Sin[c])/a^3 - (d^2*CosIntegral[d*x]*Sin[c])/(2*a) - (b^2*CosIn
tegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^3 - Sin[c + d*x]/(2*a*x^2) + (b*S
in[c + d*x])/(a^2*x) + (b^2*Cos[c]*SinIntegral[d*x])/a^3 - (d^2*Cos[c]*SinI
ntegral[d*x])/(2*a) + (b*d*Sin[c]*SinIntegral[d*x])/a^2 - (b^2*Cos[c - (a*d
)/b]*SinIntegral[(a*d)/b + d*x])/a^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx &= \int \left(\frac{\sin(c+dx)}{ax^3} - \frac{b\sin(c+dx)}{a^2x^2} + \frac{b^2\sin(c+dx)}{a^3x} - \frac{b^3\sin(c+dx)}{a^3(a+bx)}\right)dx \\
&= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{x^2}\,dx}{a^2} + \frac{b^2\int \frac{\sin(c+dx)}{x}\,dx}{a^3} - \frac{b^3\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^3} \\
&= -\frac{\sin(c+dx)}{2ax^2} + \frac{b\sin(c+dx)}{a^2x} + \frac{d\int \frac{\cos(c+dx)}{x^2}\,dx}{2a} - \frac{(bd)\int \frac{\cos(c+dx)}{x}\,dx}{a^2} + \frac{(b^2\cos(c))\int \frac{\sin(d}{x} \\
&= -\frac{d\cos(c+dx)}{2ax} + \frac{b^2\text{Ci}(dx)\sin(c)}{a^3} - \frac{b^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} - \frac{\sin(c+dx)}{2ax^2} + \frac{b\sin(c}{a^2} \\
&= -\frac{d\cos(c+dx)}{2ax} - \frac{bd\cos(c)\text{Ci}(dx)}{a^2} + \frac{b^2\text{Ci}(dx)\sin(c)}{a^3} - \frac{b^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} - \sin \\
&= -\frac{d\cos(c+dx)}{2ax} - \frac{bd\cos(c)\text{Ci}(dx)}{a^2} + \frac{b^2\text{Ci}(dx)\sin(c)}{a^3} - \frac{d^2\text{Ci}(dx)\sin(c)}{2a} - \frac{b^2\text{Ci}\left(\frac{ad}{b}+dx\right)}{a^3}
\end{aligned}$$

Mathematica [A]
time = 0.38, size = 176, normalized size = 0.93

$$-\frac{a^2dx\cos(c+dx)+x^2\text{Ci}(dx)\left(2abd\cos(c)+\left(-2b^2+a^2d^2\right)\sin(c)\right)+2b^2x^2\text{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right)+a^2\sin(c+dx)-2abx\sin(c+dx)-2b^2x^2\cos(c)\text{Si}(dx)+a^2d^2x^2\cos(c)\text{Si}(dx)-2abdx^2\sin(c)\text{Si}(dx)+2b^2x^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x)),x]
```

```
[Out] -1/2*(a^2*d*x*Cos[c + d*x] + x^2*CosIntegral[d*x]*(2*a*b*d*Cos[c] + (-2*b^2
 + a^2*d^2)*Sin[c]) + 2*b^2*x^2*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] +
 a^2*Sin[c + d*x] - 2*a*b*x*Sin[c + d*x] - 2*b^2*x^2*Cos[c]*SinIntegral[d*x
] + a^2*d^2*x^2*Cos[c]*SinIntegral[d*x] - 2*a*b*d*x^2*Sin[c]*SinIntegral[d*
x] + 2*b^2*x^2*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(a^3*x^2)
```

Maple [A]
time = 0.14, size = 202, normalized size = 1.07

method	result
derivativedivides	$d^2\left(-\frac{b\left(-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}(dx)\sin(c)+\text{cosineIntegral}(dx)\cos(c)\right)}{a^2d}+\frac{b^2(\text{sinIntegral}(dx)\cos(c)+\text{cosineIntegral}}{a^3d^2}\right.$
default	$d^2\left(-\frac{b\left(-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}(dx)\sin(c)+\text{cosineIntegral}(dx)\cos(c)\right)}{a^2d}+\frac{b^2(\text{sinIntegral}(dx)\cos(c)+\text{cosineIntegral}}{a^3d^2}\right.$
risch	$-\frac{id^2\mathrm{e}^{ic}\,\text{expIntegral}(1,-idx)}{4a}+\frac{db\,\mathrm{e}^{ic}\,\text{expIntegral}(1,-idx)}{2a^2}-\frac{ib^2\mathrm{e}^{-\frac{i(da-cb)}{b}}\,\text{expIntegral}\left(1,-idx-ic-\frac{iad-ibc}{b}\right)}{2a^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/a^2*b/d*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+1/a^3*b^2/d
^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+1/a*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x
+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c))-1/d^2*b^3/a^3*(Si(d*x+c+(a*d
-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)*x^3), x)
```

Fricas [A]
time = 0.39, size = 245, normalized size = 1.30

$$-\frac{4\,b^2x^2\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)+2\,a^2dx\cos\left(dx+c\right)+2\left(abdx^2\operatorname{Ci}\left(dx\right)+abdx^2\operatorname{Ci}\left(-dx\right)+(a^2d^2-2\,b^2)x^2\operatorname{Si}\left(dx\right)\right)\cos\left(c\right)-2\left(2\,abx-a^2\right)\sin\left(dx+c\right)-\left(4\,abdx^2\operatorname{Si}\left(dx\right)-(a^2d^2-2\,b^2)x^2\operatorname{Ci}\left(dx\right)-(a^2d^2-2\,b^2)x^2\operatorname{Ci}\left(-dx\right)\right)\sin\left(c\right)-2\left(b^2x^2\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+b^2x^2\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{4\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^2*x^2*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + 2*a^2*d
*x*cos(d*x + c) + 2*(a*b*d*x^2*cos_integral(d*x) + a*b*d*x^2*cos_integral(-
d*x) + (a^2*d^2 - 2*b^2)*x^2*sin_integral(d*x))*cos(c) - 2*(2*a*b*x - a^2)*
sin(d*x + c) - (4*a*b*d*x^2*sin_integral(d*x) - (a^2*d^2 - 2*b^2)*x^2*cos_i
```

```
ntegral(d*x) - (a^2*d^2 - 2*b^2)*x^2*cos_integral(-d*x))*sin(c) - 2*(b^2*x^
2*cos_integral((b*d*x + a*d)/b) + b^2*x^2*cos_integral(-(b*d*x + a*d)/b))*s
in(-(b*c - a*d)/b))/(a^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{x^3\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**3/(b*x+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x**3*(a + b*x)), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.89, size = 4565, normalized size = 24.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(1/2*a*d/b)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x
)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*d^2*x^2*real_part(cos_integral(d*
x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d^2*x^2*real_part(co
s_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a*b*d*x^2*
real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 2*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(
1/2*a*d/b)^2 + a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(
1/2*c)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d
^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*d
^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^
2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*a*b*d*x^2*i
mag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*
a*b*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a
*d/b)^2 + 8*a*b*d*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a
*d/b)^2 + a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d
/b)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 + 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^
2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(
1/2*a*d/b)^2 - 2*b^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*t
```

```
an(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*x^2*imag_part(cos_integ
ral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*b^2*x^2*sin_int
egral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*b^2*x^2*sin_int
egral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2
*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^2
*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*b*d*x^2*
real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*d*x^2*real
_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^2*x^2*real_part
(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*
b^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(1/2*a*d/b) - 2*a*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan
(1/2*a*d/b)^2 - 2*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*ta
n(1/2*a*d/b)^2 - 2*a^2*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*c)*tan(
1/2*a*d/b)^2 - 2*a^2*d^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*c)*tan(1
/2*a*d/b)^2 + 4*b^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_part(cos_integral(d*x))*tan(1
/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a*b*d
*x^2*real_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b*d*x
^2*real_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*d*x*
tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*d^2*x^2*imag_part(cos_in
tegral(d*x))*tan(1/2*d*x)^2 + a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan
(1/2*d*x)^2 - 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + 4*a*b*d*x^2*
imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b*d*x^2*imag_p
art(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*b*d*x^2*sin_integra
l(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^2*x^2*imag_part(cos_integral(d*x))
*tan(1/2*c)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*
a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*x^2*imag_part(cos_inte
gral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*x^2*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_integra
l(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^2*x^2*sin_integral(d*x)*tan(1/2*
d*x)^2*tan(1/2*c)^2 + 4*b^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^
2*tan(1/2*c)^2 - 8*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*b^2*x^2*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 16*b^2*x^2*sin_integral((b*
d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - a^2*d^2*x^2*imag_p
art(cos_integral(d*x))*tan(1/2*a*d/b)^2 + a^2*d^2*x^2*imag_part(cos_integra
l(-d*x))*tan(1/2*a*d/b)^2 - 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*a*d/b)^
2 + 2*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a
*d/b)^2 + 2*b^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d
/b)^2 - 2*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(
1/2*a*d/b)^2 - 2*b^2*x^2*imag_part(cos_integral...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)}{x^3\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x^3*(a + b*x)),x)
[Out] int(sin(c + d*x)/(x^3*(a + b*x)), x)
```

3.26 $\int \frac{x^4 \sin(c+dx)}{(a+bx)^2} \, dx$

Optimal. Leaf size=233

$$\frac{2\cos(c+dx)}{b^2d^3} - \frac{3a^2\cos(c+dx)}{b^4d} + \frac{2ax\cos(c+dx)}{b^3d} - \frac{x^2\cos(c+dx)}{b^2d} + \frac{a^4d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{b^6} - \frac{4a^3\text{Ci}\left(\frac{ad}{b}\right.}{}$$

```
[Out] a^4*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^6+2*cos(d*x+c)/b^2/d^3-3*a^2*cos(d*x+c)
/b^4/d+2*a*x*cos(d*x+c)/b^3/d-x^2*cos(d*x+c)/b^2/d-4*a^3*cos(-c+a*d/b)*Si(a
*d/b+d*x)/b^5+4*a^3*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^5+a^4*d*Si(a*d/b+d*x)*sin
(-c+a*d/b)/b^6-2*a*sin(d*x+c)/b^3/d^2+2*x*sin(d*x+c)/b^2/d^2-a^4*sin(d*x+c)
/b^5/(b*x+a)
```

Rubi [A]
time = 0.36, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {6874, 2718, 3377, 2717, 3378, 3384, 3380, 3383}

$$\frac{a^4d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^6} - \frac{a^4d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^6} - \frac{a^4\sin(c+dx)}{b^5(a+bx)} - \frac{4a^3\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{4a^3\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{3a^2\cos(c+dx)}{b^4d} - \frac{2a\sin(c+dx)}{b^3d^2} + \frac{2ax\cos(c+dx)}{b^3d} + \frac{2\cos(c+dx)}{b^2d^3} + \frac{2x\sin(c+dx)}{b^2d^2} - \frac{x^2\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (2*Cos[c + d*x])/(b^2*d^3) - (3*a^2*Cos[c + d*x])/(b^4*d) + (2*a*x*Cos[c +
d*x])/(b^3*d) - (x^2*Cos[c + d*x])/(b^2*d) + (a^4*d*Cos[c - (a*d)/b]*CosInt
egral[(a*d)/b + d*x])/b^6 - (4*a^3*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)
/b])/b^5 - (2*a*Sin[c + d*x])/(b^3*d^2) + (2*x*Sin[c + d*x])/(b^2*d^2) - (a
^4*Sin[c + d*x])/(b^5*(a + b*x)) - (4*a^3*Cos[c - (a*d)/b]*SinIntegral[(a*d
)/b + d*x])/b^5 - (a^4*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^6
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^4\sin(c+dx)}{(a+bx)^2}\,dx = \int \left(\frac{3a^2\sin(c+dx)}{b^4} - \frac{2ax\sin(c+dx)}{b^3} + \frac{x^2\sin(c+dx)}{b^2} + \frac{a^4\sin(c+dx)}{b^4(a+bx)^2} - \frac{4a^3\sin(c$$

$$= \frac{(3a^2)\int\sin(c+dx)\,dx}{b^4} - \frac{(4a^3)\int\frac{\sin(c+dx)}{a+bx}\,dx}{b^4} + \frac{a^4\int\frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^4} - \frac{(2a)\int x\sin(c+dx)}{b^3}$$

$$= -\frac{3a^2\cos(c+dx)}{b^4d} + \frac{2ax\cos(c+dx)}{b^3d} - \frac{x^2\cos(c+dx)}{b^2d} - \frac{a^4\sin(c+dx)}{b^5(a+bx)} - \frac{(2a)\int\cos(}{b^3}$$

$$= -\frac{3a^2\cos(c+dx)}{b^4d} + \frac{2ax\cos(c+dx)}{b^3d} - \frac{x^2\cos(c+dx)}{b^2d} - \frac{4a^3\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^5}$$

$$= \frac{2\cos(c+dx)}{b^2d^3} - \frac{3a^2\cos(c+dx)}{b^4d} + \frac{2ax\cos(c+dx)}{b^3d} - \frac{x^2\cos(c+dx)}{b^2d} + \frac{a^4d\cos\left(c-\frac{a}{}\right.}{}$$

Mathematica [A]
time = 0.65, size = 177, normalized size = 0.76

$$\frac{a^3\text{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)-4b\sin\left(c-\frac{ad}{b}\right)\right)-\frac{b(b(a+bx)(3a^2d^2-2abd^2x+b^2(-2+d^2x^2))\cos(c+dx)+d(2a^2b^2+a^4d^2-2b^4x^2)\sin(c+dx))}{d^3(a+bx)}-a^3\left(4b\cos\left(c-\frac{ad}{b}\right)+ad\sin\left(c-\frac{ad}{b}\right)\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a^3*CosIntegral[d*(a/b + x)]*(a*d*Cos[c - (a*d)/b] - 4*b*Sin[c - (a*d)/b])
 - (b*(b*(a + b*x)*(3*a^2*d^2 - 2*a*b*d^2*x + b^2*(-2 + d^2*x^2))*Cos[c + d
*x] + d*(2*a^2*b^2 + a^4*d^2 - 2*b^4*x^2)*Sin[c + d*x]))/(d^3*(a + b*x)) -
a^3*(4*b*Cos[c - (a*d)/b] + a*d*Sin[c - (a*d)/b])*SinIntegral[d*(a/b + x)])
/b^6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1215 vs. $2(236) = 472$.
time = 0.24, size = 1216, normalized size = 5.22

method	result
risch	$-\frac{i\left(-2ib^6d^5x^5+4iab^5d^5x^4-6ib^6cd^4x^4-8ia^3b^3d^5x^2+10ia^4b^2d^5x-24ia^3b^3cd^4x+4ib^6d^3x^3+12ia^5bd^5-18ia^4b^2cd^4+1\right.}{2d^4b^5(bx+a)(-dxb-da)(-dxb+2da-3cb)}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^5*(d^2*c^4*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*si
n((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+4*d^2*c^3*(a*
d-b*c)/b*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d
-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-4*d^2*c^3/b*(Si(d*x
+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b
)+6*(a^2*d^2-2*a*b*c*d+b^2*c^2)*d^2*c^2/b^2*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c)
)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*
d-b*c)/b)/b)/b)-12/b^2*(a*d-b*c)*d^2*c^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*
c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-6*d^2*c^2/b^2*cos(d*x+c)+
4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*d^2*c/b^3*(-sin(d*x+c)/(d*a
-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b
*c)/b)*cos((a*d-b*c)/b)/b)/b)-12/b^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*d^2*c*(Si(
d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b
)/b)+4*(2*a*d-2*b*c-b)*d^2*c/b^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+(a^4*d^4-4
*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*d^2/b^4*(-sin(d*x+c)/
(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a
*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-4/b^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*
```

```
d-b^3*c^3)*d^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)
/b)*sin((a*d-b*c)/b)/b)+(3*a^2*d^2-6*a*b*c*d+3*b^2*c^2-2*a*b*d+2*b^2*c+b^2)
*d^2/b^4*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*((2*a^2*b*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3
, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + 2*a^2*b*(exp_integral_e(3, (I*b*d*x + I
*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2 + (a^3*(-I*exp
_integral_e(3, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(3, -(I*b*d*x + I*a*d
)/b))*cos(c)^2 + a^3*(-I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) + I*exp_int
egral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) + (2*a^2
*b*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*
x + I*a*d)/b))*cos(c)^2 + 2*a^2*b*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b)
 - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2 + (a^3*(exp_integral
_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c
)^2 + a^3*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b
*d*x + I*a*d)/b))*sin(c)^2)*d)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + 2*((2*
a^2*b*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x
 + I*a*d)/b))*cos(c)^2 + 2*a^2*b*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) +
exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2 + (a^3*(-I*exp_integral_e
(3, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c
)^2 + a^3*(-I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(3,
-(I*b*d*x + I*a*d)/b))*sin(c)^2)*d)*cos(-(b*c - a*d)/b) + (2*a^2*b*(I*exp_i
ntegral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/
b))*cos(c)^2 + 2*a^2*b*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_in
tegral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2 + (a^3*(exp_integral_e(3, (I*b*
d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a^3*(
exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*
d)/b))*sin(c)^2)*d)*sin(-(b*c - a*d)/b))*sin(d*x + c)^2 - ((b^3*d^2*x^4*cos
(c) + 2*b^3*d*x^3*sin(c) + 2*(a*b^2*d*sin(c) - b^3*cos(c))*x^2 - 2*(a^2*b*d
*sin(c) + 2*a*b^2*cos(c))*x)*cos(d*x + c)^2 + (b^3*d^2*x^4*cos(c) + 2*b^3*d
*x^3*sin(c) + 2*(a*b^2*d*sin(c) - b^3*cos(c))*x^2 - 2*(a^2*b*d*sin(c) + 2*a
*b^2*cos(c))*x)*sin(d*x + c)^2)*cos(d*x + 2*c) - ((b^3*cos(c)^2 + b^3*sin(c
)^2)*d^2*x^4 - 2*(b^3*cos(c)^2 + b^3*sin(c)^2)*x^2 - 4*(a*b^2*cos(c)^2 + a*
b^2*sin(c)^2)*x)*cos(d*x + c) + 2*(((a^3*b^5*cos(c)^2 + a^3*b^5*sin(c)^2)*d
^5*x^2 + 2*(a^4*b^4*cos(c)^2 + a^4*b^4*sin(c)^2)*d^5*x + (a^5*b^3*cos(c)^2
+ a^5*b^3*sin(c)^2)*d^5)*cos(d*x + c)^2 + ((a^3*b^5*cos(c)^2 + a^3*b^5*sin(
c)^2)*d^5*x^2 + 2*(a^4*b^4*cos(c)^2 + a^4*b^4*sin(c)^2)*d^5*x + (a^5*b^3*co
```

```
s(c)^2 + a^5*b^3*sin(c)^2)*d^5)*sin(d*x + c)^2)*integrate(x*cos(d*x + c)/(b
^5*d^3*x^3 + 3*a*b^4*d^3*x^2 + 3*a^2*b^3*d^3*x + a^3*b^2*d^3), x) + 2*(((a^
3*b^5*cos(c)^2 + a^3*b^5*sin(c)^2)*d^5*x^2 + 2*(a^4*b^4*cos(c)^2 + a^4*b^4*
sin(c)^2)*d^5*x + (a^5*b^3*cos(c)^2 + a^5*b^3*sin(c)^2)*d^5)*cos(d*x + c)^2
 + ((a^3*b^5*cos(c)^2 + a^3*b^5*sin(c)^2)*d^5*x^2 + 2*(a^4*b^4*cos(c)^2 + a
^4*b^4*sin(c)^2)*d^5*x + (a^5*b^3*cos(c)^2 + a^5*b^3*sin(c)^2)*d^5)*sin(d*x
 + c)^2)*integrate(x*cos(d*x + c)/((b^5*d^3*x^3 + 3*a*b^4*d^3*x^2 + 3*a^2*b
^3*d^3*x + a^3*b^2*d^3)*cos(d*x + c)^2 + (b^5*d^3*x^3 + 3*a*b^4*d^3*x^2 + 3
*a^2*b^3*d^3*x + a^3*b^2*d^3)*sin(d*x + c)^2), x) - 2*(((a^2*b^6*cos(c)^2 +
 a^2*b^6*sin(c)^2)*d^4*x^2 + 2*(a^3*b^5*cos(c)^2 + a^3*b^5*sin(c)^2)*d^4*x
+ (a^4*b^4*cos(c)^2 + a^4*b^4*sin(c)^2)*d^4)*cos(d*x + c)^2 + ((a^2*b^6*cos
(c)^2 + a^2*b^6*sin(c)^2)*d^4*x^2 + 2*(a^3*b^5*cos(c)^2 + a^3*b^5*sin(c)^2)
*d^4*x + (a^4*b^4*cos(c)^2 + a^4*b^4*sin(c)^2)*d^4)*sin(d*x + c)^2)*integra
te(x*sin(d*x + c)/(b^5*d^3*x^3 + 3*a*b^4*d^3*x^2 + 3*a^2*b^3*d^3*x + a^3*b^
2*d^3), x) - 2*(((a^2*b^6*cos(c)^2 + a^2*b^6*sin(c)^2)*d^4*x^2 + 2*(a^3*b^5
*cos(c)^2 + a^3*b^5*sin(c)^2)*d^4*x + (a^4*b^4*cos(c)^2 + a^4*b^4*sin(c)^2)
*d^4)*cos(d*x + c)^2 + ((a^2*b^6*cos(c)^2 + a^2*b^6*sin(c)^2)*d^4*x^2 + 2*(
a^3*b^5*cos(c)^2 + a^3*b^5*sin(c)^2)*d^4*x + (a^4*b^4*cos(c)^2 + a^4*b^4*si
n(c)^2)*d^4)*sin(d*x + c)^2)*integrate(x*sin(d*x + c)/((b^5*d^3*x^3 + 3*a*b
^4*d^3*x^2 + 3*a^2*b^3*d^3*x + a^3*b^2*d^3)*cos(d*x + c)^2 + (b^5*d^3*x^3 +
 3*a*b^4*d^3*x^2 + 3*a^2*b^3*d^3*x + a^3*b^2*d^3)*sin(d*x + c)^2), x) - ((b
^3*d^2*x^4*sin(c) - 2*b^3*d*x^3*cos(c) - 2*(a*b^2*d*cos(c) + b^3*sin(c))*x^
2 + 2*(a^2*b*d*cos(c) - 2*a*b^2*sin(c))*x)*cos(d*x + c)^2 + (b^3*d^2*x^4*si
n(c) - 2*b^3*d*x^3*cos(c) - 2*(a*b^2*d*cos(c) + b^3*sin(c))*x^2 + 2*(a^2*b*
d*cos(c) - 2*a*b^2*sin(c))*x)*sin(d*x + c)^2)*sin(d*x + 2*c) + 2*((b^3*cos(
c)^2 + b^3*sin(c)^2)*d*x^3 + (a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^2 - (a^2
*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x)*sin(d*x + c))/(((b^5*cos(c)^2 + b^5*sin(
c)^2)*d^3*x^2 + 2*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^3*x + (a^2*b^3*cos(c)
^2 + a^2*b^3*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^5*cos(c)^2 + b^5*sin(c)^2)
*d^3*x^2 + 2*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^3*x + (a^2*b^3*cos(c)^2 +
a^2*b^3*sin(c)^2)*d^3)*sin(d*x + c)^2)
```

Fricas [A]
time = 0.36, size = 357, normalized size = 1.53

$$-\frac{2\left(b^5d^2x^3 - ab^4d^2x^2 + 3\,a^3b^2d^2 - 2\,ab^4 + \left(a^2b^3d^2 - 2\,b^5\right)x\right)\cos\left(dx+c\right) - \left(\left(a^4bd^4x + a^5d^4\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right) + \left(a^4bd^4x + a^5d^4\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right) - 8\left(a^3b^2d^3x + a^4bd^3\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right) + 2\left(a^4bd^3 - 2\,b^5dx^2 + 2\,a^2b^3d\right)\sin\left(dx+c\right) - 2\left(2\left(a^3b^2d^3x + a^4bd^3\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right) + 2\left(a^3b^2d^3x + a^4bd^3\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right) + \left(a^4bd^4x + a^5d^4\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\left(b^7d^3x + ab^6d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^5*d^2*x^3 - a*b^4*d^2*x^2 + 3*a^3*b^2*d^2 - 2*a*b^4 + (a^2*b^3*d
^2 - 2*b^5)*x)*cos(d*x + c) - ((a^4*b*d^4*x + a^5*d^4)*cos_integral((b*d*x
+ a*d)/b) + (a^4*b*d^4*x + a^5*d^4)*cos_integral(-(b*d*x + a*d)/b) - 8*(a^3
*b^2*d^3*x + a^4*b*d^3)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b)
+ 2*(a^4*b*d^3 - 2*b^5*d*x^2 + 2*a^2*b^3*d)*sin(d*x + c) - 2*(2*(a^3*b^2*d^
```

```
3*x + a^4*b*d^3)*cos_integral((b*d*x + a*d)/b) + 2*(a^3*b^2*d^3*x + a^4*b*d
^3)*cos_integral(-(b*d*x + a*d)/b) + (a^4*b*d^4*x + a^5*d^4)*sin_integral((
b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^7*d^3*x + a*b^6*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sin (c+dx)}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sin(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**4*sin(c + d*x)/(a + b*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1973 vs. 2(236) = 472.
time = 3.52, size = 1973, normalized size = 8.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)*a^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^4*cos(-(b*c - a*d)/b)*
cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)
 - a^4*b*c*d^4*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d) - b*c + a*d)/b) + a^5*d^5*cos(-(b*c - a*d)/b)*cos_integ
ral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x +
 a)*a^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^4*sin(-(b*c - a*d)/b)*sin_int
egral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a^4*
b*c*d^4*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b
*x + a) + d) - b*c + a*d)/b) + a^5*d^5*sin(-(b*c - a*d)/b)*sin_integral(((b
*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 4*(b*x + a)*a
^3*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*cos_integral(((b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 4*a^3*b
^2*c*d^3*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c
+ a*d)/b)*sin(-(b*c - a*d)/b) + 4*a^4*b*d^4*cos_integral(((b*x + a)*(b*c/(b
*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 4*(b*x +
 a)*a^3*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*cos(-(b*c - a*d)/b)*sin_i
ntegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 4*
a^3*b^2*c*d^3*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) -
a*d/(b*x + a) + d) - b*c + a*d)/b) - 4*a^4*b*d^4*cos(-(b*c - a*d)/b)*sin_in
tegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a^4
*b*d^4*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)^3*
b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^3*cos(-(b*x + a)*(b*c/(b*x + a) - a
```

```
*d/(b*x + a) + d)/b) + 3*(b*x + a)^2*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d
)^2*c*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 3*(b*x + a)*b
^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c^2*cos(-(b*x + a)*(b*c/(b*x + a) -
a*d/(b*x + a) + d)/b) + b^5*c^3*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
a) + d)/b) + (b*x + a)^2*a*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d*cos(
-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*(b*x + a)*a*b^3*(b*c/
(b*x + a) - a*d/(b*x + a) + d)*c*d*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x
 + a) + d)/b) + a*b^4*c^2*d*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
 d)/b) - (b*x + a)*a^2*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + a^2*b^3*c*d^2*cos(-(b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 3*a^3*b^2*d^3*cos(-(b*x + a)*(b*
c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*(b*x + a)^2*b^3*(b*c/(b*x + a) - a*
d/(b*x + a) + d)^2*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) +
4*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*sin(-(b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d)/b) - 2*b^5*c^2*sin(-(b*x + a)*(b*c/(b*x + a)
- a*d/(b*x + a) + d)/b) + 2*a^2*b^3*d^2*sin(-(b*x + a)*(b*c/(b*x + a) - a*d
/(b*x + a) + d)/b) + 2*(b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*co
s(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2*b^5*c*cos(-(b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + 2*a*b^4*d*cos(-(b*x + a)*(b*c/(b
*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a)*b^8*(b*c/(b*x + a) - a*d/(
b*x + a) + d)*d^2 - b^9*c*d^2 + a*b^8*d^3)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \sin(c + d\,x)}{(a + b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*sin(c + d*x))/(a + b*x)^2,x)
```

```
[Out] int((x^4*sin(c + d*x))/(a + b*x)^2, x)
```

3.27 $\int \frac{x^3 \sin(c+dx)}{(a+bx)^2} \, dx$

Optimal. Leaf size=181

$$\frac{2a\cos(c+dx)}{b^3d}-\frac{x\cos(c+dx)}{b^2d}-\frac{a^3d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{b^5}+\frac{3a^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4}+\frac{\sin(c+dx)}{b^2d^2}+\frac{a^3}{}$$

```
[Out] -a^3*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^5+2*a*cos(d*x+c)/b^3/d-x*cos(d*x+c)/b^
2/d+3*a^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^4-3*a^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)
/b^4-a^3*d*Si(a*d/b+d*x)*sin(-c+a*d/b)/b^5+sin(d*x+c)/b^2/d^2+a^3*sin(d*x+c
)/b^4/(b*x+a)
```

Rubi [A]
time = 0.28, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {6874, 2718, 3377, 2717, 3378, 3384, 3380, 3383}

$$-\frac{a^3d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5}+\frac{a^3d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{a^3\sin(c+dx)}{b^4(a+bx)}+\frac{3a^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4}+\frac{3a^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{2a\cos(c+dx)}{b^3d}+\frac{\sin(c+dx)}{b^2d^2}-\frac{x\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (2*a*Cos[c + d*x])/(b^3*d) - (x*Cos[c + d*x])/(b^2*d) - (a^3*d*Cos[c - (a*d
)/b]*CosIntegral[(a*d)/b + d*x])/b^5 + (3*a^2*CosIntegral[(a*d)/b + d*x]*Si
n[c - (a*d)/b])/b^4 + Sin[c + d*x]/(b^2*d^2) + (a^3*Sin[c + d*x])/(b^4*(a +
 b*x)) + (3*a^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4 + (a^3*d*S
in[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^5
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3\sin(c+dx)}{(a+bx)^2}\,dx = \int \left(-\frac{2a\sin(c+dx)}{b^3}+\frac{x\sin(c+dx)}{b^2}-\frac{a^3\sin(c+dx)}{b^3(a+bx)^2}+\frac{3a^2\sin(c+dx)}{b^3(a+bx)}\right)\,dx$$

$$= -\frac{(2a)\int\sin(c+dx)\,dx}{b^3}+\frac{(3a^2)\int\frac{\sin(c+dx)}{a+bx}\,dx}{b^3}-\frac{a^3\int\frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^3}+\frac{\int x\sin(c+dx)\,dx}{b^2}$$

$$= \frac{2a\cos(c+dx)}{b^3d}-\frac{x\cos(c+dx)}{b^2d}+\frac{a^3\sin(c+dx)}{b^4(a+bx)}+\frac{\int\cos(c+dx)\,dx}{b^2d}-\frac{(a^3d)\int\frac{\cos(c+}{a+}}{b^4}$$

$$= \frac{2a\cos(c+dx)}{b^3d}-\frac{x\cos(c+dx)}{b^2d}+\frac{3a^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4}+\frac{\sin(c+dx)}{b^2d^2}+\frac{a^3\text{s}}{b^4}$$

$$= \frac{2a\cos(c+dx)}{b^3d}-\frac{x\cos(c+dx)}{b^2d}-\frac{a^3d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{b^5}+\frac{3a^2\text{Ci}\left(\frac{ad}{b}+dx\right)\text{si}}{b^4}$$

Mathematica [A]
time = 0.58, size = 153, normalized size = 0.85

$$\frac{-a^2\text{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)-3b\sin\left(c-\frac{ad}{b}\right)\right)+\frac{b\left(bd\left(2a^2+abx-b^2x^2\right)\cos(c+dx)+\left(ab^2+a^3d^2+b^3x\right)\sin(c+dx)\right)}{d^2(a+bx)}+a^2\left(3b\cos\left(c-\frac{ad}{b}\right)+ad\sin\left(c-\frac{ad}{b}\right)\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (-(a^2*CosIntegral[d*(a/b + x)]*(a*d*Cos[c - (a*d)/b] - 3*b*Sin[c - (a*d)/b
])) + (b*(b*d*(2*a^2 + a*b*x - b^2*x^2)*Cos[c + d*x] + (a*b^2 + a^3*d^2 + b
^3*x)*Sin[c + d*x]))/(d^2*(a + b*x)) + a^2*(3*b*Cos[c - (a*d)/b] + a*d*Sin[
c - (a*d)/b])*SinIntegral[d*(a/b + x)])/b^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(186) = 372$.
time = 0.09, size = 851, normalized size = 4.70 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-d^2*c^3*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*s
in((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-3*d^2*c^2*(a
*d-b*c)/b*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*
d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+3*d^2*c^2/b*(Si(d*
x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/
b)-3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*d^2*c/b^2*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))
/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d
-b*c)/b)/b)/b)+6/b^2*(a*d-b*c)*d^2*c*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b
)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+3*d^2*c/b^2*cos(d*x+c)-(a^3*d
^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*d^2/b^3*(-sin(d*x+c)/(d*a-c*b+b*(d*
x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos
((a*d-b*c)/b)/b)/b)+3/b^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*d^2*(Si(d*x+c+(a*d-b*
c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-d^2*(2*a
*d-2*b*c-b)/b^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^3*cos(d*x + c) - 2*((a^2*(I*exp_int
egral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/b)
)*cos(c)^2 + a^2*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral
```

```
_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) - (a^2*(exp_inte
gral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*c
os(c)^2 + a^2*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -
(I*b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 - 2*((a
^2*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*
x + I*a*d)/b))*cos(c)^2 + a^2*(I*exp_integral_e(3, (I*b*d*x + I*a*d)/b) - I
*exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) - (
a^2*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x +
 I*a*d)/b))*cos(c)^2 + a^2*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_in
tegral_e(3, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*sin(d*x +
 c)^2 + ((b^2*d*x^3*cos(c) + b^2*x^2*sin(c) + 2*a*b*x*sin(c))*cos(d*x + c)^
2 + (b^2*d*x^3*cos(c) + b^2*x^2*sin(c) + 2*a*b*x*sin(c))*sin(d*x + c)^2)*co
s(d*x + 2*c) + 2*(((a^2*b^4*cos(c)^2 + a^2*b^4*sin(c)^2)*d^3*x^2 + 2*(a^3*b
^3*cos(c)^2 + a^3*b^3*sin(c)^2)*d^3*x + (a^4*b^2*cos(c)^2 + a^4*b^2*sin(c)^
2)*d^3)*cos(d*x + c)^2 + ((a^2*b^4*cos(c)^2 + a^2*b^4*sin(c)^2)*d^3*x^2 + 2
*(a^3*b^3*cos(c)^2 + a^3*b^3*sin(c)^2)*d^3*x + (a^4*b^2*cos(c)^2 + a^4*b^2*
sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(x*cos(d*x + c)/(b^4*d^2*x^3 + 3*a*
b^3*d^2*x^2 + 3*a^2*b^2*d^2*x + a^3*b*d^2), x) + 2*(((a^2*b^4*cos(c)^2 + a^
2*b^4*sin(c)^2)*d^3*x^2 + 2*(a^3*b^3*cos(c)^2 + a^3*b^3*sin(c)^2)*d^3*x + (
a^4*b^2*cos(c)^2 + a^4*b^2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((a^2*b^4*cos(c)
^2 + a^2*b^4*sin(c)^2)*d^3*x^2 + 2*(a^3*b^3*cos(c)^2 + a^3*b^3*sin(c)^2)*d^
3*x + (a^4*b^2*cos(c)^2 + a^4*b^2*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(
x*cos(d*x + c)/((b^4*d^2*x^3 + 3*a*b^3*d^2*x^2 + 3*a^2*b^2*d^2*x + a^3*b*d^
2)*cos(d*x + c)^2 + (b^4*d^2*x^3 + 3*a*b^3*d^2*x^2 + 3*a^2*b^2*d^2*x + a^3*
b*d^2)*sin(d*x + c)^2), x) + ((b^2*d*x^3*sin(c) - b^2*x^2*cos(c) - 2*a*b*x*
cos(c))*cos(d*x + c)^2 + (b^2*d*x^3*sin(c) - b^2*x^2*cos(c) - 2*a*b*x*cos(c
))*sin(d*x + c)^2)*sin(d*x + 2*c) - ((b^2*cos(c)^2 + b^2*sin(c)^2)*x^2 + 2*
(a*b*cos(c)^2 + a*b*sin(c)^2)*x)*sin(d*x + c))/(((b^4*cos(c)^2 + b^4*sin(c)
^2)*d^2*x^2 + 2*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d^2*x + (a^2*b^2*cos(c)^2
 + a^2*b^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^4*cos(c)^2 + b^4*sin(c)^2)*d
^2*x^2 + 2*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d^2*x + (a^2*b^2*cos(c)^2 + a^
2*b^2*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

<u>**Fricas**</u> [A]
time = 0.35, size = 316, normalized size = 1.75

$$-\frac{2\left(b^{4}dx^{2}-ab^{3}dx-2\,a^{2}b^{2}d\right)\cos\left(dx+c\right)+\left(\left(a^{3}bd^{3}x+a^{4}d^{3}\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+\left(a^{3}bd^{3}x+a^{4}d^{3}\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)-6\left(a^{2}b^{2}d^{2}x+a^{3}bd^{2}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)-2\left(a^{3}bd^{2}+b^{4}x+ab^{3}\right)\sin\left(dx+c\right)+\left(3\left(a^{2}b^{2}d^{2}x+a^{3}bd^{2}\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+3\left(a^{2}b^{2}d^{2}x+a^{3}bd^{2}\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)+2\left(a^{3}bd^{3}x+a^{4}d^{3}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\left(b^{6}d^{2}x+ab^{5}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^4*d*x^2 - a*b^3*d*x - 2*a^2*b^2*d)*cos(d*x + c) + ((a^3*b*d^3*x
+ a^4*d^3)*cos_integral((b*d*x + a*d)/b) + (a^3*b*d^3*x + a^4*d^3)*cos_inte
gral(-(b*d*x + a*d)/b) - 6*(a^2*b^2*d^2*x + a^3*b*d^2)*sin_integral((b*d*x
+ a*d)/b))*cos(-(b*c - a*d)/b) - 2*(a^3*b*d^2 + b^4*x + a*b^3)*sin(d*x + c)
```

```
 + (3*(a^2*b^2*d^2*x + a^3*b*d^2)*cos_integral((b*d*x + a*d)/b) + 3*(a^2*b^
2*d^2*x + a^3*b*d^2)*cos_integral(-(b*d*x + a*d)/b) + 2*(a^3*b*d^3*x + a^4*
d^3)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^6*d^2*x + a*b^5
*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1474 vs. 2(186) = 372.
time = 3.79, size = 1474, normalized size = 8.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -((b*x + a)*a^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*cos(-(b*c - a*d)/b)
*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b
) - a^3*b*c*d^3*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a)
- a*d/(b*x + a) + d) - b*c + a*d)/b) + a^4*d^4*cos(-(b*c - a*d)/b)*cos_inte
gral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x
+ a)*a^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*sin(-(b*c - a*d)/b)*sin_in
tegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a^3
*b*c*d^3*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(
b*x + a) + d) - b*c + a*d)/b) + a^4*d^4*sin(-(b*c - a*d)/b)*sin_integral(((
b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 3*(b*x + a)*
a^2*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos_integral(((b*x + a)*(b*c/
(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 3*a^2*
b^2*c*d^2*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c
 + a*d)/b)*sin(-(b*c - a*d)/b) + 3*a^3*b*d^3*cos_integral(((b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 3*(b*x
+ a)*a^2*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*sin_
integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 3
*a^2*b^2*c*d^2*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d) - b*c + a*d)/b) - 3*a^3*b*d^3*cos(-(b*c - a*d)/b)*sin_i
ntegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a^
3*b*d^3*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x + a)^2
```

```
*b^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*cos(-(b*x + a)*(b*c/(b*x + a) -
a*d/(b*x + a) + d)/b) - 2*(b*x + a)*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)
*c*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + b^4*c^2*cos(-(b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)*a*b^2*(b*c/(b*x +
 a) - a*d/(b*x + a) + d)*d*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
d)/b) + a*b^3*c*d*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - 2
*a^2*b^2*d^2*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) + (b*x +
 a)*b^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*sin(-(b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d)/b) - b^4*c*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a
) + d)/b) + a*b^3*d*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*
b^2/(((b*x + a)*b^7*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d - b^8*c*d + a*b^7
*d^2)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sin\left(c+d\,x\right)}{{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sin(c + d*x))/(a + b*x)^2,x)
```

```
[Out] int((x^3*sin(c + d*x))/(a + b*x)^2, x)
```

3.28 $\int \frac{x^2 \sin(c+dx)}{(a+bx)^2} \, dx$

Optimal. Leaf size=149

$$-\frac{\cos(c+dx)}{b^2 d}+\frac{a^2 d \cos\left(c-\frac{ad}{b}\right) \text{Ci}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{2a\text{Ci}\left(\frac{ad}{b}+dx\right) \sin\left(c-\frac{ad}{b}\right)}{b^3}-\frac{a^2 \sin(c+dx)}{b^3(a+bx)}-\frac{2a\cos\left(c-\frac{ad}{b}\right) \text{S}}{b^3}$$

```
[Out] a^2*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^4-cos(d*x+c)/b^2/d-2*a*cos(-c+a*d/b)*Si
(a*d/b+d*x)/b^3+2*a*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^3+a^2*d*Si(a*d/b+d*x)*sin
(-c+a*d/b)/b^4-a^2*sin(d*x+c)/b^3/(b*x+a)
```

Rubi [A]
time = 0.25, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6874, 2718, 3378, 3384, 3380, 3383}

$$\frac{a^2 d \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{a^2 d \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^2 \sin(c+dx)}{b^3(a+bx)}-\frac{2a \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3}-\frac{2a \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^3}-\frac{\cos(c+dx)}{b^2 d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] -(Cos[c + d*x]/(b^2*d)) + (a^2*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x
])/b^4 - (2*a*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b^3 - (a^2*Sin[c
 + d*x])/(b^3*(a + b*x)) - (2*a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x]
)/b^3 - (a^2*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^2}\,dx = \int \left(\frac{\sin(c+dx)}{b^2}+\frac{a^2\sin(c+dx)}{b^2(a+bx)^2}-\frac{2a\sin(c+dx)}{b^2(a+bx)}\right)dx$$

$$= \frac{\int \sin(c+dx)\,dx}{b^2}-\frac{(2a)\int \frac{\sin(c+dx)}{a+bx}\,dx}{b^2}+\frac{a^2\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^2}$$

$$= -\frac{\cos(c+dx)}{b^2d}-\frac{a^2\sin(c+dx)}{b^3(a+bx)}+\frac{(a^2d)\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^3}-\frac{\left(2a\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}}{b^2}$$

$$= -\frac{\cos(c+dx)}{b^2d}-\frac{2a\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3}-\frac{a^2\sin(c+dx)}{b^3(a+bx)}-\frac{2a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{a}{b}\right.}{b^3}$$

$$= -\frac{\cos(c+dx)}{b^2d}+\frac{a^2d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{2a\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3}-\frac{a^2\sin}{b^3(}$$

Mathematica [A]
time = 0.55, size = 117, normalized size = 0.79

$$\frac{a\text{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)-2b\sin\left(c-\frac{ad}{b}\right)\right)+b\left(-\frac{b\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{a+bx}\right)-a\left(2b\cos\left(c-\frac{ad}{b}\right)+ad\sin\left(c-\frac{ad}{b}\right)\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a*CosIntegral[d*(a/b + x)]*(a*d*Cos[c - (a*d)/b] - 2*b*Sin[c - (a*d)/b]) +
 b*(-((b*Cos[c + d*x])/d) - (a^2*Sin[c + d*x])/(a + b*x)) - a*(2*b*Cos[c -
(a*d)/b] + a*d*Sin[c - (a*d)/b])*SinIntegral[d*(a/b + x)])/b^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(152) = 304$.
time = 0.07, size = 553, normalized size = 3.71

method	result
risch	$-\frac{i(2ib^3d^2x^2+4iab^2d^2x+2ia^2bd^2)\cos(dx+c)}{2d^2b^3(bx+a)(-dxb-da)}+\frac{(2a^2bd^3x+2d^3a^3)\sin(dx+c)}{2d^2b^3(bx+a)(-dxb-da)}-\frac{i\cos\left(\frac{da-cb}{b}\right)\mathrm{expIntegral}\left(1,-\frac{id(bx}{b}\right.}{b^3}$
derivativedivides	$\frac{d^2c^2\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)+\frac{2d^2(da-cb)}{}}{}$
default	$\frac{d^2c^2\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)+\frac{2d^2(da-cb)}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(d^2*c^2*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*si
n((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+2*d^2*(a*d-b*
c)/b*c*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b
*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-2*d^2*c/b*(Si(d*x+c+(
a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+(a
^2*d^2-2*a*b*c*d+b^2*c^2)*d^2/b^2*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*
x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/
b)/b)-2/b^2*(a*d-b*c)*d^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+
c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-d^2/b^2*cos(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x^2*cos(d*x + c) + (x^2*cos(d*x + c)^2*cos(c) +
 x^2*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) - 2*(((a*b^2*cos(c)^2 + a*b^2*si
n(c)^2)*d*x^2 + 2*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x + (a^3*cos(c)^2 + a
^3*sin(c)^2)*d)*cos(d*x + c)^2 + ((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^2 +
 2*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)
*sin(d*x + c)^2)*integrate(x*cos(d*x + c)/(b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^
2*b*d*x + a^3*d), x) - 2*(((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^2 + 2*(a^2
```

```
*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*cos(d*
x + c)^2 + ((a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^2 + 2*(a^2*b*cos(c)^2 + a
^2*b*sin(c)^2)*d*x + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*sin(d*x + c)^2)*integ
rate(x*cos(d*x + c)/((b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^3*d)*cos(
d*x + c)^2 + (b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^3*d)*sin(d*x + c)
^2), x) + (x^2*cos(d*x + c)^2*sin(c) + x^2*sin(d*x + c)^2*sin(c))*sin(d*x +
 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^2 + 2*(a*b*cos(c)^2 + a*b*sin(c)
^2)*d*x + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^2*cos(c)^2
+ b^2*sin(c)^2)*d*x^2 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x + (a^2*cos(c)^2
 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)
```

Fricas [**A**]
time = 0.35, size = 264, normalized size = 1.77

$$-\frac{2\,a^{2}bd\sin\left(dx+c\right)+2\left(b^{3}x+ab^{2}\right)\cos\left(dx+c\right)-\left(\left(a^{2}bd^{2}x+a^{3}d^{2}\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+\left(a^{2}bd^{2}x+a^{3}d^{2}\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)-4\left(ab^{2}dx+a^{2}bd\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)-2\left(\left(ab^{2}dx+a^{2}bd\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+\left(ab^{2}dx+a^{2}bd\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)+\left(a^{2}bd^{2}x+a^{3}d^{2}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\left(b^{5}dx+ab^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b*d*sin(d*x + c) + 2*(b^3*x + a*b^2)*cos(d*x + c) - ((a^2*b*d^2
*x + a^3*d^2)*cos_integral((b*d*x + a*d)/b) + (a^2*b*d^2*x + a^3*d^2)*cos_i
ntegral(-(b*d*x + a*d)/b) - 4*(a*b^2*d*x + a^2*b*d)*sin_integral((b*d*x + a
*d)/b))*cos(-(b*c - a*d)/b) - 2*((a*b^2*d*x + a^2*b*d)*cos_integral((b*d*x
+ a*d)/b) + (a*b^2*d*x + a^2*b*d)*cos_integral(-(b*d*x + a*d)/b) + (a^2*b*d
^2*x + a^3*d^2)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^5*d*
x + a*b^4*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sin\left(c+dx\right)}{\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. $2(152) = 304$.
time = 4.12, size = 1120, normalized size = 7.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*
cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)
 - a^2*b*c*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d) - b*c + a*d)/b) + a^3*d^3*cos(-(b*c - a*d)/b)*cos_integ
ral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x +
 a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*sin(-(b*c - a*d)/b)*sin_int
egral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a^2*
b*c*d^2*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b
*x + a) + d) - b*c + a*d)/b) + a^3*d^3*sin(-(b*c - a*d)/b)*sin_integral(((b
*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 2*(b*x + a)*a
*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*cos_integral(((b*x + a)*(b*c/(b*x
+ a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 2*a*b^2*c*d
*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b
)*sin(-(b*c - a*d)/b) + 2*a^2*b*d^2*cos_integral(((b*x + a)*(b*c/(b*x + a)
- a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 2*(b*x + a)*a*b*
(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*cos(-(b*c - a*d)/b)*sin_integral(((b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 2*a*b^2*c*d*co
s(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) +
d) - b*c + a*d)/b) - 2*a^2*b*d^2*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a^2*b*d^2*sin(-(b*x
 + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - (b*x + a)*b^2*(b*c/(b*x + a)
 - a*d/(b*x + a) + d)*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b)
 + b^3*c*cos(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a*b^2*d*co
s(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a)*b^6*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b^7*c + a*b^6*d)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \sin (c + d\,x)}{(a + b\,x)^2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sin(c + d*x))/(a + b*x)^2,x)
```

```
[Out] int((x^2*sin(c + d*x))/(a + b*x)^2, x)
```

3.29 $\int \frac{x \sin(c+dx)}{(a+bx)^2} \, dx$

Optimal. Leaf size=124

$$-\frac{ad\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^2}+\frac{a\sin(c+dx)}{b^2(a+bx)}+\frac{\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(\frac{ad}{b}+dx\right)}{b^2}+\frac{ad\sin\left(\right.}{}$$

```
[Out] -a*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^3+cos(-c+a*d/b)*Si(a*d/b+d*x)/b^2-Ci(a*d
/b+d*x)*sin(-c+a*d/b)/b^2-a*d*Si(a*d/b+d*x)*sin(-c+a*d/b)/b^3+a*sin(d*x+c)/
b^2/(b*x+a)
```

Rubi [A]
time = 0.20, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$-\frac{ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{ad\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^2}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{a\sin(c+dx)}{b^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] -((a*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/b^3) + (CosIntegral[(a*
d)/b + d*x]*Sin[c - (a*d)/b])/b^2 + (a*Sin[c + d*x])/(b^2*(a + b*x)) + (Cos
[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^2 + (a*d*Sin[c - (a*d)/b]*SinIn
tegral[(a*d)/b + d*x])/b^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx &= \int \left(-\frac{a\sin(c+dx)}{b(a+bx)^2}+\frac{\sin(c+dx)}{b(a+bx)}\right)dx\\ &= \frac{\int \frac{\sin(c+dx)}{a+bx}\,dx}{b}-\frac{a\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b}\\ &= \frac{a\sin(c+dx)}{b^2(a+bx)}-\frac{(ad)\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^2}+\frac{\cos\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}+\frac{\sin\left(c-\frac{ad}{b}\right)\int \frac{\cos(\cdots}{a\cdots}}{b}\\ &= \frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^2}+\frac{a\sin(c+dx)}{b^2(a+bx)}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{\left(ad\cos\left(c-\frac{a\cdots}{b}\right.\right.}{\cdots}\\ &= -\frac{ad\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^2}+\frac{a\sin(c+dx)}{b^2(a+bx)}+\frac{\cos\left(c-\cdots\right.}{\cdots}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 96, normalized size = 0.77

$$\frac{\text{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\left(-ad\cos\left(c-\frac{ad}{b}\right)+b\sin\left(c-\frac{ad}{b}\right)\right)+\frac{ab\sin(c+dx)}{a+bx}+\left(b\cos\left(c-\frac{ad}{b}\right)+ad\sin\left(c-\frac{ad}{b}\right)\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (CosIntegral[d*(a/b + x)]*(-(a*d*Cos[c - (a*d)/b]) + b*Sin[c - (a*d)/b]) +
(a*b*Sin[c + d*x])/(a + b*x) + (b*Cos[c - (a*d)/b] + a*d*Sin[c - (a*d)/b])*
SinIntegral[d*(a/b + x)])/b^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(130) = 260$.
time = 0.06, size = 315, normalized size = 2.54

method	result
risch	$\frac{(-2abdx-2a^2d)\sin(dx+c)}{2b^2(bx+a)(-dxb-da)}+\frac{\cos\left(\frac{da-cb}{b}\right)\text{expIntegral}\left(1,\frac{id(bx+a)}{b}\right)ad}{2b^3}+\frac{\cos\left(\frac{da-cb}{b}\right)\text{expIntegral}\left(1,-\frac{id(bx+a)}{b}\right)ad}{2b^3}$
derivativedivides	$-\frac{d^2(da-cb)\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\text{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\text{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{b}+\frac{d^2}{}$
default	$-\frac{d^2(da-cb)\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\text{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\text{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{b}+\frac{d^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(-d^2*(a*d-b*c)/b*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b
*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+d^2/
b*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-
b*c)/b)/b)-d^2*c*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*
sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b*cos(c)^2 + b*sin(c)^2)*x*cos(d*x + c) + ((a*(exp_integral_e(3, (I*
b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(
exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*
d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a*(I*exp_integral_e(3, (I*b*d*x + I
*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*exp_i
ntegral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/
b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((a*(exp_integral_e(3,
(I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c)^2 +
a*(exp_integral_e(3, (I*b*d*x + I*a*d)/b) + exp_integral_e(3, -(I*b*d*x + I
*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a*(I*exp_integral_e(3, (I*b*d*x
+ I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*ex
p_integral_e(3, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(3, -(I*b*d*x + I*a*
d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*sin(d*x + c)^2 + (b*x*cos(d*x + c)^2*
cos(c) + b*x*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^4*cos(c)^2 + b^
```

```
4*sin(c)^2)*d*x^2 + 2*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d*x + (a^2*b^2*cos(
c)^2 + a^2*b^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^4*cos(c)^2 + b^4*sin(c)^2)
*d*x^2 + 2*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d*x + (a^2*b^2*cos(c)^2 + a^2*
b^2*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*x*cos(d*x + c)/(b^3*d*x^3 +
3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^3*d), x) + 2*(((b^4*cos(c)^2 + b^4*sin(c)^2
)*d*x^2 + 2*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d*x + (a^2*b^2*cos(c)^2 + a^2
*b^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^4*cos(c)^2 + b^4*sin(c)^2)*d*x^2 + 2
*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d*x + (a^2*b^2*cos(c)^2 + a^2*b^2*sin(c)
^2)*d)*sin(d*x + c)^2)*integrate(1/2*x*cos(d*x + c)/((b^3*d*x^3 + 3*a*b^2*d
*x^2 + 3*a^2*b*d*x + a^3*d)*cos(d*x + c)^2 + (b^3*d*x^3 + 3*a*b^2*d*x^2 + 3
*a^2*b*d*x + a^3*d)*sin(d*x + c)^2), x) + (b*x*cos(d*x + c)^2*sin(c) + b*x*
sin(d*x + c)^2*sin(c))*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^
2 + 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x + (a^2*b*cos(c)^2 + a^2*b*sin(c
)^2)*d)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^2 + 2*(a*b^2*co
s(c)^2 + a*b^2*sin(c)^2)*d*x + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d)*sin(d*x
 + c)^2)
```

Fricas [A]
time = 0.36, size = 208, normalized size = 1.68

$$\frac{2\,ab\sin\left(dx+c\right)-\left(\left(abdx+a^{2}d\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+\left(abdx+a^{2}d\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)-2\left(b^{2}x+ab\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)-\left(\left(b^{2}x+ab\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+\left(b^{2}x+ab\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)+2\left(abdx+a^{2}d\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\left(b^{4}x+ab^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*sin(d*x + c) - ((a*b*d*x + a^2*d)*cos_integral((b*d*x + a*d)/b)
+ (a*b*d*x + a^2*d)*cos_integral(-(b*d*x + a*d)/b) - 2*(b^2*x + a*b)*sin_in
tegral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - ((b^2*x + a*b)*cos_integral(
(b*d*x + a*d)/b) + (b^2*x + a*b)*cos_integral(-(b*d*x + a*d)/b) + 2*(a*b*d*
x + a^2*d)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^4*x + a*b
^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin\left(c+dx\right)}{\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 951 vs. 2(130) = 260.
time = 5.02, size = 951, normalized size = 7.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -((b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*c
os_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)
- a*b*c*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d) - b*c + a*d)/b) + a^2*d^3*cos(-(b*c - a*d)/b)*cos_integral
(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a)
*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*sin(-(b*c - a*d)/b)*sin_integral
(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a*b*c*d^2
*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
 + d) - b*c + a*d)/b) + a^2*d^3*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a)*b*(b*c/(b*
x + a) - a*d/(b*x + a) + d)*d*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/
(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - b^2*c*d*cos_integral((
(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a
*d)/b) + a*b*d^2*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d
) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - (b*x + a)*b*(b*c/(b*x + a) - a*d/(b
*x + a) + d)*d*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d) - b*c + a*d)/b) + b^2*c*d*cos(-(b*c - a*d)/b)*sin_integ
ral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a*b*d^
2*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a
) + d) - b*c + a*d)/b) + a*b*d^2*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
 a) + d)/b))*b/(((b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b^5*c
+ a*b^4*d)*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin\left(c+d\,x\right)}{{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(c + d*x))/(a + b*x)^2,x)
```

```
[Out] int((x*sin(c + d*x))/(a + b*x)^2, x)
```

3.30 $\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx$

Optimal. Leaf size=72

$$\frac{d\cos\left(c-\frac{ad}{b}\right)\operatorname{Ci}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{\sin(c+dx)}{b(a+bx)}-\frac{d\sin\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+dx\right)}{b^2}$$

```
[Out] d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^2+d*Si(a*d/b+d*x)*sin(-c+a*d/b)/b^2-sin(d*x
+c)/b/(b*x+a)
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3378, 3384, 3380, 3383}

$$\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{d\sin\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(xd+\frac{ad}{b}\right)}{b^2}-\frac{\sin(c+dx)}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x)^2,x]
```

```
[Out] (d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/b^2 - Sin[c + d*x]/(b*(a +
b*x)) - (d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx &= -\frac{\sin(c+dx)}{b(a+bx)}+\frac{d\int \frac{\cos(c+dx)}{a+bx}\,dx}{b}\\ &= -\frac{\sin(c+dx)}{b(a+bx)}+\frac{\left(d\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}-\frac{\left(d\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}\\ &= \frac{d\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{\sin(c+dx)}{b(a+bx)}-\frac{d\sin\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(\frac{ad}{b}+dx\right)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 66, normalized size = 0.92

$$\frac{d\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(d\left(\frac{a}{b}+x\right)\right)-\frac{b\sin(c+dx)}{a+bx}-d\sin\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*x)^2,x]
```

```
[Out] (d*Cos[c - (a*d)/b]*CosIntegral[d*(a/b + x)] - (b*Sin[c + d*x])/(a + b*x) -
 d*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/b^2
```

Maple [A]
time = 0.05, size = 107, normalized size = 1.49

method	result
derivativedivides	$d\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)$
default	$d\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)$
risch	$-\frac{d\,\mathrm{e}^{-\frac{i(da-cb)}{b}}\,\mathrm{expIntegral}\left(1,-idx-ic-\frac{iad-ibc}{b}\right)}{2b^2}-\frac{d\,\mathrm{e}^{\frac{i(da-cb)}{b}}\,\mathrm{expIntegral}\left(1,idx+ic+\frac{i(da-cb)}{b}\right)}{2b^2}-\frac{(-2dxb-2da}{2b(bx+a)(-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

[Out] d*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b
)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)

Maxima [C] Result contains complex when optimal does not.
time = 0.37, size = 164, normalized size = 2.28

$$\frac{d^2\left(-i\,E_2\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+i\,E_2\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+d^2\left(E_2\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+E_2\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\,((dx+c)b^2-b^2c+abd)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")

[Out] 1/2*(d^2*(-I*exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_i
ntegral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d^2
(exp_integral_e(2, (I(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(2,
-(I*(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))/(((d*x + c)*b^2 -
b^2*c + a*b*d)*d)

Fricas [A]
time = 0.34, size = 123, normalized size = 1.71

$$\frac{2\,(bdx+ad)\sin\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)+\left((bdx+ad)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+(bdx+ad)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)-2\,b\sin\left(dx+c\right)}{2\,(b^3x+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")

[Out] 1/2*(2*(b*d*x + a*d)*sin(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + ((
b*d*x + a*d)*cos_integral((b*d*x + a*d)/b) + (b*d*x + a*d)*cos_integral(-(b
*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - 2*b*sin(d*x + c))/(b^3*x + a*b^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(c+dx\right)}{\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/(b*x+a)**2,x)

[Out] Integral(sin(c + d*x)/(a + b*x)**2, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(73) =
146.
time = 5.99, size = 518, normalized size = 7.19

$$\frac{\left((bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d^2\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Ci}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)-bcd^2\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Ci}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)+ad^3\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Ci}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)+(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d^2\sin\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)-bcd^2\sin\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)+ad^3\sin\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)+bd^2\sin\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)}{b}\right)\right)b^2}{\left((bx+a)b^4\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-b^5c+ab^4d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*cos_
integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - b
*c*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*
x + a) + d) - b*c + a*d)/b) + a*d^3*cos(-(b*c - a*d)/b)*cos_integral(((b*x
+ a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d)*d^2*sin(-(b*c - a*d)/b)*sin_integral(((b*x +
a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - b*c*d^2*sin(-(b*c
- a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c
 + a*d)/b) + a*d^3*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x +
a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + b*d^2*sin(-(b*x + a)*(b*c/(b*x +
a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x +
 a) + d) - b^5*c + a*b^4*d)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)}{(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*x)^2,x)
```

```
[Out] int(sin(c + d*x)/(a + b*x)^2, x)
```

3.31 $\int \frac{\sin(c+dx)}{x(a+bx)^2} dx$

Optimal. Leaf size=149

$$-\frac{d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{ab}+\frac{\text{Ci}(dx)\sin(c)}{a^2}-\frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2}+\frac{\sin(c+dx)}{a(a+bx)}+\frac{\cos(c)\text{Si}(dx)}{a^2}-\frac{\cos\left(c-\right.}{}$$

```
[Out] -d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/a/b+cos(c)*Si(d*x)/a^2-cos(-c+a*d/b)*Si(a*d/
b+d*x)/a^2+Ci(d*x)*sin(c)/a^2+Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^2-d*Si(a*d/b+d*
x)*sin(-c+a*d/b)/a/b+sin(d*x+c)/a/(b*x+a)
```

Rubi [A]

time = 0.28, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3384, 3380, 3383, 3378}

$$-\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2}-\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{\sin(c)\text{CosIntegral}(dx)}{a^2}+\frac{\cos(c)\text{Si}(dx)}{a^2}-\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{ab}+\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{ab}+\frac{\sin(c+dx)}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x)^2),x]
```

```
[Out] -((d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/(a*b)) + (CosIntegral[d*x
]*Sin[c])/a^2 - (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^2 + Sin[c +
 d*x]/(a*(a + b*x)) + (Cos[c]*SinIntegral[d*x])/a^2 - (Cos[c - (a*d)/b]*Sin
Integral[(a*d)/b + d*x])/a^2 + (d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*
x])/(a*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx &= \int \left(\frac{\sin(c+dx)}{a^2x} - \frac{b\sin(c+dx)}{a(a+bx)^2} - \frac{b\sin(c+dx)}{a^2(a+bx)}\right) dx \\
&= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^2} - \frac{b\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^2} - \frac{b\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{a} \\
&= \frac{\sin(c+dx)}{a(a+bx)} - \frac{d\int \frac{\cos(c+dx)}{a+bx}\,dx}{a} + \frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a^2} - \frac{\left(b\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2} + \\
&= \frac{\text{Ci}(dx)\sin(c)}{a^2} - \frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2} + \frac{\sin(c+dx)}{a(a+bx)} + \frac{\cos(c)\text{Si}(dx)}{a^2} - \frac{\cos\left(c-\frac{ad}{b}\right)}{a} \\
&= -\frac{d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{ab} + \frac{\text{Ci}(dx)\sin(c)}{a^2} - \frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2} + \frac{\sin(c+dx)}{a(a+bx)}
\end{aligned}
$$

Mathematica [A]
time = 0.70, size = 138, normalized size = 0.93

$$
\frac{\frac{a\cos(dx)\sin(c)}{a+bx} + \text{Ci}(dx)\sin(c) - \frac{\text{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)+b\sin\left(c-\frac{ad}{b}\right)\right)}{b} + \frac{a\cos(c)\sin(dx)}{a+bx} + \cos(c)\text{Si}(dx) - \cos\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right) + \frac{ad\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{b}}{a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x)^2),x]
```

```
[Out] ((a*Cos[d*x]*Sin[c])/(a + b*x) + CosIntegral[d*x]*Sin[c] - (CosIntegral[d*(
a/b + x)]*(a*d*Cos[c - (a*d)/b] + b*Sin[c - (a*d)/b]))/b + (a*Cos[c]*Sin[d*
x])/(a + b*x) + Cos[c]*SinIntegral[d*x] - Cos[c - (a*d)/b]*SinIntegral[d*(a
/b + x)] + (a*d*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/b)/a^2
```

Maple [A]
time = 0.11, size = 210, normalized size = 1.41

method	result
derivativedivides	$-\frac{db\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\text{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\text{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{a}+\frac{\text{sinInteg}}{}$
default	$-\frac{db\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\text{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\text{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{a}+\frac{\text{sinInteg}}{}$
risch	$\frac{d\,\mathrm{e}^{-\frac{i(da-cb)}{b}}\text{expIntegral}\left(1,-idx-ic-\frac{iad-ibc}{b}\right)}{2ab}-\frac{i\mathrm{e}^{-\frac{i(da-cb)}{b}}\text{expIntegral}\left(1,-idx-ic-\frac{iad-ibc}{b}\right)}{2a^2}+\frac{i\mathrm{e}^{ic}\,\text{expIntegral}(}{2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d*b/a*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b
*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+1/a^2*(Si(d*x)*cos(c)
+Ci(d*x)*sin(c))-b/a^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(
a*d-b*c)/b)*sin((a*d-b*c)/b)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)^2*x), x)
```

Fricas [A]
time = 0.36, size = 260, normalized size = 1.74

$$\frac{2\,ab\sin\left(dx+c\right)+2\left(b^2x+ab\right)\cos\left(c\right)\mathrm{Si}\left(dx\right)-\left(\left(abdx+a^2d\right)\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+\left(abdx+a^2d\right)\mathrm{Ci}\left(-\frac{bdx+ad}{b}\right)+2\left(b^2x+ab\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+\left(\left(b^2x+ab\right)\mathrm{Ci}\left(dx\right)+\left(b^2x+ab\right)\mathrm{Ci}\left(-dx\right)\right)\sin\left(c\right)+\left(\left(b^2x+ab\right)\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+\left(b^2x+ab\right)\mathrm{Ci}\left(-\frac{bdx+ad}{b}\right)-2\left(abdx+a^2d\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\left(a^2b^2x+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*sin(d*x + c) + 2*(b^2*x + a*b)*cos(c)*sin_integral(d*x) - ((a*b*
d*x + a^2*d)*cos_integral((b*d*x + a*d)/b) + (a*b*d*x + a^2*d)*cos_integral
(-(b*d*x + a*d)/b) + 2*(b^2*x + a*b)*sin_integral((b*d*x + a*d)/b))*cos(-(b
*c - a*d)/b) + ((b^2*x + a*b)*cos_integral(d*x) + (b^2*x + a*b)*cos_integra
l(-d*x))*sin(c) + ((b^2*x + a*b)*cos_integral((b*d*x + a*d)/b) + (b^2*x + a
*b)*cos_integral(-(b*d*x + a*d)/b) - 2*(a*b*d*x + a^2*d)*sin_integral((b*d*
x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^2*b^2*x + a^3*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)**2,x)
```

```
[Out] Integral(sin(c + d*x)/(x*(a + b*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1281 vs. $2(153) = 306$.
time = 4.15, size = 1281, normalized size = 8.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -((b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*c
os_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)
- a*b*c*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d) - b*c + a*d)/b) + a^2*d^3*cos(-(b*c - a*d)/b)*cos_integral
(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a)
*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*sin(-(b*c - a*d)/b)*sin_integral
(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - a*b*c*d^2
*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
 + d) - b*c + a*d)/b) + a^2*d^3*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - (b*x + a)*b*(b*c/(b*
x + a) - a*d/(b*x + a) + d)*d*cos_integral((b*x + a)*(b*c/(b*x + a) - a*d/(
b*x + a) + d)/b - c)*sin(c) + b^2*c*d*cos_integral((b*x + a)*(b*c/(b*x + a)
 - a*d/(b*x + a) + d)/b - c)*sin(c) - a*b*d^2*cos_integral((b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d)/b - c)*sin(c) - (b*x + a)*b*(b*c/(b*x + a) -
a*d/(b*x + a) + d)*d*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
 + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) + b^2*c*d*cos_integral(((b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) -
a*b*d^2*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
 a*d)/b)*sin(-(b*c - a*d)/b) + (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) +
 d)*d*cos(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b
+ c) - b^2*c*d*cos(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a
) + d)/b + c) + a*b*d^2*cos(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*d
/(b*x + a) + d)/b + c) + (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*
cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
+ d) - b*c + a*d)/b) - b^2*c*d*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*
```

```
(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a*b*d^2*cos(-(b*c - a
*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
a*d)/b) + a*b*d^2*sin(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^
3/(((b*x + a)*a^2*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d) - a^2*b^5*c + a^3
*b^4*d)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x*(a + b*x)^2),x)
```

```
[Out] int(sin(c + d*x)/(x*(a + b*x)^2), x)
```

3.32 $\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx$

Optimal. Leaf size=188

$$\frac{d\cos(c)\text{Ci}(dx)}{a^2}+\frac{d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{a^2}-\frac{2b\text{Ci}(dx)\sin(c)}{a^3}+\frac{2b\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3}-\frac{\sin(c+dx)}{a^2x}-\frac{b}{a}$$

```
[Out] d*Ci(d*x)*cos(c)/a^2+d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/a^2-2*b*cos(c)*Si(d*x)/a
^3+2*b*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^3-2*b*Ci(d*x)*sin(c)/a^3-d*Si(d*x)*sin
(c)/a^2-2*b*Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^3+d*Si(a*d/b+d*x)*sin(-c+a*d/b)/a
^2-sin(d*x+c)/a^2/x-b*sin(d*x+c)/a^2/(b*x+a)
```

Rubi [A]
time = 0.35, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$-\frac{2b\sin(c)\text{CosIntegral}(dx)}{a^3}+\frac{2b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^3}-\frac{2b\cos(c)\text{Si}(dx)}{a^3}+\frac{2b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^3}+\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2}-\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{b\sin(c+dx)}{a^2(a+bx)}+\frac{d\cos(c)\text{CosIntegral}(dx)}{a^2}-\frac{d\sin(c)\text{Si}(dx)}{a^2}-\frac{\sin(c+dx)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x)^2),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a^2 + (d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b +
 d*x])/a^2 - (2*b*CosIntegral[d*x]*Sin[c])/a^3 + (2*b*CosIntegral[(a*d)/b +
 d*x]*Sin[c - (a*d)/b])/a^3 - Sin[c + d*x]/(a^2*x) - (b*Sin[c + d*x])/(a^2*
(a + b*x)) - (2*b*Cos[c]*SinIntegral[d*x])/a^3 - (d*Sin[c]*SinIntegral[d*x]
)/a^2 + (2*b*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^3 - (d*Sin[c -
(a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx = \int \left(\frac{\sin(c+dx)}{a^2x^2} - \frac{2b\sin(c+dx)}{a^3x} + \frac{b^2\sin(c+dx)}{a^2(a+bx)^2} + \frac{2b^2\sin(c+dx)}{a^3(a+bx)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a^2} - \frac{(2b)\int \frac{\sin(c+dx)}{x}\,dx}{a^3} + \frac{(2b^2)\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^3} + \frac{b^2\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{a^2x} - \frac{b\sin(c+dx)}{a^2(a+bx)} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{a^2} + \frac{(bd)\int \frac{\cos(c+dx)}{a+bx}\,dx}{a^2} - \frac{(2b\cos(c))\int \frac{\sin(}{x}}{a^3}$$

$$= -\frac{2b\operatorname{Ci}(dx)\sin(c)}{a^3} + \frac{2b\operatorname{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} - \frac{\sin(c+dx)}{a^2x} - \frac{b\sin(c+dx)}{a^2(a+bx)} - \frac{2b\cos(}{}$$

$$= \frac{d\cos(c)\operatorname{Ci}(dx)}{a^2} + \frac{d\cos\left(c-\frac{ad}{b}\right)\operatorname{Ci}\left(\frac{ad}{b}+dx\right)}{a^2} - \frac{2b\operatorname{Ci}(dx)\sin(c)}{a^3} + \frac{2b\operatorname{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\right.}{a^3}$$

Mathematica [A]
time = 1.22, size = 184, normalized size = 0.98

$$-\frac{-ad\cos(c)\operatorname{Ci}(dx) - ad\cos\left(c-\frac{ad}{b}\right)\operatorname{Ci}\left(d\left(\frac{a}{b}+x\right)\right) + \frac{a(a+2bx)\cos(dx)\sin(c)}{x(a+bx)} + 2b\operatorname{Ci}(dx)\sin(c) - 2b\operatorname{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right) + \frac{a(a+2bx)\cos(c)\sin(dx)}{x(a+bx)} + 2b\cos(c)\operatorname{Si}(dx) + ad\sin(c)\operatorname{Si}(dx) - 2b\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(d\left(\frac{a}{b}+x\right)\right) + ad\sin\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x)^2),x]
```

```
[Out] -((-(a*d*Cos[c]*CosIntegral[d*x]) - a*d*Cos[c - (a*d)/b]*CosIntegral[d*(a/b
 + x)] + (a*(a + 2*b*x)*Cos[d*x]*Sin[c])/(x*(a + b*x)) + 2*b*CosIntegral[d*
x]*Sin[c] - 2*b*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + (a*(a + 2*b*x)*
Cos[c]*Sin[d*x])/(x*(a + b*x)) + 2*b*Cos[c]*SinIntegral[d*x] + a*d*Sin[c]*S
```

```
inIntegral[d*x] - 2*b*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + a*d*Sin[c
 - (a*d)/b]*SinIntegral[d*(a/b + x)])/a^3)
```

Maple [A]
time = 0.12, size = 256, normalized size = 1.36

method	result
derivativedivides	$d\left(\frac{b^2\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{a^2}+\frac{-\sin\cdots}{\cdots}\right.$
default	$d\left(\frac{b^2\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{a^2}+\frac{-\sin\cdots}{\cdots}\right.$
risch	$-\frac{d\,\mathrm{e}^{-\frac{i(da-cb)}{b}}\,\mathrm{expIntegral}\left(1,-idx-ic-\frac{iad-ibc}{b}\right)}{2a^2}-\frac{d\,\mathrm{e}^{ic}\,\mathrm{expIntegral}(1,-idx)}{2a^2}+\frac{ib\,\mathrm{e}^{-\frac{i(da-cb)}{b}}\,\mathrm{expIntegral}\left(1,-idx\cdots\right.}{a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(b^2/a^2*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a
*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+1/a^2*(-sin(d*x+c
)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))-2/d/a^3*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(
c))+2/d*b^2/a^3*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c
)/b)*sin((a*d-b*c)/b)/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)^2*x^2), x)
```

Fricas [A]
time = 0.38, size = 355, normalized size = 1.89

$$\frac{\left(\left(abdx^2+a^2dx\right)\mathrm{Ci}\left(dx\right)+\left(abdx^2+a^2dx\right)\mathrm{Ci}\left(-dx\right)-4\left(b^2x^2+abx\right)\mathrm{Si}\left(dx\right)\right)\cos\left(c\right)+\left(\left(abdx^2+a^2dx\right)\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+\left(abdx^2+a^2dx\right)\mathrm{Ci}\left(-\frac{bdx+ad}{b}\right)+4\left(b^2x^2+abx\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)-2\left(2\,abx+a^2\right)\sin\left(dx+c\right)-2\left(\left(b^2x^2+abx\right)\mathrm{Ci}\left(dx\right)+\left(b^2x^2+abx\right)\mathrm{Ci}\left(-dx\right)+\left(abdx^2+a^2dx\right)\mathrm{Si}\left(dx\right)\right)\sin\left(c\right)-2\left(\left(b^2x^2+abx\right)\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+\left(b^2x^2+abx\right)\mathrm{Ci}\left(-\frac{bdx+ad}{b}\right)-\left(abdx^2+a^2dx\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\left(a^3bx^2+a^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(((a*b*d*x^2 + a^2*d*x)*cos_integral(d*x) + (a*b*d*x^2 + a^2*d*x)*cos_i
ntegral(-d*x) - 4*(b^2*x^2 + a*b*x)*sin_integral(d*x))*cos(c) + ((a*b*d*x^2
 + a^2*d*x)*cos_integral((b*d*x + a*d)/b) + (a*b*d*x^2 + a^2*d*x)*cos_integ
ral(-(b*d*x + a*d)/b) + 4*(b^2*x^2 + a*b*x)*sin_integral((b*d*x + a*d)/b))*
cos(-(b*c - a*d)/b) - 2*(2*a*b*x + a^2)*sin(d*x + c) - 2*((b^2*x^2 + a*b*x)
*cos_integral(d*x) + (b^2*x^2 + a*b*x)*cos_integral(-d*x) + (a*b*d*x^2 + a^
2*d*x)*sin_integral(d*x))*sin(c) - 2*((b^2*x^2 + a*b*x)*cos_integral((b*d*x
 + a*d)/b) + (b^2*x^2 + a*b*x)*cos_integral(-(b*d*x + a*d)/b) - (a*b*d*x^2
+ a^2*d*x)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^3*b*x^2 +
 a^4*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{x^2\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x+a)**2,x)
```

```
[Out] Integral(sin(c + d*x)/(x**2*(a + b*x)**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3180 vs. $2(191) = 382$.
time = 3.94, size = 3180, normalized size = 16.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*cos(c)*cos_integra
l((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)/b - 2*(b*x + a)*a*(b
*c/(b*x + a) - a*d/(b*x + a) + d)*c*d^2*cos(c)*cos_integral((b*x + a)*(b*c/
(b*x + a) - a*d/(b*x + a) + d)/b - c) + a*b*c^2*d^2*cos(c)*cos_integral((b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c) + (b*x + a)*a^2*(b*c/(b*x
 + a) - a*d/(b*x + a) + d)*d^3*cos(c)*cos_integral((b*x + a)*(b*c/(b*x + a)
 - a*d/(b*x + a) + d)/b - c)/b - a^2*c*d^3*cos(c)*cos_integral((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d)/b - c) + (b*x + a)^2*a*(b*c/(b*x + a) - a
*d/(b*x + a) + d)^2*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b
*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)/b - 2*(b*x + a)*a*(b*c/(b*x +
a) - a*d/(b*x + a) + d)*c*d^2*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(
b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + a*b*c^2*d^2*cos(-(b*c
- a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c
```

```
 + a*d)/b) + (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*cos(-(b*
c - a*d)/b)*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b
*c + a*d)/b)/b - a^2*c*d^3*cos(-(b*c - a*d)/b)*cos_integral(((b*x + a)*(b*c
/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a)^2*a*(b*c/(b*x +
 a) - a*d/(b*x + a) + d)^2*d^2*sin(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a
) - a*d/(b*x + a) + d)/b + c)/b - 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x +
 a) + d)*c*d^2*sin(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a
) + d)/b + c) + a*b*c^2*d^2*sin(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) -
 a*d/(b*x + a) + d)/b + c) + (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) +
 d)*d^3*sin(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/
b + c)/b - a^2*c*d^3*sin(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*d/(b
*x + a) + d)/b + c) + (b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d
^2*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x +
a) + d) - b*c + a*d)/b)/b - 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) +
d)*c*d^2*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(
b*x + a) + d) - b*c + a*d)/b) + a*b*c^2*d^2*sin(-(b*c - a*d)/b)*sin_integra
l(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + (b*x + a
)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*sin(-(b*c - a*d)/b)*sin_integ
ral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)/b - a^2*
c*d^3*sin(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x
 + a) + d) - b*c + a*d)/b) - 2*(b*x + a)^2*(b*c/(b*x + a) - a*d/(b*x + a) +
 d)^2*d*cos_integral((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*s
in(c) + 4*(b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*cos_integral(
(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*sin(c) - 2*b^2*c^2*d*c
os_integral((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*sin(c) - 2
*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos_integral((b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*sin(c) + 2*a*b*c*d^2*cos_integr
al((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*sin(c) - 2*(b*x + a
)^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d*cos_integral(((b*x + a)*(b*c/(b
*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) + 4*(b*x +
 a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d*cos_integral(((b*x + a)*(b*c/
(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) - 2*b^2*
c^2*d*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a
*d)/b)*sin(-(b*c - a*d)/b) - 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) +
 d)*d^2*cos_integral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
 a*d)/b)*sin(-(b*c - a*d)/b) + 2*a*b*c*d^2*cos_integral(((b*x + a)*(b*c/(b*
x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*sin(-(b*c - a*d)/b) + 2*(b*x +
a)^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d*cos(c)*sin_integral(-(b*x + a)
*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c) - 4*(b*x + a)*b*(b*c/(b*x + a)
- a*d/(b*x + a) + d)*c*d*cos(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d)/b + c) + 2*b^2*c^2*d*cos(c)*sin_integral(-(b*x + a)*(b*c/(
b*x + a) - a*d/(b*x + a) + d)/b + c) + 2*(b*x + a)*a*(b*c/(b*x + a) - a*d/(
b*x + a) + d)*d^2*cos(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x
+ a) + d)/b + c) - 2*a*b*c*d^2*cos(c)*sin_integral(-(b*x + a)*(b*c/(b*x + a
) - a*d/(b*x + a) + d)/b + c) + 2*(b*x + a)^2*(b*c/(b*x + a) - a*d/(b*x + a
```

```
) + d)^2*d*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d
/(b*x + a) + d) - b*c + a*d)/b) - 4*(b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x +
 a) + d)*c*d*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a
*d/(b*x + a) + d) - b*c + a*d)/b) + 2*b^2*c^2*d*cos(-(b*c - a*d)/b)*sin_int
egral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) + 2*(b
*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*cos(-(b*c - a*d)/b)*sin_i
ntegral(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b) - 2*
a*b*c*d^2*cos(-(b*c - a*d)/b)*sin_integral(((b*x + a)*(b*c/(b*x + a) - a*d/
(b*x + a) + d) - b*c + a*d)/b) + 2*(b*x + a)*a*...
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)}{x^2\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x^2*(a + b*x)^2),x)
```

```
[Out] int(sin(c + d*x)/(x^2*(a + b*x)^2), x)
```

3.33 $\int \frac{x^3 \sin(c+dx)}{(a+bx)^3}\,dx$

Optimal. Leaf size=265

$$-\frac{\cos(c+dx)}{b^3 d}+\frac{a^3 d\cos(c+dx)}{2b^5(a+bx)}+\frac{3a^2 d\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{b^5}-\frac{3a\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4}+\frac{a^3 d^2\mathrm{Ci}\left(\frac{ad}{b}+\right.}{}$$

```
[Out] 3*a^2*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^5-cos(d*x+c)/b^3/d+1/2*a^3*d*cos(d*x+
c)/b^5/(b*x+a)-3*a*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^4+1/2*a^3*d^2*cos(-c+a*d/b
)*Si(a*d/b+d*x)/b^6+3*a*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^4-1/2*a^3*d^2*Ci(a*d/
b+d*x)*sin(-c+a*d/b)/b^6+3*a^2*d*Si(a*d/b+d*x)*sin(-c+a*d/b)/b^5+1/2*a^3*si
n(d*x+c)/b^4/(b*x+a)^2-3*a^2*sin(d*x+c)/b^4/(b*x+a)
```

Rubi [A]
time = 0.42, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6874, 2718, 3378, 3384, 3380, 3383}

$$\frac{a^3 d^2 \sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2b^6}+\frac{a^3 d^2\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(xd+\frac{ad}{b}\right)}{2b^6}+\frac{a^3 d\cos(c+dx)}{2b^5(a+bx)}+\frac{a^3\sin(c+dx)}{2b^4(a+bx)^2}+\frac{3a^2 d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5}-\frac{3a^2 d\sin\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(xd+\frac{ad}{b}\right)}{b^5}-\frac{3a^2\sin(c+dx)}{b^4(a+bx)}-\frac{3a\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{3a\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{\cos(c+dx)}{b^3 d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -(Cos[c + d*x]/(b^3*d)) + (a^3*d*Cos[c + d*x])/(2*b^5*(a + b*x)) + (3*a^2*d
*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/b^5 - (3*a*CosIntegral[(a*d)/
b + d*x]*Sin[c - (a*d)/b])/b^4 + (a^3*d^2*CosIntegral[(a*d)/b + d*x]*Sin[c
- (a*d)/b])/(2*b^6) + (a^3*Sin[c + d*x])/(2*b^4*(a + b*x)^2) - (3*a^2*Sin[c
 + d*x])/(b^4*(a + b*x)) - (3*a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x]
)/b^4 + (a^3*d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*b^6) - (3*
a^2*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^5
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sin(c+dx)}{(a+bx)^3}\,dx &= \int \left(\frac{\sin(c+dx)}{b^3}-\frac{a^3\sin(c+dx)}{b^3(a+bx)^3}+\frac{3a^2\sin(c+dx)}{b^3(a+bx)^2}-\frac{3a\sin(c+dx)}{b^3(a+bx)}\right)dx\\
&= \frac{\int \sin(c+dx)\,dx}{b^3}-\frac{(3a)\int \frac{\sin(c+dx)}{a+bx}\,dx}{b^3}+\frac{(3a^2)\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^3}-\frac{a^3\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{b^3}\\
&= -\frac{\cos(c+dx)}{b^3d}+\frac{a^3\sin(c+dx)}{2b^4(a+bx)^2}-\frac{3a^2\sin(c+dx)}{b^4(a+bx)}+\frac{(3a^2d)\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^4}-\frac{(a^3d)\int \frac{\cos}{(}}{2b}\\
&= -\frac{\cos(c+dx)}{b^3d}+\frac{a^3d\cos(c+dx)}{2b^5(a+bx)}-\frac{3a\operatorname{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4}+\frac{a^3\sin(c+dx)}{2b^4(a+bx)^2}-3a\\
&= -\frac{\cos(c+dx)}{b^3d}+\frac{a^3d\cos(c+dx)}{2b^5(a+bx)}+\frac{3a^2d\cos\left(c-\frac{ad}{b}\right)\operatorname{Ci}\left(\frac{ad}{b}+dx\right)}{b^5}-\frac{3a\operatorname{Ci}\left(\frac{ad}{b}+dx\right)\operatorname{si}}{b^4}\\
&= -\frac{\cos(c+dx)}{b^3d}+\frac{a^3d\cos(c+dx)}{2b^5(a+bx)}+\frac{3a^2d\cos\left(c-\frac{ad}{b}\right)\operatorname{Ci}\left(\frac{ad}{b}+dx\right)}{b^5}-\frac{3a\operatorname{Ci}\left(\frac{ad}{b}+dx\right)\operatorname{si}}{b^4}
\end{aligned}$$

Mathematica [A]
time = 0.64, size = 235, normalized size = 0.89

$$-\frac{b\cos(dx)\left(-\left((a+bx)\left(-2ab^2+a^3d^2-2b^3x\right)\cos(c)\right)+a^2bd(5a+6bx)\sin(c)\right)+b\left(a^2bd(5a+6bx)\cos(c)+(a+bx)\left(-2ab^2+a^3d^2-2b^3x\right)\sin(c)\right)\sin(dx)-ad(a+bx)^2\left(\operatorname{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\left(6abd\cos\left(c-\frac{ad}{b}\right)+\left(-6b^2+a^2d^2\right)\sin\left(c-\frac{ad}{b}\right)\right)+\left(\left(-6b^2+a^2d^2\right)\cos\left(c-\frac{ad}{b}\right)-6abd\sin\left(c-\frac{ad}{b}\right)\right)\operatorname{Si}\left(d\left(\frac{a}{b}+x\right)\right)\right)}{2b^6d(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -1/2*(b*Cos[d*x]*(-((a + b*x)*(-2*a*b^2 + a^3*d^2 - 2*b^3*x)*Cos[c]) + a^2*
b*d*(5*a + 6*b*x)*Sin[c]) + b*(a^2*b*d*(5*a + 6*b*x)*Cos[c] + (a + b*x)*(-2
*a*b^2 + a^3*d^2 - 2*b^3*x)*Sin[c])*Sin[d*x] - a*d*(a + b*x)^2*(CosIntegral
[d*(a/b + x)]*(6*a*b*d*Cos[c - (a*d)/b] + (-6*b^2 + a^2*d^2)*Sin[c - (a*d)/
b]) + ((-6*b^2 + a^2*d^2)*Cos[c - (a*d)/b] - 6*a*b*d*Sin[c - (a*d)/b])*SinI
ntegral[d*(a/b + x)]))/(b^6*d*(a + b*x)^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1207 vs. $2(263) = 526$.
time = 0.24, size = 1208, normalized size = 4.56

method	result
risch	$\frac{i\left(2ia^3b^3d^6x^3+6ia^4b^2d^6x^2-4ib^6d^4x^4+6ia^5b\,d^6x-16ia\,b^5d^4x^3+2ia^6d^6-24ia^2b^4d^4x^2-16ia^3b^3d^4x-4ia^4b^2d^4\right)\cos(dx+}{4b^5d^3(bx+a)^2\left(-d^2x^2b^2-2ab\,d^2x-d^2a^2\right)}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-d^3*c^3*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b+1/2*(-cos(d*x+c)/(
d*a-c*b+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*
d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)+3*d^3*c^2/b*(-sin(d*x+c)/(d*a-c*b+b*(d*
x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos
((a*d-b*c)/b)/b)/b)-3*d^3*c^2*(a*d-b*c)/b*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+
c))^2/b+1/2*(-cos(d*x+c)/(d*a-c*b+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((
a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)+6*d^3*c*(a*d-
b*c)/b^2*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d
-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-3*d^3*c*(a*d-b*c)^2
/b^2*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b+1/2*(-cos(d*x+c)/(d*a-c*b+b*(
d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*s
in((a*d-b*c)/b)/b)/b)/b)-3*d^3*c/b^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b
)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+3/b^3*(a^2*d^2-2*a*b*c*d+b^2*
c^2)*d^3*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d
-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-(a^3*d^3-3*a^2*b*c*
d^2+3*a*b^2*c^2*d-b^3*c^3)*d^3/b^3*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b
+1/2*(-cos(d*x+c)/(d*a-c*b+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c
)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)-3/b^3*(a*d-b*c)*d^3*
(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*
c)/b)/b)-d^3/b^3*cos(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^3*cos(d*x + c) + 3*((a^2*(-I*exp_in
tegral_e(4, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b
))*cos(c)^2 + a^2*(-I*exp_integral_e(4, (I*b*d*x + I*a*d)/b) + I*exp_integr
al_e(4, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a^2*(exp_in
tegral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))
*cos(c)^2 + a^2*(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4,
 -(I*b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 - 3*(
a*b*cos(c)^2 + a*b*sin(c)^2)*x*sin(d*x + c) + 3*((a^2*(-I*exp_integral_e(4,
 (I*b*d*x + I*a*d)/b) + I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2
 + a^2*(-I*exp_integral_e(4, (I*b*d*x + I*a*d)/b) + I*exp_integral_e(4, -(I
*b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a^2*(exp_integral_e(4,
 (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 +
 a^2*(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x
+ I*a*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*sin(d*x + c)^2 + ((b^2*d*x^3*co
s(c) + 3*a*b*x*sin(c))*cos(d*x + c)^2 + (b^2*d*x^3*cos(c) + 3*a*b*x*sin(c))
*sin(d*x + c)^2)*cos(d*x + 2*c) + 6*(((a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)
*d^3*x^3 + 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^3*x^2 + 3*(a^4*b^3*cos
(c)^2 + a^4*b^3*sin(c)^2)*d^3*x + (a^5*b^2*cos(c)^2 + a^5*b^2*sin(c)^2)*d^3
)*cos(d*x + c)^2 + ((a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^3*x^3 + 3*(a^3*
b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^3*x^2 + 3*(a^4*b^3*cos(c)^2 + a^4*b^3*si
n(c)^2)*d^3*x + (a^5*b^2*cos(c)^2 + a^5*b^2*sin(c)^2)*d^3)*sin(d*x + c)^2)*
integrate(1/2*x*cos(d*x + c)/(b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^3*d^2
*x^2 + 4*a^3*b^2*d^2*x + a^4*b*d^2), x) + 6*(((a^2*b^5*cos(c)^2 + a^2*b^5*s
in(c)^2)*d^3*x^3 + 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^3*x^2 + 3*(a^4
*b^3*cos(c)^2 + a^4*b^3*sin(c)^2)*d^3*x + (a^5*b^2*cos(c)^2 + a^5*b^2*sin(c
)^2)*d^3)*cos(d*x + c)^2 + ((a^2*b^5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^3*x^3 +
 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^3*x^2 + 3*(a^4*b^3*cos(c)^2 + a^
4*b^3*sin(c)^2)*d^3*x + (a^5*b^2*cos(c)^2 + a^5*b^2*sin(c)^2)*d^3)*sin(d*x
+ c)^2)*integrate(1/2*x*cos(d*x + c)/((b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^
2*b^3*d^2*x^2 + 4*a^3*b^2*d^2*x + a^4*b*d^2)*cos(d*x + c)^2 + (b^5*d^2*x^4
+ 4*a*b^4*d^2*x^3 + 6*a^2*b^3*d^2*x^2 + 4*a^3*b^2*d^2*x + a^4*b*d^2)*sin(d*
x + c)^2), x) - 12*(((a*b^6*cos(c)^2 + a*b^6*sin(c)^2)*d^2*x^3 + 3*(a^2*b^5
*cos(c)^2 + a^2*b^5*sin(c)^2)*d^2*x^2 + 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(c
)^2)*d^2*x + (a^4*b^3*cos(c)^2 + a^4*b^3*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((
a*b^6*cos(c)^2 + a*b^6*sin(c)^2)*d^2*x^3 + 3*(a^2*b^5*cos(c)^2 + a^2*b^5*si
n(c)^2)*d^2*x^2 + 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^2*x + (a^4*b^3*
cos(c)^2 + a^4*b^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(1/2*x*sin(d*x +
```

```
 c)/(b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^3*d^2*x^2 + 4*a^3*b^2*d^2*x +
a^4*b*d^2), x) - 12*(((a*b^6*cos(c)^2 + a*b^6*sin(c)^2)*d^2*x^3 + 3*(a^2*b^
5*cos(c)^2 + a^2*b^5*sin(c)^2)*d^2*x^2 + 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(
c)^2)*d^2*x + (a^4*b^3*cos(c)^2 + a^4*b^3*sin(c)^2)*d^2)*cos(d*x + c)^2 + (
(a*b^6*cos(c)^2 + a*b^6*sin(c)^2)*d^2*x^3 + 3*(a^2*b^5*cos(c)^2 + a^2*b^5*s
in(c)^2)*d^2*x^2 + 3*(a^3*b^4*cos(c)^2 + a^3*b^4*sin(c)^2)*d^2*x + (a^4*b^3
*cos(c)^2 + a^4*b^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(1/2*x*sin(d*x
+ c)/((b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^3*d^2*x^2 + 4*a^3*b^2*d^2*x
+ a^4*b*d^2)*cos(d*x + c)^2 + (b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^3*d^
2*x^2 + 4*a^3*b^2*d^2*x + a^4*b*d^2)*sin(d*x + c)^2), x) + ((b^2*d*x^3*sin(
c) - 3*a*b*x*cos(c))*cos(d*x + c)^2 + (b^2*d*x^3*sin(c) - 3*a*b*x*cos(c))*s
in(d*x + c)^2)*sin(d*x + 2*c))/(((b^5*cos(c)^2 + b^5*sin(c)^2)*d^2*x^3 + 3*
(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^2*x^2 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*s
in(c)^2)*d^2*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^2)*cos(d*x + c)^2
+ ((b^5*cos(c)^2 + b^5*sin(c)^2)*d^2*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)
^2)*d^2*x^2 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^2*x + (a^3*b^2*cos(
c)^2 + a^3*b^2*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Fricas [A]
time = 0.38, size = 515, normalized size = 1.94

$$\frac{2\left(a^4bd^2-2b^5x^2-2a^2b^3+\left(a^3b^2d^2-4ab^4\right)x\right)\cos\left(dx+c\right)+2\left(3\left(a^2b^3d^2x^2+2a^3b^2d^2x+a^4bd^2\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+3\left(a^2b^3d^2x^2+2a^3b^2d^2x+a^4bd^2\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)+\left(a^5d^3-6a^3b^2d+\left(a^3b^2d^3-6ab^4d\right)x^2+2\left(a^4bd^3-6a^2b^3d\right)x\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)-2\left(6a^2b^3dx+5a^3b^2d\right)\sin\left(dx+c\right)-\left(\left(a^5d^3-6a^3b^2d+\left(a^3b^2d^3-6ab^4d\right)x^2+2\left(a^4bd^3-6a^2b^3d\right)x\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+\left(a^5d^3-6a^3b^2d+\left(a^3b^2d^3-6ab^4d\right)x^2+2\left(a^4bd^3-6a^2b^3d\right)x\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)-12\left(a^2b^3d^2x^2+2a^3b^2d^2x+a^4bd^2\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{4\left(b^8dx^2+2ab^7dx+a^2b^6d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^4*b*d^2 - 2*b^5*x^2 - 2*a^2*b^3 + (a^3*b^2*d^2 - 4*a*b^4)*x)*cos(
d*x + c) + 2*(3*(a^2*b^3*d^2*x^2 + 2*a^3*b^2*d^2*x + a^4*b*d^2)*cos_integra
l((b*d*x + a*d)/b) + 3*(a^2*b^3*d^2*x^2 + 2*a^3*b^2*d^2*x + a^4*b*d^2)*cos_
integral(-(b*d*x + a*d)/b) + (a^5*d^3 - 6*a^3*b^2*d + (a^3*b^2*d^3 - 6*a*b^
4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^2*b^3*d)*x)*sin_integral((b*d*x + a*d)/b))*co
s(-(b*c - a*d)/b) - 2*(6*a^2*b^3*d*x + 5*a^3*b^2*d)*sin(d*x + c) - ((a^5*d^
3 - 6*a^3*b^2*d + (a^3*b^2*d^3 - 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^2*b^3*
d)*x)*cos_integral((b*d*x + a*d)/b) + (a^5*d^3 - 6*a^3*b^2*d + (a^3*b^2*d^3
 - 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^2*b^3*d)*x)*cos_integral(-(b*d*x + a
*d)/b) - 12*(a^2*b^3*d^2*x^2 + 2*a^3*b^2*d^2*x + a^4*b*d^2)*sin_integral((b
*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^8*d*x^2 + 2*a*b^7*d*x + a^2*b^6*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sin\left(c+dx\right)}{\left(a+bx\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.25, size = 16724, normalized size = 63.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^3*b^2*d^3*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 - a^3*b^2*d^3*x^2*imag_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b^2*d^3*x^2*s
in_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 2*a^3*b^2*d^3*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*b^2*d^3*x^2*real_part(cos_integral(-d*x - a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^3*b^2*d^3*x^2*real_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2
- 2*a^3*b^2*d^3*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^4*b*d^3*x*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^4*b*d^3*x*imag_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
6*a^2*b^3*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 + 6*a^2*b^3*d^2*x^2*real_part(cos_integral(-d*x -
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^4*b*d^3*x*sin_in
tegral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^3*
b^2*d^3*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^
2 + a^3*b^2*d^3*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*a^3*b^2*d^3*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 4*a^3*b^2*d^3*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^3*b^2*d^3*x^2*imag_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^3*b^2*d^
3*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b
) - 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(1/2*a*d/b) + 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^4*b*d^3*x*real
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
 + 4*a^4*b*d^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b) - 24*a^2*b^3*d^2*x^2*sin_integral((b*d*x + a*d)/b)*t
an(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - a^3*b^2*d^3*x^2*imag_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^3*b^2*d^3*x^2*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^3*
b^2*d^3*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 +
 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)*tan(1/2*a*d/b)^2 - 12*a^2*b^3*d^2*x^2*imag_part(cos_integral(-d*x -
```

```
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^4*b*d^3*x*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 -
4*a^4*b*d^3*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c)*tan(1/2*a*d/b)^2 + 24*a^2*b^3*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(
1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a^3*b^2*d^3*x^2*imag_part(cos_inte
gral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^3*b^2*d^3*x^2*imag_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b^2*d^3
*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^5*d^3*
imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*
d/b)^2 - 6*a*b^4*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^5*d^3*imag_part(cos_integral(-d*x - a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a*b^4*d*x^2*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 1
2*a^3*b^2*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c)^2*tan(1/2*a*d/b)^2 + 12*a^3*b^2*d^2*x*real_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^5*d^3*sin_integral(
(b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*a*b^4*d*
x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b
)^2 + 2*a^3*b^2*d^3*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c) + 2*a^3*b^2*d^3*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c) - 2*a^4*b*d^3*x*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^4*b*d^3*x*imag_part(cos_integral(-d*x - a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a^2*b^3*d^2*x^2*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a^2*b^3*d^2*x^2*real_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^4*b*d^3*x*sin_
integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^3*b^2*d^3*x^2*r
eal_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^3*b
^2*d^3*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d
/b) + 8*a^4*b*d^3*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)*tan(1/2*a*d/b) - 8*a^4*b*d^3*x*imag_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 24*a^2*b^3*d^2*x^2*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sin(c + d\,x)}{(a + b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sin(c + d*x))/(a + b*x)^3,x)
[Out] int((x^3*sin(c + d*x))/(a + b*x)^3, x)
```

3.34 $\int \frac{x^2 \sin(c+dx)}{(a+bx)^3} \, dx$

Optimal. Leaf size=241

$$-\frac{a^2 d \cos(c+dx)}{2b^4(a+bx)}-\frac{2ad\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{b^4}+\frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3}-\frac{a^2d^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^5}-$$

```
[Out] -2*a*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^4-1/2*a^2*d*cos(d*x+c)/b^4/(b*x+a)+cos
(-c+a*d/b)*Si(a*d/b+d*x)/b^3-1/2*a^2*d^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^5-Ci
(a*d/b+d*x)*sin(-c+a*d/b)/b^3+1/2*a^2*d^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^5-2
*a*d*Si(a*d/b+d*x)*sin(-c+a*d/b)/b^4-1/2*a^2*sin(d*x+c)/b^3/(b*x+a)^2+2*a*s
in(d*x+c)/b^3/(b*x+a)
```

Rubi [A]
time = 0.37, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$-\frac{a^2d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2b^5}-\frac{a^2d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^5}-\frac{a^2d\cos(c+dx)}{2b^4(a+bx)}-\frac{a^2\sin(c+dx)}{2b^3(a+bx)^2}-\frac{2ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4}+\frac{2ad\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{2a\sin(c+dx)}{b^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -1/2*(a^2*d*Cos[c + d*x])/(b^4*(a + b*x)) - (2*a*d*Cos[c - (a*d)/b]*CosInte
gral[(a*d)/b + d*x])/b^4 + (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b^
3 - (a^2*d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*b^5) - (a^2*Si
n[c + d*x])/(2*b^3*(a + b*x)^2) + (2*a*Sin[c + d*x])/(b^3*(a + b*x)) + (Cos
[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^3 - (a^2*d^2*Cos[c - (a*d)/b]*S
inIntegral[(a*d)/b + d*x])/(2*b^5) + (2*a*d*Sin[c - (a*d)/b]*SinIntegral[(a
*d)/b + d*x])/b^4
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \sin(c+dx)}{(a+bx)^3}\,dx &= \int \left(\frac{a^2 \sin(c+dx)}{b^2(a+bx)^3} - \frac{2a\sin(c+dx)}{b^2(a+bx)^2} + \frac{\sin(c+dx)}{b^2(a+bx)}\right) dx \\
&= \frac{\int \frac{\sin(c+dx)}{a+bx}\,dx}{b^2} - \frac{(2a)\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^2} + \frac{a^2\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{b^2} \\
&= -\frac{a^2\sin(c+dx)}{2b^3(a+bx)^2} + \frac{2a\sin(c+dx)}{b^3(a+bx)} - \frac{(2ad)\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^3} + \frac{(a^2d)\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2b^3} + \frac{\cos}{} \\
&= -\frac{a^2d\cos(c+dx)}{2b^4(a+bx)} + \frac{\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3} - \frac{a^2\sin(c+dx)}{2b^3(a+bx)^2} + \frac{2a\sin(c+dx)}{b^3(a+bx)} + \\
&= -\frac{a^2d\cos(c+dx)}{2b^4(a+bx)} - \frac{2ad\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3} - \frac{a^2 s}{2b^3} \\
&= -\frac{a^2d\cos(c+dx)}{2b^4(a+bx)} - \frac{2ad\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3} - \frac{a^2d}{}
\end{aligned}$$

Mathematica [A]
time = 0.80, size = 154, normalized size = 0.64

$$-\frac{-\mathrm{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\left(-4abd\cos\left(c-\frac{ad}{b}\right)+\left(2b^2-a^2d^2\right)\sin\left(c-\frac{ad}{b}\right)\right)+\frac{ab(ad(a+bx)\cos(c+dx)-b(3a+4bx)\sin(c+dx))}{(a+bx)^2}+\left(\left(-2b^2+a^2d^2\right)\cos\left(c-\frac{ad}{b}\right)-4abd\sin\left(c-\frac{ad}{b}\right)\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -1/2*(-(CosIntegral[d*(a/b + x)]*(-4*a*b*d*Cos[c - (a*d)/b] + (2*b^2 - a^2*
d^2)*Sin[c - (a*d)/b])) + (a*b*(a*d*(a + b*x)*Cos[c + d*x] - b*(3*a + 4*b*x
```

```
)*Sin[c + d*x]))/(a + b*x)^2 + ((-2*b^2 + a^2*d^2)*Cos[c - (a*d)/b] - 4*a*b
*d*Sin[c - (a*d)/b])*SinIntegral[d*(a/b + x)])/b^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(240) = 480$.
time = 0.09, size = 779, normalized size = 3.23 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(d^3*c^2*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b+1/2*(-cos(d*x+c)/(d
*a-c*b+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d
-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)-2*d^3*c/b*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c
))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a
*d-b*c)/b)/b)/b)+2*d^3*(a*d-b*c)/b*c*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2
/b+1/2*(-cos(d*x+c)/(d*a-c*b+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b
*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)-2*d^3*(a*d-b*c)/b^
2*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b
)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+d^3*(a*d-b*c)^2/b^2*(-1/2*
sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b+1/2*(-cos(d*x+c)/(d*a-c*b+b*(d*x+c))/b-(
Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c
)/b)/b)/b)/b)+d^3/b^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a
*d-b*c)/b)*sin((a*d-b*c)/b)/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((b*cos(c)^2 + b*sin(c)^2)*d*x^2*cos(d*x + c) + ((a*(I*exp_integral_e(
4, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)
^2 + a*(I*exp_integral_e(4, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*
b*d*x + I*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) - (a*(exp_integral_e(4, (I
*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*
(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a
*d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + (b*cos(c)^2 + b*sin
(c)^2)*x*sin(d*x + c) + ((a*(I*exp_integral_e(4, (I*b*d*x + I*a*d)/b) - I*e
xp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*exp_integral_e(4, (
I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*
cos(-(b*c - a*d)/b) - (a*(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_inte
gral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(exp_integral_e(4, (I*b*d*x +
 I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*sin(c)^2)*sin(-(b*c -
 a*d)/b))*sin(d*x + c)^2 + ((b*d*x^2*cos(c) - b*x*sin(c))*cos(d*x + c)^2 +
```

```
(b*d*x^2*cos(c) - b*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) - 6*(((a*b^4*c
os(c)^2 + a*b^4*sin(c)^2)*d^3*x^3 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)
*d^3*x^2 + 3*(a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^3*x + (a^4*b*cos(c)^2
+ a^4*b*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*
d^3*x^3 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^3*x^2 + 3*(a^3*b^2*cos(
c)^2 + a^3*b^2*sin(c)^2)*d^3*x + (a^4*b*cos(c)^2 + a^4*b*sin(c)^2)*d^3)*sin
(d*x + c)^2)*integrate(1/2*x*cos(d*x + c)/(b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 +
6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2), x) - 6*(((a*b^4*cos(c)^2 + a*
b^4*sin(c)^2)*d^3*x^3 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^3*x^2 + 3
*(a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^3*x + (a^4*b*cos(c)^2 + a^4*b*sin(
c)^2)*d^3)*cos(d*x + c)^2 + ((a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^3*x^3 + 3*
(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^3*x^2 + 3*(a^3*b^2*cos(c)^2 + a^3*b
^2*sin(c)^2)*d^3*x + (a^4*b*cos(c)^2 + a^4*b*sin(c)^2)*d^3)*sin(d*x + c)^2)
*integrate(1/2*x*cos(d*x + c)/((b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 + 6*a^2*b^2*d
^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2)*cos(d*x + c)^2 + (b^4*d^2*x^4 + 4*a*b^3*d
^2*x^3 + 6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2)*sin(d*x + c)^2), x) +
 4*(((b^5*cos(c)^2 + b^5*sin(c)^2)*d^2*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(
c)^2)*d^2*x^2 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^2*x + (a^3*b^2*co
s(c)^2 + a^3*b^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^5*cos(c)^2 + b^5*sin(c
)^2)*d^2*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^2*x^2 + 3*(a^2*b^3*cos
(c)^2 + a^2*b^3*sin(c)^2)*d^2*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^2
)*sin(d*x + c)^2)*integrate(1/2*x*sin(d*x + c)/(b^4*d^2*x^4 + 4*a*b^3*d^2*x
^3 + 6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2), x) + 4*(((b^5*cos(c)^2 +
 b^5*sin(c)^2)*d^2*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^2*x^2 + 3*(a
^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d^2*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin
(c)^2)*d^2)*cos(d*x + c)^2 + ((b^5*cos(c)^2 + b^5*sin(c)^2)*d^2*x^3 + 3*(a*
b^4*cos(c)^2 + a*b^4*sin(c)^2)*d^2*x^2 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(
c)^2)*d^2*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d^2)*sin(d*x + c)^2)*in
tegrate(1/2*x*sin(d*x + c)/((b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 + 6*a^2*b^2*d^2*
x^2 + 4*a^3*b*d^2*x + a^4*d^2)*cos(d*x + c)^2 + (b^4*d^2*x^4 + 4*a*b^3*d^2*
x^3 + 6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2)*sin(d*x + c)^2), x) + ((
b*d*x^2*sin(c) + b*x*cos(c))*cos(d*x + c)^2 + (b*d*x^2*sin(c) + b*x*cos(c))
*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^4*cos(c)^2 + b^4*sin(c)^2)*d^2*x^3 +
3*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d^2*x^2 + 3*(a^2*b^2*cos(c)^2 + a^2*b^2
*sin(c)^2)*d^2*x + (a^3*b*cos(c)^2 + a^3*b*sin(c)^2)*d^2)*cos(d*x + c)^2 +
((b^4*cos(c)^2 + b^4*sin(c)^2)*d^2*x^3 + 3*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2
)*d^2*x^2 + 3*(a^2*b^2*cos(c)^2 + a^2*b^2*sin(c)^2)*d^2*x + (a^3*b*cos(c)^2
 + a^3*b*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Fricas [A]
time = 0.38, size = 438, normalized size = 1.82

$$-\frac{2\left(a^{2}b^{2}dx+a^{3}bd\right)\cos\left(dx+c\right)+2\left(2\left(ab^{3}dx^{2}+2\,a^{2}b^{2}dx+a^{3}bd\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+2\left(ab^{3}dx^{2}+2\,a^{2}b^{2}dx+a^{3}bd\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)+\left(a^{4}d^{2}-2\,a^{2}b^{2}+\left(a^{2}b^{2}d^{2}-2\,b^{4}\right)x^{2}+2\left(a^{3}bd^{2}-2\,ab^{3}\right)x\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)-2\left(4\,ab^{3}x+3\,a^{2}b^{2}\right)\sin\left(dx+c\right)-\left(\left(a^{4}d^{2}-2\,a^{2}b^{2}+\left(a^{2}b^{2}d^{2}-2\,b^{4}\right)x^{2}+2\left(a^{3}bd^{2}-2\,ab^{3}\right)x\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+\left(a^{4}d^{2}-2\,a^{2}b^{2}+\left(a^{2}b^{2}d^{2}-2\,b^{4}\right)x^{2}+2\left(a^{3}bd^{2}-2\,ab^{3}\right)x\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)-8\left(ab^{3}dx^{2}+2\,a^{2}b^{2}dx+a^{3}bd\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{4\left(b^{7}x^{2}+2\,ab^{6}x+a^{2}b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^2*b^2*d*x + a^3*b*d)*cos(d*x + c) + 2*(2*(a*b^3*d*x^2 + 2*a^2*b^
2*d*x + a^3*b*d)*cos_integral((b*d*x + a*d)/b) + 2*(a*b^3*d*x^2 + 2*a^2*b^2
*d*x + a^3*b*d)*cos_integral(-(b*d*x + a*d)/b) + (a^4*d^2 - 2*a^2*b^2 + (a^
2*b^2*d^2 - 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*sin_integral((b*d*x + a
*d)/b))*cos(-(b*c - a*d)/b) - 2*(4*a*b^3*x + 3*a^2*b^2)*sin(d*x + c) - ((a^
4*d^2 - 2*a^2*b^2 + (a^2*b^2*d^2 - 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*
cos_integral((b*d*x + a*d)/b) + (a^4*d^2 - 2*a^2*b^2 + (a^2*b^2*d^2 - 2*b^4
)*x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*cos_integral(-(b*d*x + a*d)/b) - 8*(a*b^
3*d*x^2 + 2*a^2*b^2*d*x + a^3*b*d)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c
 - a*d)/b))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sin(c + dx)}{(a + bx)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.16, size = 15410, normalized size = 63.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^2*
sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
+ 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x -
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^2*real
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2
 - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x*imag_part
(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 4*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + 4*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))
```

*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*d^2*x*sin_integral(
(b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2
*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^2
*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/
2*c)^2 + 4*a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*b^2*d^2*x^2*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*b^2*d^2*x^2*s
in_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a
*b^3*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2
*tan(1/2*a*d/b) + 8*a*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x*
real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a
*d/b) - 16*a*b^3*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2
*c)^2*tan(1/2*a*d/b) - a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))
*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^2*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 8*a*b^3*d*x^2*imag_p
art(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 -
8*a*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x*real_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 16*a*b^3*
d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b
)^2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan
(1/2*a*d/b)^2 - a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d^2*imag_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^4*x^2*imag_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^4*d^2
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*
a*d/b)^2 + 2*b^4*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 + 8*a^2*b^2*d*x*real_part(cos_integral(d*x + a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 8*a^2*b^2*d*x*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
+ 2*a^4*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(
1/2*a*d/b)^2 - 4*b^4*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*b^2*d^2*x^2*real_part(cos_integral
(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^3*b*d^2*x*imag_part(cos_int
egral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^3*b*d^2*x*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b^3*d*x^2*real
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b^3*d*x^
2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^3

```
*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*
b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d
/b) - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^
2*tan(1/2*a*d/b) + 8*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^3*b*d^2*x*imag_part(cos_integral(
-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a*b^3*d*x^2*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)
 + 16*a*b^3*d*x^2*real_part(cos_integral(-d*x -...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sin\left(c+d\,x\right)}{{\left(a+b\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sin(c + d*x))/(a + b*x)^3,x)
```

```
[Out] int((x^2*sin(c + d*x))/(a + b*x)^3, x)
```

3.35 $\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx$

Optimal. Leaf size=179

$$\frac{ad\cos(c+dx)}{2b^3(a+bx)}+\frac{d\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{ad^2\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^4}+\frac{a\sin(c+dx)}{2b^2(a+bx)^2}-\frac{\sin(c+dx)}{b^2(a+bx)}+\frac{ad^2}{}$$

```
[Out] d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/b^3+1/2*a*d*cos(d*x+c)/b^3/(b*x+a)+1/2*a*d^2*
cos(-c+a*d/b)*Si(a*d/b+d*x)/b^4-1/2*a*d^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^4+d
*Si(a*d/b+d*x)*sin(-c+a*d/b)/b^3+1/2*a*sin(d*x+c)/b^2/(b*x+a)^2-sin(d*x+c)/
b^2/(b*x+a)
```

Rubi [A]
time = 0.24, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\frac{ad^2\sin\left(c-\frac{ad}{b}\right)\mathrm{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2b^4}+\frac{ad^2\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(xd+\frac{ad}{b}\right)}{2b^4}+\frac{d\cos\left(c-\frac{ad}{b}\right)\mathrm{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3}-\frac{d\sin\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{ad\cos(c+dx)}{2b^3(a+bx)}-\frac{\sin(c+dx)}{b^2(a+bx)}+\frac{a\sin(c+dx)}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] (a*d*Cos[c + d*x])/(2*b^3*(a + b*x)) + (d*Cos[c - (a*d)/b]*CosIntegral[(a*d
)/b + d*x])/b^3 + (a*d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*b^
4) + (a*Sin[c + d*x])/(2*b^2*(a + b*x)^2) - Sin[c + d*x]/(b^2*(a + b*x)) +
(a*d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*b^4) - (d*Sin[c - (a
*d)/b]*SinIntegral[(a*d)/b + d*x])/b^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx &= \int \left(-\frac{a\sin(c+dx)}{b(a+bx)^3}+\frac{\sin(c+dx)}{b(a+bx)^2}\right)dx \\
&= \frac{\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b}-\frac{a\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{b} \\
&= \frac{a\sin(c+dx)}{2b^2(a+bx)^2}-\frac{\sin(c+dx)}{b^2(a+bx)}+\frac{d\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^2}-\frac{(ad)\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2b^2} \\
&= \frac{ad\cos(c+dx)}{2b^3(a+bx)}+\frac{a\sin(c+dx)}{2b^2(a+bx)^2}-\frac{\sin(c+dx)}{b^2(a+bx)}+\frac{(ad^2)\int \frac{\sin(c+dx)}{a+bx}\,dx}{2b^3}+\frac{\left(d\cos\left(c-\frac{ad}{b}\right)\right)}{b} \\
&= \frac{ad\cos(c+dx)}{2b^3(a+bx)}+\frac{d\cos\left(c-\frac{ad}{b}\right)\operatorname{Ci}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{a\sin(c+dx)}{2b^2(a+bx)^2}-\frac{\sin(c+dx)}{b^2(a+bx)}-\frac{d\sin\left(c\right.}{} \\
&= \frac{ad\cos(c+dx)}{2b^3(a+bx)}+\frac{d\cos\left(c-\frac{ad}{b}\right)\operatorname{Ci}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{ad^2\operatorname{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^4}+\frac{a\sin(c}{2b^2(a+}
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 157, normalized size = 0.88

$$\frac{b\cos(dx)(ad(a+bx)\cos(c)-b(a+2bx)\sin(c))-b(b(a+2bx)\cos(c)+ad(a+bx)\sin(c))\sin(dx)+d(a+bx)^2\left(\operatorname{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\left(2b\cos\left(c-\frac{ad}{b}\right)+ad\sin\left(c-\frac{ad}{b}\right)\right)+\left(ad\cos\left(c-\frac{ad}{b}\right)-2b\sin\left(c-\frac{ad}{b}\right)\right)\operatorname{Si}\left(d\left(\frac{a}{b}+x\right)\right)\right)}{2b^4(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] (b*Cos[d*x]*(a*d*(a + b*x)*Cos[c] - b*(a + 2*b*x)*Sin[c]) - b*(b*(a + 2*b*x
)*Cos[c] + a*d*(a + b*x)*Sin[c])*Sin[d*x] + d*(a + b*x)^2*(CosIntegral[d*(a
/b + x)]*(2*b*Cos[c - (a*d)/b] + a*d*Sin[c - (a*d)/b]) + (a*d*Cos[c - (a*d)
/b] - 2*b*Sin[c - (a*d)/b])*SinIntegral[d*(a/b + x)]))/(2*b^4*(a + b*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(174) = 348$.
time = 0.07, size = 419, normalized size = 2.34

method	result
derivativedivides	$\frac{\frac{d^3\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\text{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\text{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{b}-\frac{d^3(da-cb)\Big(}{}}{}$
default	$\frac{\frac{d^3\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\text{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\text{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{b}-\frac{d^3(da-cb)\Big(}{}}{}$
risch	$\frac{i(2ia\,b^3d^3x^3+6ia^2b^2d^3x^2+6ia^3b\,d^3x+2ia^4d^3)\cos(dx+c)}{4b^3(bx+a)^2(-d^2x^2b^2-2ab\,d^2x-d^2a^2)}+\frac{(4b^4d^2x^3+10a\,b^3d^2x^2+8a^2b^2d^2x+2a^3b\,d^2)\sin(dx+c)}{4b^3(bx+a)^2(-d^2x^2b^2-2ab\,d^2x-d^2a^2)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(d^3/b*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin(
(a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-d^3*(a*d-b*c)/b
*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b+1/2*(-cos(d*x+c)/(d*a-c*b+b*(d*x+
c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((
a*d-b*c)/b)/b)/b)/b)-d^3*c*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b+1/2*(-c
os(d*x+c)/(d*a-c*b+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-C
i(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((b*cos(c)^2 + b*sin(c)^2)*x*cos(d*x + c) + ((a*(exp_integral_e(4, (I*
b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(
exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*
d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a*(I*exp_integral_e(4, (I*b*d*x + I
*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*exp_i
ntegral_e(4, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/
b))*sin(c)^2)*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((a*(exp_integral_e(4,
```

```
(I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 +
a*(exp_integral_e(4, (I*b*d*x + I*a*d)/b) + exp_integral_e(4, -(I*b*d*x + I
*a*d)/b))*sin(c)^2)*cos(-(b*c - a*d)/b) + (a*(I*exp_integral_e(4, (I*b*d*x
+ I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*d)/b))*cos(c)^2 + a*(I*ex
p_integral_e(4, (I*b*d*x + I*a*d)/b) - I*exp_integral_e(4, -(I*b*d*x + I*a*
d)/b))*sin(c)^2)*sin(-(b*c - a*d)/b))*sin(d*x + c)^2 + (b*x*cos(d*x + c)^2*
cos(c) + b*x*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 4*(((b^5*cos(c)^2 + b^
5*sin(c)^2)*d*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d*x^2 + 3*(a^2*b^3*
cos(c)^2 + a^2*b^3*sin(c)^2)*d*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d)
*cos(d*x + c)^2 + ((b^5*cos(c)^2 + b^5*sin(c)^2)*d*x^3 + 3*(a*b^4*cos(c)^2
+ a*b^4*sin(c)^2)*d*x^2 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d*x + (a^
3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*x*cos(d
*x + c)/(b^4*d*x^4 + 4*a*b^3*d*x^3 + 6*a^2*b^2*d*x^2 + 4*a^3*b*d*x + a^4*d)
, x) + 4*(((b^5*cos(c)^2 + b^5*sin(c)^2)*d*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*
sin(c)^2)*d*x^2 + 3*(a^2*b^3*cos(c)^2 + a^2*b^3*sin(c)^2)*d*x + (a^3*b^2*co
s(c)^2 + a^3*b^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^5*cos(c)^2 + b^5*sin(c)^
2)*d*x^3 + 3*(a*b^4*cos(c)^2 + a*b^4*sin(c)^2)*d*x^2 + 3*(a^2*b^3*cos(c)^2
+ a^2*b^3*sin(c)^2)*d*x + (a^3*b^2*cos(c)^2 + a^3*b^2*sin(c)^2)*d)*sin(d*x
+ c)^2)*integrate(1/2*x*cos(d*x + c)/((b^4*d*x^4 + 4*a*b^3*d*x^3 + 6*a^2*b^
2*d*x^2 + 4*a^3*b*d*x + a^4*d)*cos(d*x + c)^2 + (b^4*d*x^4 + 4*a*b^3*d*x^3
+ 6*a^2*b^2*d*x^2 + 4*a^3*b*d*x + a^4*d)*sin(d*x + c)^2), x) + (b*x*cos(d*x
 + c)^2*sin(c) + b*x*sin(d*x + c)^2*sin(c))*sin(d*x + 2*c))/(((b^4*cos(c)^2
 + b^4*sin(c)^2)*d*x^3 + 3*(a*b^3*cos(c)^2 + a*b^3*sin(c)^2)*d*x^2 + 3*(a^2
*b^2*cos(c)^2 + a^2*b^2*sin(c)^2)*d*x + (a^3*b*cos(c)^2 + a^3*b*sin(c)^2)*d
)*cos(d*x + c)^2 + ((b^4*cos(c)^2 + b^4*sin(c)^2)*d*x^3 + 3*(a*b^3*cos(c)^2
 + a*b^3*sin(c)^2)*d*x^2 + 3*(a^2*b^2*cos(c)^2 + a^2*b^2*sin(c)^2)*d*x + (a
^3*b*cos(c)^2 + a^3*b*sin(c)^2)*d)*sin(d*x + c)^2)
```

Fricas [**A**]
time = 0.35, size = 346, normalized size = 1.93

$$\frac{2\,(ab^2dx + a^2bd)\cos\left(dx + c\right) + 2\,\left(\left(b^3dx^2 + 2\,ab^2dx + a^2bd\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right) + \left(b^3dx^2 + 2\,ab^2dx + a^2bd\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right) + \left(ab^2d^2x^2 + 2\,a^2bd^2x + a^3d^2\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right) - 2\,(2\,b^3x + ab^2)\sin\left(dx + c\right) - \left(\left(ab^2d^2x^2 + 2\,a^2bd^2x + a^3d^2\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right) + \left(ab^2d^2x^2 + 2\,a^2bd^2x + a^3d^2\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right) - 4\,\left(b^3dx^2 + 2\,ab^2dx + a^2bd\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{4\,(b^6x^2 + 2\,ab^5x + a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*b^2*d*x + a^2*b*d)*cos(d*x + c) + 2*((b^3*d*x^2 + 2*a*b^2*d*x + a
^2*b*d)*cos_integral((b*d*x + a*d)/b) + (b^3*d*x^2 + 2*a*b^2*d*x + a^2*b*d)
*cos_integral(-(b*d*x + a*d)/b) + (a*b^2*d^2*x^2 + 2*a^2*b*d^2*x + a^3*d^2)
*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - 2*(2*b^3*x + a*b^2)*s
in(d*x + c) - ((a*b^2*d^2*x^2 + 2*a^2*b*d^2*x + a^3*d^2)*cos_integral((b*d*
x + a*d)/b) + (a*b^2*d^2*x^2 + 2*a^2*b*d^2*x + a^3*d^2)*cos_integral(-(b*d*
x + a*d)/b) - 4*(b^3*d*x^2 + 2*a*b^2*d*x + a^2*b*d)*sin_integral((b*d*x + a
*d)/b))*sin(-(b*c - a*d)/b))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin (c + dx)}{(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.50, size = 10535, normalized size = 58.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(1/2*a*d/b)^2 - a*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^2*d^2*x^2*sin_int
egral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b
^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2
*tan(1/2*a*d/b) + 2*a*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a*b^2*d^2*x^2*real_part(cos_int
egral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b^2*d^
2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1
/2*a*d/b)^2 + 2*a^2*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*
x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*b*d^2*x*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^3*d*x^2*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 + 2*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^2*b*d^2*x*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a*b^2*d^2*x^2*imag_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*b^2*d^2*x^2*imag_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*b^2*d^2*x^2*s
in_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b^2*d^2*x^2*
imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/
b) - 4*a*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)*tan(1/2*a*d/b) + 8*a*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*b^3*d*x^2*imag_part(cos_integral
(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*b^3*d*x^2*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/
b) + 4*a^2*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(1/2*a*d/b) + 4*a^2*b*d^2*x*real_part(cos_integral(-d*x - a*d/b
```

```
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*b^3*d*x^2*sin_integral((b
*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - a*b^2*d^2*x^2*i
mag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a*b^2
*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b
)^2 - 2*a*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*
a*d/b)^2 + 4*b^3*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*
tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^2*b*d^2*x*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^
2*b*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*t
an(1/2*a*d/b)^2 + 8*b^3*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*
tan(1/2*c)*tan(1/2*a*d/b)^2 + a*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*
d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a*b^2*d^2*x^2*imag_part(cos_integral(
-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^2*d^2*x^2*sin_integral
((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*d^2*imag_part(cos_int
egral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^3*d^2*
imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a
*d/b)^2 + 4*a*b^2*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b^2*d*x*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*d^2*sin_integral
((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^2*d^
2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*
b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
 - 2*a^2*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*a^2*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)^2 - 4*a^2*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*a*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a*b^2*d^2*x^2*real_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a^2*b*d^2*x*imag_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^2*b*d^2
*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*
a*d/b) + 8*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b) + 8*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a^2*b*d^2*x*sin_integral((b*
d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 2*a*b^2*d^2*x^2*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a*b^2*d^
2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 8
*a*b^2*d*x*imag_part(cos_integral(d*x + a*d/b))...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin\left(c+d\,x\right)}{{\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(c + d*x))/(a + b*x)^3,x)
```

```
[Out] int((x*sin(c + d*x))/(a + b*x)^3, x)
```

3.36 $\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx$

Optimal. Leaf size=104

$$-\frac{d\cos(c+dx)}{2b^2(a+bx)}-\frac{d^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^3}-\frac{\sin(c+dx)}{2b(a+bx)^2}-\frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{2b^3}$$

```
[Out] -1/2*d*cos(d*x+c)/b^2/(b*x+a)-1/2*d^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/b^3+1/2*d
^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)/b^3-1/2*sin(d*x+c)/b/(b*x+a)^2
```

Rubi [A]
time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3378, 3384, 3380, 3383}

$$-\frac{d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2b^3}-\frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^3}-\frac{d\cos(c+dx)}{2b^2(a+bx)}-\frac{\sin(c+dx)}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x)^3,x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(b^2*(a + b*x)) - (d^2*CosIntegral[(a*d)/b + d*x]*Sin
[c - (a*d)/b])/(2*b^3) - Sin[c + d*x]/(2*b*(a + b*x)^2) - (d^2*Cos[c - (a*d
)/b]*SinIntegral[(a*d)/b + d*x])/(2*b^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx &= -\frac{\sin(c+dx)}{2b(a+bx)^2}+\frac{d\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2b}\\ &= -\frac{d\cos(c+dx)}{2b^2(a+bx)}-\frac{\sin(c+dx)}{2b(a+bx)^2}-\frac{d^2\int \frac{\sin(c+dx)}{a+bx}\,dx}{2b^2}\\ &= -\frac{d\cos(c+dx)}{2b^2(a+bx)}-\frac{\sin(c+dx)}{2b(a+bx)^2}-\frac{\left(d^2\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^2}-\frac{\left(d^2\sin\left(c-\frac{ad}{b}\right)\right)\int}{2b^2}\\ &= -\frac{d\cos(c+dx)}{2b^2(a+bx)}-\frac{d^2\operatorname{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^3}-\frac{\sin(c+dx)}{2b(a+bx)^2}-\frac{d^2\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\frac{ad}{b}+\right.}{2b^3}\end{aligned}$$

Mathematica [A]
time = 0.53, size = 87, normalized size = 0.84

$$-\frac{d^2\operatorname{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\sin\left(c-\frac{ad}{b}\right)+\frac{b(d(a+bx)\cos(c+dx)+b\sin(c+dx))}{(a+bx)^2}+d^2\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*x)^3,x]
[Out] -1/2*(d^2*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + (b*(d*(a + b*x)*Cos[c
 + d*x] + b*Sin[c + d*x]))/(a + b*x)^2 + d^2*Cos[c - (a*d)/b]*SinIntegral[d
*(a/b + x)])/b^3
```

Maple [A]
time = 0.05, size = 145, normalized size = 1.39

method	result
derivativedivides	$d^2\left(-\frac{\sin(dx+c)}{2(da-cb+b(dx+c))^2b}+\frac{-\frac{\cos(dx+c)}{(da-cb+b(dx+c))b}-\frac{\frac{\text{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\text{cosineIntegral}\left(dx+c+\frac{da-}{b}\right.}{b}}{b}}{2b}\right.$
default	$d^2\left(-\frac{\sin(dx+c)}{2(da-cb+b(dx+c))^2b}+\frac{-\frac{\cos(dx+c)}{(da-cb+b(dx+c))b}-\frac{\frac{\text{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}-\frac{\text{cosineIntegral}\left(dx+c+\frac{da-}{b}\right.}{b}}{b}}{2b}\right.$

risch	$-\frac{i d^{2} \mathrm{e}^{-\frac{i(da-cb)}{b}} \operatorname{expIntegral}\left(1,-idx-ic-\frac{iad-ibc}{b}\right)}{4 b^{3}}+\frac{i d^{2} \mathrm{e}^{\frac{i(da-cb)}{b}} \operatorname{expIntegral}\left(1,idx+ic+\frac{i(da-cb)}{b}\right)}{4 b^{3}}+\frac{i\left(-2ib^{3}d^{3}x^{3}\right.}{4b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b+1/2*(-cos(d*x+c)/(d*a-c*b+b*(d
*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*si
n((a*d-b*c)/b)/b)/b)/b)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.39, size = 199, normalized size = 1.91

$$\frac{d^{3}\left(-i\,E_{3}\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+i\,E_{3}\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+d^{3}\left(E_{3}\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+E_{3}\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\left((dx+c)^{2}b^{3}+b^{3}c^{2}-2\,ab^{2}cd+a^{2}bd^{2}-2\left(b^{3}c-ab^{2}d\right)(dx+c)\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(d^3*(-I*exp_integral_e(3, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_i
ntegral_e(3, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d^3
*(exp_integral_e(3, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(3,
-(I*(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))/(((d*x + c)^2*b^3
 + b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - 2*(b^3*c - a*b^2*d)*(d*x + c))*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(98) = 196.
time = 0.36, size = 210, normalized size = 2.02

$$-\frac{2\,b^{2}\sin\left(dx+c\right)+2\left(b^{2}d^{2}x^{2}+2\,abd^{2}x+a^{2}d^{2}\right)\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)+2\left(b^{2}dx+abd\right)\cos\left(dx+c\right)-\left(\left(b^{2}d^{2}x^{2}+2\,abd^{2}x+a^{2}d^{2}\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+\left(b^{2}d^{2}x^{2}+2\,abd^{2}x+a^{2}d^{2}\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{4\left(b^{5}x^{2}+2\,ab^{4}x+a^{2}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*sin(d*x + c) + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*cos(-(b*
c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + 2*(b^2*d*x + a*b*d)*cos(d*x + c
) - ((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*cos_integral((b*d*x + a*d)/b) +
(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*cos_integral(-(b*d*x + a*d)/b))*sin(-
(b*c - a*d)/b))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(c+dx\right)}{\left(a+bx\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.04, size = 5727, normalized size = 55.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 - b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d^2*x^2*sin_integral
((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d^2*
x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
2*a*d/b) + 2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*b^2*d^2*x^2*real_part(cos_integral(d*x +
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2*d^2*x^2*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 +
 2*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 - 2*a*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b*d^2*x*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b^2*d^2*x^2*imag_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*d^2*x^2*i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*d^
2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*b^2*d^2
*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2
*a*d/b) - 4*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^
2*tan(1/2*c)*tan(1/2*a*d/b) + 8*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*t
an(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a*b*d^2*x*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a*b*d^2*x*re
al_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d
/b) - b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*a*d/b)^2 + b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x
)^2*tan(1/2*a*d/b)^2 - 2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*
d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a*b*d^2*x*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + b^2*d^2*x^
2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b^2*
d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
 + 2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^
2 + a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^
```

```
2*tan(1/2*a*d/b)^2 - a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*sin_integral((b*d*x + a*d)
/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d^2*x^2*real_part(
cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*b^2*d^2*x^2*real_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*b*d^2*x*ima
g_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*d^2*x
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*
d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*d^2
*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2
*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a
*d/b) + 8*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)*tan(1/2*a*d/b) - 8*a*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a*b*d^2*x*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 2*b^2*d^2*x^2*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*b^2*d^2*x^2*r
eal_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*d^
2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*
a*d/b) + 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)^2*tan(1/2*a*d/b) - 2*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))
*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*b*d^2*x*imag_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b*d^2*x*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*b^2*d^2*x^2*real_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2*d^2*x^2*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d^2*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 -
2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*t
an(1/2*a*d/b)^2 + 2*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 - 2*a*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b*d^2*x*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2
*a*d/b)^2 + b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
 - b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 2*b^2
*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 4*a*b*d^2*x*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*b*d^2*x*real_
part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)}{(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*x)^3,x)
[Out] int(sin(c + d*x)/(a + b*x)^3, x)
```

3.37 $\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx$

Optimal. Leaf size=261

$$\frac{d\cos(c+dx)}{2ab(a+bx)}-\frac{d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{a^2b}+\frac{\text{Ci}(dx)\sin(c)}{a^3}-\frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3}+\frac{d^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c\right.}{2ab^2}$$

```
[Out] -d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/a^2/b+1/2*d*cos(d*x+c)/a/b/(b*x+a)+cos(c)*Si
(d*x)/a^3-cos(-c+a*d/b)*Si(a*d/b+d*x)/a^3+1/2*d^2*cos(-c+a*d/b)*Si(a*d/b+d*
x)/a/b^2+Ci(d*x)*sin(c)/a^3+Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^3-1/2*d^2*Ci(a*d/
b+d*x)*sin(-c+a*d/b)/a/b^2-d*Si(a*d/b+d*x)*sin(-c+a*d/b)/a^2/b+1/2*sin(d*x+
c)/a/(b*x+a)^2+sin(d*x+c)/a^2/(b*x+a)
```

Rubi [A]
time = 0.37, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3384, 3380, 3383, 3378}

$$-\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^3}-\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^3}+\frac{\sin(c)\text{CosIntegral}(dx)}{a^3}+\frac{\cos(c)\text{Si}(dx)}{a^3}-\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2b}+\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2b}+\frac{\sin(c+dx)}{a^2(a+bx)}+\frac{d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2ab^2}+\frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2ab^2}+\frac{\sin(c+dx)}{2a(a+bx)^2}+\frac{d\cos(c+dx)}{2ab(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x)^3),x]
```

```
[Out] (d*Cos[c + d*x])/(2*a*b*(a + b*x)) - (d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/
b + d*x])/(a^2*b) + (CosIntegral[d*x]*Sin[c])/a^3 - (CosIntegral[(a*d)/b +
d*x]*Sin[c - (a*d)/b])/a^3 + (d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/
b])/(2*a*b^2) + Sin[c + d*x]/(2*a*(a + b*x)^2) + Sin[c + d*x]/(a^2*(a + b*x
)) + (Cos[c]*SinIntegral[d*x])/a^3 - (Cos[c - (a*d)/b]*SinIntegral[(a*d)/b
+ d*x])/a^3 + (d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*a*b^2) +
 (d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(a^2*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx &= \int \left(\frac{\sin(c+dx)}{a^3x} - \frac{b\sin(c+dx)}{a(a+bx)^3} - \frac{b\sin(c+dx)}{a^2(a+bx)^2} - \frac{b\sin(c+dx)}{a^3(a+bx)}\right)dx \\
&= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^3} - \frac{b\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^3} - \frac{b\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{a^2} - \frac{b\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{a} \\
&= \frac{\sin(c+dx)}{2a(a+bx)^2} + \frac{\sin(c+dx)}{a^2(a+bx)} - \frac{d\int \frac{\cos(c+dx)}{a+bx}\,dx}{a^2} - \frac{d\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2a} + \frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a^3} - \frac{(b
\\
&= \frac{d\cos(c+dx)}{2ab(a+bx)} + \frac{\mathrm{Ci}(dx)\sin(c)}{a^3} - \frac{\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} + \frac{\sin(c+dx)}{2a(a+bx)^2} + \frac{\sin(c+dx)}{a^2(a+bx)}
\\
&= \frac{d\cos(c+dx)}{2ab(a+bx)} - \frac{d\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{a^2b} + \frac{\mathrm{Ci}(dx)\sin(c)}{a^3} - \frac{\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3}
\\
&= \frac{d\cos(c+dx)}{2ab(a+bx)} - \frac{d\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{a^2b} + \frac{\mathrm{Ci}(dx)\sin(c)}{a^3} - \frac{\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3}
\end{aligned}$$

Mathematica [A]
time = 0.62, size = 449, normalized size = 1.72

$$\frac{a^3bd\cos(c+dx) + a^2b^2dx\cos(c+dx) + 2b^2(a+bx)^2\mathrm{Ci}(dx)\sin(c) + (a+bx)^2\mathrm{Ci}\left(d\left(\frac{a}{b}+x\right)\right)\left(-2abd\cos\left(c-\frac{ad}{b}\right) + (-2b^2+a^2d^2)\sin\left(c-\frac{ad}{b}\right)\right) + 3a^2b^2\sin(c+dx) + 2ab^3x\sin(c+dx) + 2a^2b^2\cos(c)\mathrm{Si}(dx) + 4ab^3x\cos(c)\mathrm{Si}(dx) + 2b^4x^2\cos(c)\mathrm{Si}(dx) - 2a^2b^2\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right) + a^4d^2\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right) - 4ab^3x\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right) + 2a^3bd^2x\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right) - 2b^4x^2\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right) + a^2b^2d^2x^2\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right) + 2a^3bd\sin\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right) + 4a^2b^2dx\sin\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right) + 2ab^3dx^2\sin\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right)}{2a^3b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x)^3),x]
```

```
[Out] (a^3*b*d*Cos[c + d*x] + a^2*b^2*d*x*Cos[c + d*x] + 2*b^2*(a + b*x)^2*CosInt
egral[d*x]*Sin[c] + (a + b*x)^2*CosIntegral[d*(a/b + x)]*(-2*a*b*d*Cos[c -
```

```
(a*d)/b] + (-2*b^2 + a^2*d^2)*Sin[c - (a*d)/b]) + 3*a^2*b^2*Sin[c + d*x] +
2*a*b^3*x*Sin[c + d*x] + 2*a^2*b^2*Cos[c]*SinIntegral[d*x] + 4*a*b^3*x*Cos[
c]*SinIntegral[d*x] + 2*b^4*x^2*Cos[c]*SinIntegral[d*x] - 2*a^2*b^2*Cos[c -
 (a*d)/b]*SinIntegral[d*(a/b + x)] + a^4*d^2*Cos[c - (a*d)/b]*SinIntegral[d
*(a/b + x)] - 4*a*b^3*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 2*a^3*b
*d^2*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] - 2*b^4*x^2*Cos[c - (a*d)/
b]*SinIntegral[d*(a/b + x)] + a^2*b^2*d^2*x^2*Cos[c - (a*d)/b]*SinIntegral[
d*(a/b + x)] + 2*a^3*b*d*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 4*a^2*
b^2*d*x*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 2*a*b^3*d*x^2*Sin[c - (
a*d)/b]*SinIntegral[d*(a/b + x)])/(2*a^3*b^2*(a + b*x)^2)
```

Maple [A]
time = 0.14, size = 359, normalized size = 1.38

method	result
derivativedivides	$-\frac{db\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{a^2}+$ sinInt
default	$-\frac{db\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\mathrm{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\mathrm{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{a^2}+$ sinInt
risch	$\frac{d\,\mathrm{e}^{-\frac{i(da-cb)}{b}}\,\mathrm{expIntegral}\left(1,-idx-ic-\frac{iad-ibc}{b}\right)}{2a^2b}+\frac{i\mathrm{e}^{\frac{i(da-cb)}{b}}\,\mathrm{expIntegral}\left(1,idx+ic+\frac{i(da-cb)}{b}\right)}{2a^3}+\frac{i\mathrm{e}^{ic}\,\mathrm{expIntegral}(}{2a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -d*b/a^2*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d
-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+1/a^3*(Si(d*x)*cos(
c)+Ci(d*x)*sin(c))-d^2*b/a*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b+1/2*(-c
os(d*x+c)/(d*a-c*b+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-C
i(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)-b/a^3*(Si(d*x+c+(a*d-b*c)/b)
*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)^3*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(259) = 518$.
time = 0.40, size = 532, normalized size = 2.04

$$\frac{4\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)\cos\left(c\right)\operatorname{Si}\left(dx\right)+2\left(a^{2}b^{2}dx+a^{3}bd\right)\cos\left(dx+c\right)-2\left(\left(ab^{3}dx^{2}+2\,a^{2}b^{2}dx+a^{3}bd\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+\left(ab^{3}dx^{2}+2\,a^{2}b^{2}dx+a^{3}bd\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)-\left(a^{4}d^{2}-2\,a^{2}b^{2}+\left(a^{2}b^{2}d^{2}-2\,b^{4}\right)x^{2}+2\left(a^{3}bd^{2}-2\,ab^{3}\right)x\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+2\left(2\,ab^{3}x+3\,a^{2}b^{2}\right)\sin\left(dx+c\right)+2\left(\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)\operatorname{Ci}\left(dx\right)+\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)\operatorname{Ci}\left(-dx\right)\right)\sin\left(c\right)-\left(\left(a^{4}d^{2}-2\,a^{2}b^{2}+\left(a^{2}b^{2}d^{2}-2\,b^{4}\right)x^{2}+2\left(a^{3}bd^{2}-2\,ab^{3}\right)x\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+\left(a^{4}d^{2}-2\,a^{2}b^{2}+\left(a^{2}b^{2}d^{2}-2\,b^{4}\right)x^{2}+2\left(a^{3}bd^{2}-2\,ab^{3}\right)x\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)+4\left(ab^{3}dx^{2}+2\,a^{2}b^{2}dx+a^{3}bd\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{4\left(a^{3}b^{4}x^{2}+2\,a^{4}b^{3}x+a^{5}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*cos(c)*sin_integral(d*x) + 2*(a^2*b^
2*d*x + a^3*b*d)*cos(d*x + c) - 2*((a*b^3*d*x^2 + 2*a^2*b^2*d*x + a^3*b*d)*
cos_integral((b*d*x + a*d)/b) + (a*b^3*d*x^2 + 2*a^2*b^2*d*x + a^3*b*d)*cos
_integral(-(b*d*x + a*d)/b) - (a^4*d^2 - 2*a^2*b^2 + (a^2*b^2*d^2 - 2*b^4)*
x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c -
 a*d)/b) + 2*(2*a*b^3*x + 3*a^2*b^2)*sin(d*x + c) + 2*((b^4*x^2 + 2*a*b^3*x
 + a^2*b^2)*cos_integral(d*x) + (b^4*x^2 + 2*a*b^3*x + a^2*b^2)*cos_integra
l(-d*x))*sin(c) - ((a^4*d^2 - 2*a^2*b^2 + (a^2*b^2*d^2 - 2*b^4)*x^2 + 2*(a^
3*b*d^2 - 2*a*b^3)*x)*cos_integral((b*d*x + a*d)/b) + (a^4*d^2 - 2*a^2*b^2
+ (a^2*b^2*d^2 - 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*cos_integral(-(b*d
*x + a*d)/b) + 4*(a*b^3*d*x^2 + 2*a^2*b^2*d*x + a^3*b*d)*sin_integral((b*d*
x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c+dx\right)}{x\left(a+bx\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)**3,x)
```

```
[Out] Integral(sin(c + d*x)/(x*(a + b*x)**3), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.62, size = 17806, normalized size = 68.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^2*s
in_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
```

2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^2*real_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2
- 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x*imag_part(
cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 -
2*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 - 2*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*d^2*x*sin_integral((
b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*
x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^2*
b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 4*a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*b^2*d^2*x^2*si
n_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a*
b^3*d*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(1/2*a*d/b) - 4*a*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x*real_part(cos_integral
(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x*r
eal_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*
d/b) + 8*a*b^3*d*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b) - a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^2*sin_integra
l((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*d*x^2*imag_par
t(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4
*a*b^3*d*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x*real_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a*b^3*d*x
^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2
 + a^2*b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/
2*a*d/b)^2 - a^2*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan
(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d^2*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^4*x^2*imag_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^4*x^2*
imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 -
 a^4*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 + 2*b^4*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^4*x^2*imag_part(cos_integral(-d*

```
x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a^2*b^2*d*x*real_part(
cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4
*a^2*b^2*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 - 4*b^4*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + 2*a^4*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)
^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*b^4*x^2*sin_integral((b*d*x + a*d)/b)*
tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^2*real_part(
cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*b^2*d^2*x^2*re
al_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^3*b*d^2
*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^3
*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*a*b^3*d*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*a*b^3*d*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*
tan(1/2*c)^2 - 4*a^3*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a
*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a^3*b*d^2*x*imag_part(cos_integral
(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)}{x\,(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x*(a + b*x)^3),x)
```

```
[Out] int(sin(c + d*x)/(x*(a + b*x)^3), x)
```

3.38 $\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx$

Optimal. Leaf size=299

$$-\frac{d\cos(c+dx)}{2a^2(a+bx)}+\frac{d\cos(c)\text{Ci}(dx)}{a^3}+\frac{2d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{a^3}-\frac{3b\text{Ci}(dx)\sin(c)}{a^4}+\frac{3b\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right.}{a^4}$$

```
[Out] d*Ci(d*x)*cos(c)/a^3+2*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/a^3-1/2*d*cos(d*x+c)/a
^2/(b*x+a)-3*b*cos(c)*Si(d*x)/a^4+3*b*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^4-1/2*d
^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^2/b-3*b*Ci(d*x)*sin(c)/a^4-d*Si(d*x)*sin(c
)/a^3-3*b*Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^4+1/2*d^2*Ci(a*d/b+d*x)*sin(-c+a*d/
b)/a^2/b+2*d*Si(a*d/b+d*x)*sin(-c+a*d/b)/a^3-sin(d*x+c)/a^3/x-1/2*b*sin(d*x
+c)/a^2/(b*x+a)^2-2*b*sin(d*x+c)/a^3/(b*x+a)
```

Rubi [A]

time = 0.46, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$-\frac{3b\sin(c)\text{CosIntegral}(dx)}{a^4}+\frac{3b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^4}-\frac{3b\cos(c)\text{Si}(dx)}{a^4}+\frac{3b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^4}+\frac{2d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^3}-\frac{2d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{2b\sin(c+dx)}{a^3(a+bx)}+\frac{d\cos(c)\text{CosIntegral}(dx)}{a^3}-\frac{d\sin(c)\text{Si}(dx)}{a^3}-\frac{\sin(c+dx)}{a^3x}-\frac{d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2a^2b}-\frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2a^2b}-\frac{b\sin(c+dx)}{2a^2(a+bx)^2}-\frac{d\cos(c+dx)}{2a^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x)^3),x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(a^2*(a + b*x)) + (d*Cos[c]*CosIntegral[d*x])/a^3 + (
2*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/a^3 - (3*b*CosIntegral[d*x
]*Sin[c])/a^4 + (3*b*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^4 - (d^
2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*a^2*b) - Sin[c + d*x]/(a^
3*x) - (b*Sin[c + d*x])/(2*a^2*(a + b*x)^2) - (2*b*Sin[c + d*x])/(a^3*(a +
b*x)) - (3*b*Cos[c]*SinIntegral[d*x])/a^4 - (d*Sin[c]*SinIntegral[d*x])/a^3
 + (3*b*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^4 - (d^2*Cos[c - (a*
d)/b]*SinIntegral[(a*d)/b + d*x])/(2*a^2*b) - (2*d*Sin[c - (a*d)/b]*SinInte
gral[(a*d)/b + d*x])/a^3
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx &= \int \left(\frac{\sin(c+dx)}{a^3x^2} - \frac{3b\sin(c+dx)}{a^4x} + \frac{b^2\sin(c+dx)}{a^2(a+bx)^3} + \frac{2b^2\sin(c+dx)}{a^3(a+bx)^2} + \frac{3b^2\sin(c+dx)}{a^4(a+bx)}\right.\\
&= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a^3} - \frac{(3b)\int \frac{\sin(c+dx)}{x}\,dx}{a^4} + \frac{(3b^2)\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^4} + \frac{(2b^2)\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{a^3} + \frac{b^2\int \frac{\sin(c}{(a+}}{a}\\
&= -\frac{\sin(c+dx)}{a^3x} - \frac{b\sin(c+dx)}{2a^2(a+bx)^2} - \frac{2b\sin(c+dx)}{a^3(a+bx)} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{a^3} + \frac{(2bd)\int \frac{\cos(c+dx)}{a+bx}\,dx}{a^3}\\
&= -\frac{d\cos(c+dx)}{2a^2(a+bx)} - \frac{3b\mathrm{Ci}(dx)\sin(c)}{a^4} + \frac{3b\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^4} - \frac{\sin(c+dx)}{a^3x} - \frac{b\sin(c}{2a^2(a}\\
&= -\frac{d\cos(c+dx)}{2a^2(a+bx)} + \frac{d\cos(c)\mathrm{Ci}(dx)}{a^3} + \frac{2d\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{3b\mathrm{Ci}(dx)\sin(c)}{a^4} + \frac{3b}{}\\
&= -\frac{d\cos(c+dx)}{2a^2(a+bx)} + \frac{d\cos(c)\mathrm{Ci}(dx)}{a^3} + \frac{2d\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{3b\mathrm{Ci}(dx)\sin(c)}{a^4} + \frac{3b}{}\end{aligned}$$

Mathematica [A]
time = 1.31, size = 540, normalized size = 1.81

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x)^3),x]
```

```
[Out] -1/2*(a^3*b*d*x*Cos[c + d*x] + a^2*b^2*d*x^2*Cos[c + d*x] + 2*b*x*(a + b*x)
^2*CosIntegral[d*x]*(-(a*d*Cos[c]) + 3*b*Sin[c]) + x*(a + b*x)^2*CosIntegra
l[d*(a/b + x)]*(-4*a*b*d*Cos[c - (a*d)/b] + (-6*b^2 + a^2*d^2)*Sin[c - (a*d
)/b]) + 2*a^3*b*Sin[c + d*x] + 9*a^2*b^2*x*Sin[c + d*x] + 6*a*b^3*x^2*Sin[c
 + d*x] + 6*a^2*b^2*x*Cos[c]*SinIntegral[d*x] + 12*a*b^3*x^2*Cos[c]*SinInte
gral[d*x] + 6*b^4*x^3*Cos[c]*SinIntegral[d*x] + 2*a^3*b*d*x*Sin[c]*SinInteg
ral[d*x] + 4*a^2*b^2*d*x^2*Sin[c]*SinIntegral[d*x] + 2*a*b^3*d*x^3*Sin[c]*S
inIntegral[d*x] - 6*a^2*b^2*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + a
^4*d^2*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] - 12*a*b^3*x^2*Cos[c - (
a*d)/b]*SinIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x^2*Cos[c - (a*d)/b]*SinInte
gral[d*(a/b + x)] - 6*b^4*x^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + a
^2*b^2*d^2*x^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 4*a^3*b*d*x*Sin[
c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 8*a^2*b^2*d*x^2*Sin[c - (a*d)/b]*Si
nIntegral[d*(a/b + x)] + 4*a*b^3*d*x^3*Sin[c - (a*d)/b]*SinIntegral[d*(a/b
+ x)])/(a^4*b*x*(a + b*x)^2)
```

Maple [A]
time = 0.17, size = 405, normalized size = 1.35

method	result
derivativedivides	$d\left(\frac{2b^2\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\text{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\text{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{a^3}+\frac{-\text{s}}{}\right.$
default	$d\left(\frac{2b^2\left(-\frac{\sin(dx+c)}{(da-cb+b(dx+c))b}+\frac{\frac{\text{sinIntegral}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)}{b}+\frac{\text{cosineIntegral}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)}{b}}{b}\right)}{a^3}+\frac{-\text{s}}{}\right.$
risch	$-\frac{d\,\mathrm{e}^{-\frac{i(da-cb)}{b}}\,\text{expIntegral}\left(1,-idx-ic-\frac{iad-ibc}{b}\right)}{a^3}-\frac{d\,\mathrm{e}^{ic}\,\text{expIntegral}(1,-idx)}{2a^3}-\frac{id^2\mathrm{e}^{-\frac{i(da-cb)}{b}}\,\text{expIntegral}\left(1,-id\right.}{4b\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d*(2*b^2/a^3*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)*sin(
(a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+1/a^3*(-sin(d*x
+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))-3/d/a^4*b*(Si(d*x)*cos(c)+Ci(d*x)*si
n(c))+d*b^2/a^2*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c))^2/b+1/2*(-cos(d*x+c)/(
```

```
d*a-c*b+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*
d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)+3/d*b^2/a^4*(Si(d*x+c+(a*d-b*c)/b)*cos(
(a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)^3*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(297) = 594$.
time = 0.40, size = 689, normalized size = 2.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^2*b^2*d*x^2 + a^3*b*d*x)*cos(d*x + c) - 2*((a*b^3*d*x^3 + 2*a^2*
b^2*d*x^2 + a^3*b*d*x)*cos_integral(d*x) + (a*b^3*d*x^3 + 2*a^2*b^2*d*x^2 +
 a^3*b*d*x)*cos_integral(-d*x) - 6*(b^4*x^3 + 2*a*b^3*x^2 + a^2*b^2*x)*sin_
integral(d*x))*cos(c) - 2*(2*(a*b^3*d*x^3 + 2*a^2*b^2*d*x^2 + a^3*b*d*x)*co
s_integral((b*d*x + a*d)/b) + 2*(a*b^3*d*x^3 + 2*a^2*b^2*d*x^2 + a^3*b*d*x)
*cos_integral(-(b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 6*b^4)*x^3 + 2*(a^3*b*d^2
 - 6*a*b^3)*x^2 + (a^4*d^2 - 6*a^2*b^2)*x)*sin_integral((b*d*x + a*d)/b))*c
os(-(b*c - a*d)/b) + 2*(6*a*b^3*x^2 + 9*a^2*b^2*x + 2*a^3*b)*sin(d*x + c) +
 2*(3*(b^4*x^3 + 2*a*b^3*x^2 + a^2*b^2*x)*cos_integral(d*x) + 3*(b^4*x^3 +
2*a*b^3*x^2 + a^2*b^2*x)*cos_integral(-d*x) + 2*(a*b^3*d*x^3 + 2*a^2*b^2*d*
x^2 + a^3*b*d*x)*sin_integral(d*x))*sin(c) - (((a^2*b^2*d^2 - 6*b^4)*x^3 +
2*(a^3*b*d^2 - 6*a*b^3)*x^2 + (a^4*d^2 - 6*a^2*b^2)*x)*cos_integral((b*d*x
+ a*d)/b) + ((a^2*b^2*d^2 - 6*b^4)*x^3 + 2*(a^3*b*d^2 - 6*a*b^3)*x^2 + (a^4
*d^2 - 6*a^2*b^2)*x)*cos_integral(-(b*d*x + a*d)/b) + 8*(a*b^3*d*x^3 + 2*a^
2*b^2*d*x^2 + a^3*b*d*x)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b)
)/(a^4*b^3*x^3 + 2*a^5*b^2*x^2 + a^6*b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c + dx\right)}{x^{2} \left(a + bx\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x+a)**3,x)
```

```
[Out] Integral(sin(c + d*x)/(x**2*(a + b*x)**3), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4. time = 5.16, size = 20808, normalized size = 69.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a^2*b^2*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^3*imag_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^3*
sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
+ 2*a^2*b^2*d^2*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b^2*d^2*x^3*real_part(cos_integral(-d*x -
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^3*real
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2
 - 2*a^2*b^2*d^2*x^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^2*imag_part(cos_integral(d*x + a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x^2*imag_
part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
^2 - 4*a*b^3*d*x^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^3*d*x^3*real_part(cos_integral(d*x))*tan(
1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^3*d*x^3*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^3*
d*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d
/b)^2 + 4*a^3*b*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^3*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^2*b^2*d^2*x^3*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*b^2*d^2*x^3*sin_integral((
b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*b^2*d^2*x^3*imag_part(c
os_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*
b^2*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
*tan(1/2*a*d/b) + 8*a^2*b^2*d^2*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d
*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a*b^3*d*x^3*imag_part(cos_integral(d*x
+ a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a*b^3*d*x^3*imag_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
+ 4*a^3*b*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x^2*real_part(cos_integral(-d*x - a*d/
b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 16*a*b^3*d*x^3*sin_integra
l((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - a^2*b^2*d^2
*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 +
 a^2*b^2*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
```

```
/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^
2*tan(1/2*a*d/b)^2 - 8*a*b^3*d*x^3*imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b^3*d*x^3*imag_part(cos_integ
ral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^3*d*x^3*imag_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2
- 4*a*b^3*d*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan
(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^2*real_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b^3*d*x^
3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 16*a*b^3*d
*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)
^2 + a^2*b^2*d^2*x^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(
1/2*a*d/b)^2 - a^2*b^2*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^3*sin_integral((b*d*x + a*d)/b)*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d^2*x*imag_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*b^4*x^3*imag_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*b^4*
x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
^2 - a^4*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c)^2*tan(1/2*a*d/b)^2 + 6*b^4*x^3*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*b^4*x^3*imag_part(cos_integr
al(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d*x^2*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 + 4*a^2*b^2*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 - 8*a^2*b^2*d*x^2*real_part(cos_integral(-d*x - a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^2*b^2*d*x^2*real_part
(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 12*b^4*
x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4*
d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d
/b)^2 - 12*b^4*x^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^
2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^3*real_part(cos_integral(d*x + a*d/b))
*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*b^2*d^2*x^3*real_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^3*b*d...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)}{x^2\,{\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x^2*(a + b*x)^3),x)
```

```
[Out] int(sin(c + d*x)/(x^2*(a + b*x)^3), x)
```

3.39 $\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx$

Optimal. Leaf size=377

$$-\frac{d\cos(c+dx)}{2a^3x}+\frac{bd\cos(c+dx)}{2a^3(a+bx)}-\frac{3bd\cos(c)\mathrm{Ci}(dx)}{a^4}-\frac{3bd\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{a^4}+\frac{6b^2\mathrm{Ci}(dx)\sin(c)}{a^5}-\frac{d^2\mathrm{Ci}(}{}$$

```
[Out] -3*b*d*Ci(d*x)*cos(c)/a^4-3*b*d*Ci(a*d/b+d*x)*cos(-c+a*d/b)/a^4-1/2*d*cos(d
*x+c)/a^3/x+1/2*b*d*cos(d*x+c)/a^3/(b*x+a)+6*b^2*cos(c)*Si(d*x)/a^5-1/2*d^2
*cos(c)*Si(d*x)/a^3-6*b^2*cos(-c+a*d/b)*Si(a*d/b+d*x)/a^5+1/2*d^2*cos(-c+a*
d/b)*Si(a*d/b+d*x)/a^3+6*b^2*Ci(d*x)*sin(c)/a^5-1/2*d^2*Ci(d*x)*sin(c)/a^3+
3*b*d*Si(d*x)*sin(c)/a^4+6*b^2*Ci(a*d/b+d*x)*sin(-c+a*d/b)/a^5-1/2*d^2*Ci(a
*d/b+d*x)*sin(-c+a*d/b)/a^3-3*b*d*Si(a*d/b+d*x)*sin(-c+a*d/b)/a^4-1/2*sin(d
*x+c)/a^3/x^2+3*b*sin(d*x+c)/a^4/x+1/2*b^2*sin(d*x+c)/a^3/(b*x+a)^2+3*b^2*s
in(d*x+c)/a^4/(b*x+a)
```

Rubi [A]
time = 0.54, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6874, 3378, 3384, 3380, 3383}

$$\frac{6b^2\sin(c)\mathrm{CosIntegral}(dx)}{a^5}-\frac{6b^2\sin\left(c-\frac{ad}{b}\right)\mathrm{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^5}+\frac{6b^2\cos(c)\mathrm{Si}(dx)}{a^5}-\frac{6b^2\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(xd+\frac{ad}{b}\right)}{a^5}+\frac{3b^2\sin(c+dx)}{a^4(a+bx)}-\frac{3bd\cos(c)\mathrm{CosIntegral}(dx)}{a^4}-\frac{3bd\cos\left(c-\frac{ad}{b}\right)\mathrm{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^4}+\frac{3bd\sin(c)\mathrm{Si}(dx)}{a^4}+\frac{3bd\sin\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(xd+\frac{ad}{b}\right)}{a^4}+\frac{3b\sin(c+dx)}{a^4x}+\frac{b^2\sin(c+dx)}{2a^3(a+bx)^2}+\frac{d^2\sin\left(c-\frac{ad}{b}\right)\mathrm{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2a^3}+\frac{d^2\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(xd+\frac{ad}{b}\right)}{2a^3}+\frac{bd\cos(c+dx)}{2a^3(a+bx)}-\frac{d^2\sin(c)\mathrm{CosIntegral}(dx)}{2a^3}-\frac{d^2\cos(c)\mathrm{Si}(dx)}{2a^3}-\frac{\sin(c+dx)}{2a^3x^2}-\frac{d\cos(c+dx)}{2a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x)^3),x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(a^3*x) + (b*d*Cos[c + d*x])/(2*a^3*(a + b*x)) - (3*b
*d*Cos[c]*CosIntegral[d*x])/a^4 - (3*b*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)
/b + d*x])/a^4 + (6*b^2*CosIntegral[d*x]*Sin[c])/a^5 - (d^2*CosIntegral[d*x
]*Sin[c])/(2*a^3) - (6*b^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^5
 + (d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*a^3) - Sin[c + d*x]
/(2*a^3*x^2) + (3*b*Sin[c + d*x])/(a^4*x) + (b^2*Sin[c + d*x])/(2*a^3*(a +
b*x)^2) + (3*b^2*Sin[c + d*x])/(a^4*(a + b*x)) + (6*b^2*Cos[c]*SinIntegral[
d*x])/a^5 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a^3) + (3*b*d*Sin[c]*SinIntegr
al[d*x])/a^4 - (6*b^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^5 + (d
^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*a^3) + (3*b*d*Sin[c - (a
*d)/b]*SinIntegral[(a*d)/b + d*x])/a^4
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx = \int \left(\frac{\sin(c+dx)}{a^3x^3} - \frac{3b\sin(c+dx)}{a^4x^2} + \frac{6b^2\sin(c+dx)}{a^5x} - \frac{b^3\sin(c+dx)}{a^3(a+bx)^3} - \frac{3b^3\sin(c+dx)}{a^4(a+bx)^2}\right.$$

$$= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a^3} - \frac{(3b)\int \frac{\sin(c+dx)}{x^2}\,dx}{a^4} + \frac{(6b^2)\int \frac{\sin(c+dx)}{x}\,dx}{a^5} - \frac{(6b^3)\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^5} - \frac{(3b^3)\int}{}$$

$$= -\frac{\sin(c+dx)}{2a^3x^2} + \frac{3b\sin(c+dx)}{a^4x} + \frac{b^2\sin(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\sin(c+dx)}{a^4(a+bx)} + \frac{d\int \frac{\cos(c+dx)}{x^2}\,dx}{2a^3} -$$

$$= -\frac{d\cos(c+dx)}{2a^3x} + \frac{bd\cos(c+dx)}{2a^3(a+bx)} + \frac{6b^2\operatorname{Ci}(dx)\sin(c)}{a^5} - \frac{6b^2\operatorname{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^5} -$$

$$= -\frac{d\cos(c+dx)}{2a^3x} + \frac{bd\cos(c+dx)}{2a^3(a+bx)} - \frac{3bd\cos(c)\operatorname{Ci}(dx)}{a^4} - \frac{3bd\cos\left(c-\frac{ad}{b}\right)\operatorname{Ci}\left(\frac{ad}{b}+dx\right)}{a^4} +$$

$$= -\frac{d\cos(c+dx)}{2a^3x} + \frac{bd\cos(c+dx)}{2a^3(a+bx)} - \frac{3bd\cos(c)\operatorname{Ci}(dx)}{a^4} - \frac{3bd\cos\left(c-\frac{ad}{b}\right)\operatorname{Ci}\left(\frac{ad}{b}+dx\right)}{a^4} +$$

Mathematica [A]
time = 1.18, size = 630, normalized size = 1.67

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x)^3),x]
```

```
[Out] (-(a^4*d*x*Cos[c + d*x]) - a^3*b*d*x^2*Cos[c + d*x] - x^2*(a + b*x)^2*CosIn
tegral[d*x]*(6*a*b*d*Cos[c] + (-12*b^2 + a^2*d^2)*Sin[c]) + x^2*(a + b*x)^2
*CosIntegral[d*(a/b + x)]*(-6*a*b*d*Cos[c - (a*d)/b] + (-12*b^2 + a^2*d^2)*
Sin[c - (a*d)/b]) - a^4*Sin[c + d*x] + 4*a^3*b*x*Sin[c + d*x] + 18*a^2*b^2*
x^2*Sin[c + d*x] + 12*a*b^3*x^3*Sin[c + d*x] + 12*a^2*b^2*x^2*Cos[c]*SinInt
egral[d*x] - a^4*d^2*x^2*Cos[c]*SinIntegral[d*x] + 24*a*b^3*x^3*Cos[c]*SinI
ntegral[d*x] - 2*a^3*b*d^2*x^3*Cos[c]*SinIntegral[d*x] + 12*b^4*x^4*Cos[c]*
SinIntegral[d*x] - a^2*b^2*d^2*x^4*Cos[c]*SinIntegral[d*x] + 6*a^3*b*d*x^2*
Sin[c]*SinIntegral[d*x] + 12*a^2*b^2*d*x^3*Sin[c]*SinIntegral[d*x] + 6*a*b^
3*d*x^4*Sin[c]*SinIntegral[d*x] - 12*a^2*b^2*x^2*Cos[c - (a*d)/b]*SinIntegr
al[d*(a/b + x)] + a^4*d^2*x^2*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] - 2
4*a*b^3*x^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x^3*Cos
[c - (a*d)/b]*SinIntegral[d*(a/b + x)] - 12*b^4*x^4*Cos[c - (a*d)/b]*SinInt
egral[d*(a/b + x)] + a^2*b^2*d^2*x^4*Cos[c - (a*d)/b]*SinIntegral[d*(a/b +
x)] + 6*a^3*b*d*x^2*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 12*a^2*b^2*
d*x^3*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 6*a*b^3*d*x^4*Sin[c - (a*
d)/b]*SinIntegral[d*(a/b + x)])/(2*a^5*x^2*(a + b*x)^2)
```

Maple [**A**]
time = 0.21, size = 466, normalized size = 1.24 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-3/d*b^3/a^4*(-sin(d*x+c)/(d*a-c*b+b*(d*x+c))/b+(Si(d*x+c+(a*d-b*c)/b)
*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-3/d/a^4*b*
(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+6/d^2/a^5*b^2*(Si(d*x)*cos(
c)+Ci(d*x)*sin(c))+1/a^3*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si
(d*x)*cos(c)-1/2*Ci(d*x)*sin(c))-b^3/a^3*(-1/2*sin(d*x+c)/(d*a-c*b+b*(d*x+c
))^2/b+1/2*(-cos(d*x+c)/(d*a-c*b+b*(d*x+c))/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a
*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)-6/d^2*b^3/a^5*
(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*
c)/b)/b))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)^3*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(367) = 734$.
time = 0.41, size = 816, normalized size = 2.16

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^3*b*d*x^2 + a^4*d*x)*cos(d*x + c) + 2*(3*(a*b^3*d*x^4 + 2*a^2*b^
2*d*x^3 + a^3*b*d*x^2)*cos_integral(d*x) + 3*(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3
 + a^3*b*d*x^2)*cos_integral(-d*x) + ((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b
*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 - 12*a^2*b^2)*x^2)*sin_integral(d*x))*cos(c
) + 2*(3*(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3 + a^3*b*d*x^2)*cos_integral((b*d*x
+ a*d)/b) + 3*(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3 + a^3*b*d*x^2)*cos_integral(-(
b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b*d^2 - 12*a*b^3)*x^
3 + (a^4*d^2 - 12*a^2*b^2)*x^2)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c -
a*d)/b) - 2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4)*sin(d*x + c)
+ (((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 -
12*a^2*b^2)*x^2)*cos_integral(d*x) + ((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b
*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 - 12*a^2*b^2)*x^2)*cos_integral(-d*x) - 12*
(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3 + a^3*b*d*x^2)*sin_integral(d*x))*sin(c) + (
((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 - 12*
a^2*b^2)*x^2)*cos_integral((b*d*x + a*d)/b) + ((a^2*b^2*d^2 - 12*b^4)*x^4 +
 2*(a^3*b*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 - 12*a^2*b^2)*x^2)*cos_integral(-(
b*d*x + a*d)/b) + 12*(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3 + a^3*b*d*x^2)*sin_inte
gral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^
7*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{x^3 (a+bx)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**3/(b*x+a)**3,x)
```

```
[Out] Integral(sin(c + d*x)/(x**3*(a + b*x)**3), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.11, size = 24116, normalized size = 63.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x))*
tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^4*imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^
2*b^2*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan
(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^4*sin_integral((b*d*x + a*d)/b)*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*b^2*d^2*x^4*real_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*
b^2*d^2*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
^2*tan(1/2*a*d/b) - 2*a^2*b^2*d^2*x^4*real_part(cos_integral(d*x + a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^4*real_part(co
s_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2
*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/
2*a*d/b)^2 - 2*a^2*b^2*d^2*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2
*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^3*imag_part(cos_integral(d*x +
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d^2*x^3*ima
g_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*
a^3*b*d^2*x^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1
/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a*b^3*d*x^4*real_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a*b^3*d*
x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
^2 - 6*a*b^3*d*x^4*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)^2*tan(1/2*a*d/b)^2 + 6*a*b^3*d*x^4*real_part(cos_integral(-d*x))*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*d^2*x^3*sin_integral(d
*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*d^2*x^3*sin_inte
gral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^
2*d^2*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)^2 - a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*a^2*b^2*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*a^2*b^2*d^2*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c
)^2 + 4*a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*b^2*d^2*x^4*sin_
integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 12*a*b
^3*d*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(1/2*a*d/b) - 12*a*b^3*d*x^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x^3*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b*d^2*x
^3*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
```

```
2*a*d/b) + 24*a*b^3*d*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(1/2*a*d/b) - a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^4*imag_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^4*imag_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*b^2*d^2*x^4*imag
_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x
^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b^2*d^2*x^4*si
n_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 12*a*b^3*d*x^
4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*
d/b)^2 + 12*a*b^3*d*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2
*c)*tan(1/2*a*d/b)^2 + 12*a*b^3*d*x^4*imag_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 12*a*b^3*d*x^4*imag_part(cos_
integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^
3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*
d/b)^2 - 4*a^3*b*d^2*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/
2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^3*real_part(cos_integral(-d*x - a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b*d^2*x^3*real_part(c
os_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 24*a*b^3*d*
x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 24*a*b^3
*d*x^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/
b)^2 + a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*ta
n(1/2*a*d/b)^2 + a^2*b^2*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^4*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*b^2*d^2*x^4*i...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)}{x^3\,(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x^3*(a + b*x)^3),x)
```

```
[Out] int(sin(c + d*x)/(x^3*(a + b*x)^3), x)
```

3.40 $\int x^3(a+bx^2)\sin(c+dx)\,dx$

Optimal. Leaf size=141

$$-\frac{120bx\cos(c+dx)}{d^5}+\frac{6ax\cos(c+dx)}{d^3}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{120b\sin(c+}{d^6}$$

```
[Out] -120*b*x*cos(d*x+c)/d^5+6*a*x*cos(d*x+c)/d^3+20*b*x^3*cos(d*x+c)/d^3-a*x^3*
cos(d*x+c)/d-b*x^5*cos(d*x+c)/d+120*b*sin(d*x+c)/d^6-6*a*sin(d*x+c)/d^4-60*
b*x^2*sin(d*x+c)/d^4+3*a*x^2*sin(d*x+c)/d^2+5*b*x^4*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.14, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3420, 3377, 2717}

$$-\frac{6a\sin(c+dx)}{d^4}+\frac{6ax\cos(c+dx)}{d^3}+\frac{3ax^2\sin(c+dx)}{d^2}-\frac{ax^3\cos(c+dx)}{d}+\frac{120b\sin(c+dx)}{d^6}-\frac{120bx\cos(c+dx)}{d^5}-\frac{60bx^2\sin(c+dx)}{d^4}+\frac{20bx^3\cos(c+dx)}{d^3}+\frac{5bx^4\sin(c+dx)}{d^2}-\frac{bx^5\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (-120*b*x*Cos[c + d*x])/d^5 + (6*a*x*Cos[c + d*x])/d^3 + (20*b*x^3*Cos[c +
d*x])/d^3 - (a*x^3*Cos[c + d*x])/d - (b*x^5*Cos[c + d*x])/d + (120*b*Sin[c
+ d*x])/d^6 - (6*a*Sin[c + d*x])/d^4 - (60*b*x^2*Sin[c + d*x])/d^4 + (3*a*x
^2*Sin[c + d*x])/d^2 + (5*b*x^4*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(a+bx^2\right)\sin(c+dx)\,dx &= \int \left(ax^3\sin(c+dx)+bx^5\sin(c+dx)\right)\,dx \\
&= a\int x^3\sin(c+dx)\,dx+b\int x^5\sin(c+dx)\,dx \\
&= -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{(3a)\int x^2\cos(c+dx)\,dx}{d}+\frac{(5b)\int x^4 \cdots}{} \\
&= -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{5bx^4\sin(c+dx)}{d^2} \\
&= \frac{6ax\cos(c+dx)}{d^3}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+ \\
&= \frac{6ax\cos(c+dx)}{d^3}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}- \\
&= -\frac{120bx\cos(c+dx)}{d^5}+\frac{6ax\cos(c+dx)}{d^3}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+d}{d} \\
&= -\frac{120bx\cos(c+dx)}{d^5}+\frac{6ax\cos(c+dx)}{d^3}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+d}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 92, normalized size = 0.65

$$\frac{-dx(ad^2(-6+d^2x^2)+b(120-20d^2x^2+d^4x^4))\cos(c+dx)+(3ad^2(-2+d^2x^2)+5b(24-12d^2x^2+d^4x^4))\sin(c+dx)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (-(d*x*(a*d^2*(-6 + d^2*x^2) + b*(120 - 20*d^2*x^2 + d^4*x^4))*Cos[c + d*x]
) + (3*a*d^2*(-2 + d^2*x^2) + 5*b*(24 - 12*d^2*x^2 + d^4*x^4))*Sin[c + d*x]
)/d^6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(141) = 282$.
time = 0.05, size = 449, normalized size = 3.18 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(a*c^3*cos(d*x+c)+3*a*c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-3*a*c*(-(d*
x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+a*(-(d*x+c)^3*cos(d*x+
c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+1/d^2*b*c^5*co
s(d*x+c)+5/d^2*b*c^4*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-10/d^2*b*c^3*(-(d*x+c)
^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+10/d^2*b*c^2*(-(d*x+c)^3*c
os(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))-5/d^2*b
```

```
*c*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24
*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))+1/d^2*b*(-(d*x+c)^5*cos(d*x+c)+5*(d*x+c)
^4*sin(d*x+c)+20*(d*x+c)^3*cos(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(d*x+c
)-120*(d*x+c)*cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(141) = 282$.
time = 0.29, size = 372, normalized size = 2.64

$$\frac{ac^3\cos(dx+c)+\frac{bc^5\cos(dx+c)}{d^2}-3\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)ac^2-\frac{5\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)bc^4}{d^2}+3\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)ac+\frac{10\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)bc^3}{d^2}-\left(\left((dx+c)^3-6dx-6c\right)\cos(dx+c)-3\left((dx+c)^2-2\right)\sin(dx+c)\right)a-\frac{10\left(\left((dx+c)^3-6dx-6c\right)\cos(dx+c)-3\left((dx+c)^2-2\right)\sin(dx+c)\right)bc^2}{d^2}+\frac{5\left(\left((dx+c)^4-12(dx+c)^2+24\right)\cos(dx+c)-4\left((dx+c)^3-6dx-6c\right)\sin(dx+c)\right)bc}{d^2}-\frac{\left(\left((dx+c)^5-20(dx+c)^3+120dx+120c\right)\cos(dx+c)-5\left((dx+c)^4-12(dx+c)^2+24\right)\sin(dx+c)\right)b}{d^2}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c^3*cos(d*x + c) + b*c^5*cos(d*x + c)/d^2 - 3*((d*x + c)*cos(d*x + c) -
sin(d*x + c))*a*c^2 - 5*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^4/d^2 +
 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*c + 10*(((
d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^3/d^2 - (((d*x
 + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a -
 10*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x
 + c))*b*c^2/d^2 + 5*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*
((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b*c/d^2 - (((d*x + c)^5 - 20*(d*x
 + c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 +
 24)*sin(d*x + c))*b/d^2)/d^4
```

Fricas [A]
time = 0.36, size = 95, normalized size = 0.67

$$-\frac{\left(bd^5x^5+(ad^5-20\,bd^3)x^3-6\,(ad^3-20\,bd)x\right)\cos(dx+c)-\left(5\,bd^4x^4-6\,ad^2+3\,(ad^4-20\,bd^2)x^2+120\,b\right)\sin(dx+c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^5*x^5 + (a*d^5 - 20*b*d^3)*x^3 - 6*(a*d^3 - 20*b*d)*x)*cos(d*x + c)
- (5*b*d^4*x^4 - 6*a*d^2 + 3*(a*d^4 - 20*b*d^2)*x^2 + 120*b)*sin(d*x + c))/
d^6
```

Sympy [A]
time = 0.44, size = 168, normalized size = 1.19

$$\begin{cases} -\frac{ax^3\cos(c+dx)}{d}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{6ax\cos(c+dx)}{d^3}-\frac{6a\sin(c+dx)}{d^4}-\frac{bx^5\cos(c+dx)}{d}+\frac{5bx^4\sin(c+dx)}{d^2}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{60bx^2\sin(c+dx)}{d^4}-\frac{120bx\cos(c+dx)}{d^5}+\frac{120b\sin(c+dx)}{d^6} & \text{for}\ d\neq 0 \\ \left(\frac{ax^4}{4}+\frac{bx^6}{6}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**3*cos(c + d*x)/d + 3*a*x**2*sin(c + d*x)/d**2 + 6*a*x*cos(
c + d*x)/d**3 - 6*a*sin(c + d*x)/d**4 - b*x**5*cos(c + d*x)/d + 5*b*x**4*si
n(c + d*x)/d**2 + 20*b*x**3*cos(c + d*x)/d**3 - 60*b*x**2*sin(c + d*x)/d**4
 - 120*b*x*cos(c + d*x)/d**5 + 120*b*sin(c + d*x)/d**6, Ne(d, 0)), ((a*x**4
/4 + b*x**6/6)*sin(c), True))
```

Giac [A]
time = 3.49, size = 97, normalized size = 0.69

$$-\frac{\left(bd^{5}x^{5}+ad^{5}x^{3}-20\,bd^{3}x^{3}-6\,ad^{3}x+120\,bdx\right)\cos\left(dx+c\right)}{d^{6}}+\frac{\left(5\,bd^{4}x^{4}+3\,ad^{4}x^{2}-60\,bd^{2}x^{2}-6\,ad^{2}+120\,b\right)\sin\left(dx+c\right)}{d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^5*x^5 + a*d^5*x^3 - 20*b*d^3*x^3 - 6*a*d^3*x + 120*b*d*x)*cos(d*x + c
)/d^6 + (5*b*d^4*x^4 + 3*a*d^4*x^2 - 60*b*d^2*x^2 - 6*a*d^2 + 120*b)*sin(d*
x + c)/d^6
```

Mupad [B]
time = 0.34, size = 121, normalized size = 0.86

$$\frac{6\,\sin\left(c+d\,x\right)\,\left(20\,b-a\,d^2\right)}{d^6}+\frac{x^3\,\cos\left(c+d\,x\right)\,\left(20\,b-a\,d^2\right)}{d^3}-\frac{3\,x^2\,\sin\left(c+d\,x\right)\,\left(20\,b-a\,d^2\right)}{d^4}-\frac{6\,x\,\cos\left(c+d\,x\right)\,\left(20\,b-a\,d^2\right)}{d^5}-\frac{b\,x^5\,\cos\left(c+d\,x\right)}{d}+\frac{5\,b\,x^4\,\sin\left(c+d\,x\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(c + d*x)*(a + b*x^2),x)
```

```
[Out] (6*sin(c + d*x)*(20*b - a*d^2))/d^6 + (x^3*cos(c + d*x)*(20*b - a*d^2))/d^3
 - (3*x^2*sin(c + d*x)*(20*b - a*d^2))/d^4 - (6*x*cos(c + d*x)*(20*b - a*d^
2))/d^5 - (b*x^5*cos(c + d*x))/d + (5*b*x^4*sin(c + d*x))/d^2
```

3.41 $\int x^2(a+bx^2)\sin(c+dx)\,dx$

Optimal. Leaf size=111

$$-\frac{24b\cos(c+dx)}{d^5}+\frac{2a\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}-\frac{24bx\sin(c+dx)}{d^4}$$

```
[Out] -24*b*cos(d*x+c)/d^5+2*a*cos(d*x+c)/d^3+12*b*x^2*cos(d*x+c)/d^3-a*x^2*cos(d
*x+c)/d-b*x^4*cos(d*x+c)/d-24*b*x*sin(d*x+c)/d^4+2*a*x*sin(d*x+c)/d^2+4*b*x
^3*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3420, 3377, 2718}

$$\frac{2a\cos(c+dx)}{d^3}+\frac{2ax\sin(c+dx)}{d^2}-\frac{ax^2\cos(c+dx)}{d}-\frac{24b\cos(c+dx)}{d^5}-\frac{24bx\sin(c+dx)}{d^4}+\frac{12bx^2\cos(c+dx)}{d^3}+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{bx^4\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (-24*b*Cos[c + d*x])/d^5 + (2*a*Cos[c + d*x])/d^3 + (12*b*x^2*Cos[c + d*x])
/d^3 - (a*x^2*Cos[c + d*x])/d - (b*x^4*Cos[c + d*x])/d - (24*b*x*Sin[c + d*
x])/d^4 + (2*a*x*Sin[c + d*x])/d^2 + (4*b*x^3*Sin[c + d*x])/d^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2(a+bx^2)\sin(c+dx)\,dx = \int \left(ax^2\sin(c+dx)+bx^4\sin(c+dx)\right)\,dx$$
$$= a\int x^2\sin(c+dx)\,dx + b\int x^4\sin(c+dx)\,dx$$
$$= -\frac{ax^2\cos(c+dx)}{d} - \frac{bx^4\cos(c+dx)}{d} + \frac{(2a)\int x\cos(c+dx)\,dx}{d} + \frac{(4b)\int x^3 \text{co}}{}$$
$$= -\frac{ax^2\cos(c+dx)}{d} - \frac{bx^4\cos(c+dx)}{d} + \frac{2ax\sin(c+dx)}{d^2} + \frac{4bx^3\sin(c+dx)}{d^2} -$$
$$= \frac{2a\cos(c+dx)}{d^3} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{ax^2\cos(c+dx)}{d} - \frac{bx^4\cos(c+dx)}{d} +$$
$$= \frac{2a\cos(c+dx)}{d^3} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{ax^2\cos(c+dx)}{d} - \frac{bx^4\cos(c+dx)}{d} -$$
$$= -\frac{24b\cos(c+dx)}{d^5} + \frac{2a\cos(c+dx)}{d^3} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{ax^2\cos(c+dx)}{d} -$$

Mathematica [A]
time = 0.10, size = 75, normalized size = 0.68

$$\frac{-((ad^2(-2+d^2x^2)+b(24-12d^2x^2+d^4x^4))\cos(c+dx))+2dx(ad^2+2b(-6+d^2x^2))\sin(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (-((a*d^2*(-2 + d^2*x^2) + b*(24 - 12*d^2*x^2 + d^4*x^4))*Cos[c + d*x]) + 2
*d*x*(a*d^2 + 2*b*(-6 + d^2*x^2))*Sin[c + d*x])/d^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(111) = 222$.
time = 0.05, size = 302, normalized size = 2.72

method	result
risch	$-\frac{(b\,x^4 d^4+a\,d^4x^2-12d^2x^2b-2d^2a+24b)\cos(dx+c)}{d^5}+\frac{2x(2d^2x^2b+d^2a-12b)\sin(dx+c)}{d^4}$
norman	$\frac{\frac{4d^2a-48b}{d^5}+\frac{b\,x^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(d^2a-12b\right)x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}-\frac{b\,x^4}{d}-\frac{\left(d^2a-12b\right)x^2}{d^3}+\frac{8b\,x^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}+\frac{4\left(d^2a-12b\right)x\tan\left(\frac{d}{2}\right.}{d^4}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
meijerg	$\frac{16b\sqrt{\pi}\,\sin(c)\left(-\frac{x\left(d^2\right)^{\frac{5}{2}}\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{10\sqrt{\pi}\,d^4}+\frac{\left(d^2\right)^{\frac{5}{2}}\left(\frac{5}{8}d^4x^4-\frac{15}{2}d^2x^2+15\right)\sin(dx)}{10\sqrt{\pi}\,d^5}\right)}{d^4\sqrt{d^2}}+\frac{16b\sqrt{\pi}\,\cos(c)\left(\frac{3}{2\sqrt{\pi}}-\right.}{}$
derivativedivides	$\frac{-a\,c^2\cos(dx+c)-2ac(\sin(dx+c)-(dx+c)\cos(dx+c))+a\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)-\frac{b\,c^4}{}}{}$

default	$\frac{-a\,c^2\cos(dx+c)-2ac(\sin(dx+c)-(dx+c)\cos(dx+c))+a\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)-\frac{b\,c}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(-a*c^2*cos(d*x+c)-2*a*c*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+a*(-(d*x+c)^
2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-1/d^2*b*c^4*cos(d*x+c)-4/d^
2*b*c^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+6/d^2*b*c^2*(-(d*x+c)^2*cos(d*x+c)+
2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-4/d^2*b*c*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+
c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+1/d^2*b*(-(d*x+c)^4*cos(
d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)-24*(d*x
+c)*sin(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(111) = 222$.
time = 0.30, size = 258, normalized size = 2.32

$$-\frac{ac^2\cos(dx+c)+\frac{bc^4\cos(dx+c)}{d^2}-2\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)ac-\frac{4\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)bc^3}{d^2}+\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)a+\frac{6\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)bc^2}{d^2}-\frac{4\left(\left((dx+c)^3-6\,dx-6\,c\right)\cos(dx+c)-3\left((dx+c)^2-2\right)\sin(dx+c)\right)bc}{d^2}+\frac{\left(\left((dx+c)^4-12\,(dx+c)^2+24\right)\cos(dx+c)-4\left((dx+c)^3-6\,dx-6\,c\right)\sin(dx+c)\right)b}{d^2}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*c^2*cos(d*x + c) + b*c^4*cos(d*x + c)/d^2 - 2*((d*x + c)*cos(d*x + c) -
 sin(d*x + c))*a*c - 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^3/d^2 +
(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a + 6*(((d*x +
c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^2/d^2 - 4*(((d*x + c
)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b*c/d^2
 + (((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d
*x - 6*c)*sin(d*x + c))*b/d^2)/d^3
```

Fricas [A]
time = 0.42, size = 77, normalized size = 0.69

$$-\frac{(bd^4x^4 - 2\,ad^2 + (ad^4 - 12\,bd^2)x^2 + 24\,b)\cos(dx+c) - 2\,(2\,bd^3x^3 + (ad^3 - 12\,bd)x)\sin(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^4*x^4 - 2*a*d^2 + (a*d^4 - 12*b*d^2)*x^2 + 24*b)*cos(d*x + c) - 2*(2
*b*d^3*x^3 + (a*d^3 - 12*b*d)*x)*sin(d*x + c))/d^5
```

Sympy [A]
time = 0.32, size = 134, normalized size = 1.21

$$\begin{cases} -\frac{ax^2\cos(c+dx)}{d} + \frac{2ax\sin(c+dx)}{d^2} + \frac{2a\cos(c+dx)}{d^3} - \frac{bx^4\cos(c+dx)}{d} + \frac{4bx^3\sin(c+dx)}{d^2} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{24bx\sin(c+dx)}{d^4} - \frac{24b\cos(c+dx)}{d^5} & \text{for}\ d \neq 0 \\ \left(\frac{ax^3}{3} + \frac{bx^5}{5}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**2*cos(c + d*x)/d + 2*a*x*sin(c + d*x)/d**2 + 2*a*cos(c + d
*x)/d**3 - b*x**4*cos(c + d*x)/d + 4*b*x**3*sin(c + d*x)/d**2 + 12*b*x**2*c
os(c + d*x)/d**3 - 24*b*x*sin(c + d*x)/d**4 - 24*b*cos(c + d*x)/d**5, Ne(d,
 0)), ((a*x**3/3 + b*x**5/5)*sin(c), True))
```

Giac [A]
time = 2.89, size = 79, normalized size = 0.71

$$-\frac{(bd^4x^4 + ad^4x^2 - 12\,bd^2x^2 - 2\,ad^2 + 24\,b)\cos\left(dx + c\right)}{d^5} + \frac{2\,(2\,bd^3x^3 + ad^3x - 12\,bdx)\sin\left(dx + c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^4*x^4 + a*d^4*x^2 - 12*b*d^2*x^2 - 2*a*d^2 + 24*b)*cos(d*x + c)/d^5 +
 2*(2*b*d^3*x^3 + a*d^3*x - 12*b*d*x)*sin(d*x + c)/d^5
```

Mupad [B]
time = 4.73, size = 97, normalized size = 0.87

$$\frac{x^2\,\cos\left(c+d\,x\right)\,\left(12\,b-a\,d^2\right)}{d^3}-\frac{2\,\cos\left(c+d\,x\right)\,\left(12\,b-a\,d^2\right)}{d^5}-\frac{2\,x\,\sin\left(c+d\,x\right)\,\left(12\,b-a\,d^2\right)}{d^4}-\frac{b\,x^4\,\cos\left(c+d\,x\right)}{d}+\frac{4\,b\,x^3\,\sin\left(c+d\,x\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(c + d*x)*(a + b*x^2),x)
```

```
[Out] (x^2*cos(c + d*x)*(12*b - a*d^2))/d^3 - (2*cos(c + d*x)*(12*b - a*d^2))/d^5
 - (2*x*sin(c + d*x)*(12*b - a*d^2))/d^4 - (b*x^4*cos(c + d*x))/d + (4*b*x^
3*sin(c + d*x))/d^2
```

3.42 $\int x(a+bx^2)\sin(c+dx)\,dx$

Optimal. Leaf size=80

$$\frac{6bx\cos(c+dx)}{d^3}-\frac{ax\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{a\sin(c+dx)}{d^2}+\frac{3bx^2\sin(c+dx)}{d^2}$$

```
[Out] 6*b*x*cos(d*x+c)/d^3-a*x*cos(d*x+c)/d-b*x^3*cos(d*x+c)/d-6*b*sin(d*x+c)/d^4
+a*sin(d*x+c)/d^2+3*b*x^2*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3420, 3377, 2717}

$$\frac{a\sin(c+dx)}{d^2}-\frac{ax\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{6bx\cos(c+dx)}{d^3}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{bx^3\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (6*b*x*Cos[c + d*x])/d^3 - (a*x*Cos[c + d*x])/d - (b*x^3*Cos[c + d*x])/d -
(6*b*Sin[c + d*x])/d^4 + (a*Sin[c + d*x])/d^2 + (3*b*x^2*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x(a+bx^2)\sin(c+dx)\,dx &= \int \left(ax\sin(c+dx)+bx^3\sin(c+dx)\right)\,dx \\
&= a\int x\sin(c+dx)\,dx+b\int x^3\sin(c+dx)\,dx \\
&= -\frac{ax\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}+\frac{a\int\cos(c+dx)\,dx}{d}+\frac{(3b)\int x^2\cos(c+ \\
&= -\frac{ax\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{(6b \\
&= \frac{6bx\cos(c+dx)}{d^3}-\frac{ax\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}+\frac{3bx^2 s \\
&= \frac{6bx\cos(c+dx)}{d^3}-\frac{ax\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{a\sin
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 57, normalized size = 0.71

$$\frac{-dx(ad^2+b(-6+d^2x^2))\cos(c+dx)+(ad^2+3b(-2+d^2x^2))\sin(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (-(d*x*(a*d^2 + b*(-6 + d^2*x^2))*Cos[c + d*x]) + (a*d^2 + 3*b*(-2 + d^2*x^
2))*Sin[c + d*x])/d^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(80) = 160$.
time = 0.04, size = 181, normalized size = 2.26

method	result
risch	$-\frac{x(d^2x^2b+d^2a-6b)\cos(dx+c)}{d^3}+\frac{(3d^2x^2b+d^2a-6b)\sin(dx+c)}{d^4}$
norman	$\frac{\frac{bx^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(d^2a-6b)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}-\frac{bx^3}{d}-\frac{(d^2a-6b)x}{d^3}+\frac{2(d^2a-6b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{6bx^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
meijerg	$\frac{8b\sqrt{\pi}\,\sin(c)\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3d^2x^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}}-\frac{dx\left(-\frac{d^2x^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4}+\frac{8b\sqrt{\pi}\,\cos(c)\left(\frac{xd\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{20\sqrt{\pi}}-\right.}{d^4}$
derivativedivides	$\frac{ac\cos(dx+c)+a(\sin(dx+c)-(dx+c)\cos(dx+c))+\frac{bc^3\cos(dx+c)}{d^2}+\frac{3bc^2(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^2}-\frac{3bc\left(-(dx+c)^2\cos(dx+\right.}{}}{d^2}$
default	$\frac{ac\cos(dx+c)+a(\sin(dx+c)-(dx+c)\cos(dx+c))+\frac{bc^3\cos(dx+c)}{d^2}+\frac{3bc^2(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^2}-\frac{3bc\left(-(dx+c)^2\cos(dx+\right.}{}}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(a*c*cos(d*x+c)+a*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+1/d^2*b*c^3*cos(d*x
+c)+3/d^2*b*c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-3/d^2*b*c*(-(d*x+c)^2*cos(d
*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+1/d^2*b*(-(d*x+c)^3*cos(d*x+c)+3*(
d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(80) = 160$.
time = 0.28, size = 165, normalized size = 2.06

$$\frac{ac\cos\left(dx+c\right)+\frac{bc^{3}\cos(dx+c)}{d^{2}}-\left(\left(dx+c\right)\cos\left(dx+c\right)-\sin\left(dx+c\right)\right)a-\frac{3\left(\left(dx+c\right)\cos(dx+c)-\sin(dx+c)\right)bc^{2}}{d^{2}}+\frac{3\left(\left(\left(dx+c\right)^{2}-2\right)\cos(dx+c)-2\left(dx+c\right)\sin(dx+c)\right)bc}{d^{2}}-\frac{\left(\left(\left(dx+c\right)^{3}-6\,dx-6\,c\right)\cos(dx+c)-3\left(\left(dx+c\right)^{2}-2\right)\sin(dx+c)\right)b}{d^{2}}}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c*cos(d*x + c) + b*c^3*cos(d*x + c)/d^2 - ((d*x + c)*cos(d*x + c) - sin(
d*x + c))*a - 3*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^2/d^2 + 3*(((d*
x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c/d^2 - (((d*x + c
)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b/d^2)/
d^2
```

Fricas [A]
time = 0.36, size = 60, normalized size = 0.75

$$-\frac{\left(bd^{3}x^{3}+\left(ad^{3}-6\,bd\right)x\right)\cos\left(dx+c\right)-\left(3\,bd^{2}x^{2}+ad^{2}-6\,b\right)\sin\left(dx+c\right)}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^3*x^3 + (a*d^3 - 6*b*d)*x)*cos(d*x + c) - (3*b*d^2*x^2 + a*d^2 - 6*b
)*sin(d*x + c))/d^4
```

Sympy [A]
time = 0.21, size = 99, normalized size = 1.24

$$\begin{cases} -\frac{ax\cos\left(c+dx\right)}{d}+\frac{a\sin\left(c+dx\right)}{d^{2}}-\frac{bx^{3}\cos\left(c+dx\right)}{d}+\frac{3bx^{2}\sin\left(c+dx\right)}{d^{2}}+\frac{6bx\cos\left(c+dx\right)}{d^{3}}-\frac{6b\sin\left(c+dx\right)}{d^{4}} & \text{for}\: d\neq 0 \\ \left(\frac{ax^{2}}{2}+\frac{bx^{4}}{4}\right)\sin\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x*cos(c + d*x)/d + a*sin(c + d*x)/d**2 - b*x**3*cos(c + d*x)/
d + 3*b*x**2*sin(c + d*x)/d**2 + 6*b*x*cos(c + d*x)/d**3 - 6*b*sin(c + d*x)
/d**4, Ne(d, 0)), ((a*x**2/2 + b*x**4/4)*sin(c), True))
```

Giac [A]
time = 3.40, size = 60, normalized size = 0.75

$$-\frac{(bd^3x^3 + ad^3x - 6\,bdx)\cos{(dx + c)}}{d^4} + \frac{(3\,bd^2x^2 + ad^2 - 6\,b)\sin{(dx + c)}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^3*x^3 + a*d^3*x - 6*b*d*x)*cos(d*x + c)/d^4 + (3*b*d^2*x^2 + a*d^2 -
6*b)*sin(d*x + c)/d^4
```

Mupad [B]
time = 0.14, size = 73, normalized size = 0.91

$$\frac{x\,\cos{(c + d\,x)}\,\left(6\,b - a\,d^2\right)}{d^3} - \frac{\sin{(c + d\,x)}\,\left(6\,b - a\,d^2\right)}{d^4} - \frac{b\,x^3\,\cos{(c + d\,x)}}{d} + \frac{3\,b\,x^2\,\sin{(c + d\,x)}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(c + d*x)*(a + b*x^2),x)
```

```
[Out] (x*cos(c + d*x)*(6*b - a*d^2))/d^3 - (sin(c + d*x)*(6*b - a*d^2))/d^4 - (b*
x^3*cos(c + d*x))/d + (3*b*x^2*sin(c + d*x))/d^2
```

3.43 $\int (a+bx^2)\sin(c+dx)\,dx$

Optimal. Leaf size=53

$$\frac{2b\cos(c+dx)}{d^3}-\frac{a\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{2bx\sin(c+dx)}{d^2}$$

```
[Out] 2*b*cos(d*x+c)/d^3-a*cos(d*x+c)/d-b*x^2*cos(d*x+c)/d+2*b*x*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3410, 2718, 3377}

$$-\frac{a\cos(c+dx)}{d}+\frac{2b\cos(c+dx)}{d^3}+\frac{2bx\sin(c+dx)}{d^2}-\frac{bx^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (2*b*Cos[c + d*x])/d^3 - (a*Cos[c + d*x])/d - (b*x^2*Cos[c + d*x])/d + (2*b
*x*Sin[c + d*x])/d^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3410

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+bx^2)\sin(c+dx)\,dx &= \int \left(a\sin(c+dx)+bx^2\sin(c+dx)\right)\,dx \\
&= a\int \sin(c+dx)\,dx + b\int x^2\sin(c+dx)\,dx \\
&= -\frac{a\cos(c+dx)}{d} - \frac{bx^2\cos(c+dx)}{d} + \frac{(2b)\int x\cos(c+dx)\,dx}{d} \\
&= -\frac{a\cos(c+dx)}{d} - \frac{bx^2\cos(c+dx)}{d} + \frac{2bx\sin(c+dx)}{d^2} - \frac{(2b)\int \sin(c+dx)\,dx}{d^2} \\
&= \frac{2b\cos(c+dx)}{d^3} - \frac{a\cos(c+dx)}{d} - \frac{bx^2\cos(c+dx)}{d} + \frac{2bx\sin(c+dx)}{d^2}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 41, normalized size = 0.77

$$\frac{-((ad^2+b(-2+d^2x^2))\cos(c+dx))+2bdx\sin(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (-((a*d^2 + b*(-2 + d^2*x^2))*Cos[c + d*x]) + 2*b*d*x*Sin[c + d*x])/d^3
```

Maple [A]
time = 0.04, size = 99, normalized size = 1.87

method	result
risch	$-\frac{(d^2x^2b+d^2a-2b)\cos(dx+c)}{d^3}+\frac{2bx\sin(dx+c)}{d^2}$
norman	$\frac{\frac{b\,x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2d^2a-4b}{d^3}-\frac{b\,x^2}{d}+\frac{4bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
derivativedivides	$\frac{-a\cos(dx+c)-\frac{b\,c^2\cos(dx+c)}{d^2}-\frac{2bc(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^2}+\frac{b\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{d^2}}{d}$
default	$\frac{-a\cos(dx+c)-\frac{b\,c^2\cos(dx+c)}{d^2}-\frac{2bc(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^2}+\frac{b\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{d^2}}{d}$
meijerg	$\frac{4b\sqrt{\pi}\,\sin(c)\left(\frac{x\left(d^2\right)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\,d^2}-\frac{\left(d^2\right)^{\frac{3}{2}}\left(-\frac{3d^2x^2}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\,d^3}\right)}{d^2\sqrt{d^2}}+\frac{4b\sqrt{\pi}\,\cos(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{d^2x^2}{2}+1\right)\cos(dx)}{2\sqrt{\pi}}+\frac{dx\,\mathrm{s}}{2\sqrt{}}\right.}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

[Out] 1/d*(-a*cos(d*x+c)-1/d^2*b*c^2*cos(d*x+c)-2/d^2*b*c*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+1/d^2*b*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c)))

Maxima [A]
time = 0.29, size = 91, normalized size = 1.72

$$-\frac{a\cos\left(dx+c\right)+\frac{bc^2\cos(dx+c)}{d^2}-\frac{2\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)bc}{d^2}+\frac{\left(\left((dx+c)^2-2\right)\cos(dx+c)-2\,(dx+c)\sin(dx+c)\right)b}{d^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*sin(d*x+c),x, algorithm="maxima")

[Out] -(a*cos(d*x + c) + b*c^2*cos(d*x + c)/d^2 - 2*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c/d^2 + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b/d^2)/d

Fricas [A]
time = 0.35, size = 41, normalized size = 0.77

$$\frac{2\,bdx\sin\left(dx+c\right)-\left(bd^2x^2+ad^2-2\,b\right)\cos\left(dx+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*sin(d*x+c),x, algorithm="fricas")

[Out] (2*b*d*x*sin(d*x + c) - (b*d^2*x^2 + a*d^2 - 2*b)*cos(d*x + c))/d^3

Sympy [A]
time = 0.14, size = 65, normalized size = 1.23

$$\begin{cases} -\frac{a\cos\left(c+dx\right)}{d}-\frac{bx^2\cos\left(c+dx\right)}{d}+\frac{2bx\sin\left(c+dx\right)}{d^2}+\frac{2b\cos\left(c+dx\right)}{d^3} & \text{for } d\neq 0 \\ \left(ax+\frac{bx^3}{3}\right)\sin\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)*sin(d*x+c),x)

[Out] Piecewise((-a*cos(c + d*x)/d - b*x**2*cos(c + d*x)/d + 2*b*x*sin(c + d*x)/d**2 + 2*b*cos(c + d*x)/d**3, Ne(d, 0)), ((a*x + b*x**3/3)*sin(c), True))

Giac [A]
time = 5.31, size = 42, normalized size = 0.79

$$\frac{2\,bx\sin\left(dx+c\right)}{d^2}-\frac{\left(bd^2x^2+ad^2-2\,b\right)\cos\left(dx+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] 2*b*x*sin(d*x + c)/d^2 - (b*d^2*x^2 + a*d^2 - 2*b)*cos(d*x + c)/d^3
```

Mupad [B]
time = 4.69, size = 49, normalized size = 0.92

$$\frac{\cos\left(c+d\,x\right)\,\left(2\,b-a\,d^2\right)}{d^3}+\frac{2\,b\,x\,\sin\left(c+d\,x\right)}{d^2}-\frac{b\,x^2\,\cos\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*x^2),x)
```

```
[Out] (cos(c + d*x)*(2*b - a*d^2))/d^3 + (2*b*x*sin(c + d*x))/d^2 - (b*x^2*cos(c
+ d*x))/d
```

3.44 $\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx$

Optimal. Leaf size=41

$$-\frac{bx\cos(c+dx)}{d}+a\text{Ci}(dx)\sin(c)+\frac{b\sin(c+dx)}{d^2}+a\cos(c)\text{Si}(dx)$$

```
[Out] -b*x*cos(d*x+c)/d+a*cos(c)*Si(d*x)+a*Ci(d*x)*sin(c)+b*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3420, 3384, 3380, 3383, 3377, 2717}

$$a\sin(c)\text{CosIntegral}(dx)+a\cos(c)\text{Si}(dx)+\frac{b\sin(c+dx)}{d^2}-\frac{bx\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sin[c + d*x])/x,x]
```

```
[Out] -((b*x*Cos[c + d*x])/d) + a*CosIntegral[d*x]*Sin[c] + (b*Sin[c + d*x])/d^2
+ a*Cos[c]*SinIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx &= \int \left(\frac{a\sin(c+dx)}{x}+bx\sin(c+dx)\right)dx\\
&= a\int \frac{\sin(c+dx)}{x}\,dx+b\int x\sin(c+dx)\,dx\\
&= -\frac{bx\cos(c+dx)}{d}+\frac{b\int\cos(c+dx)\,dx}{d}+(a\cos(c))\int\frac{\sin(dx)}{x}\,dx+(a\sin(c))\int\\
&= -\frac{bx\cos(c+dx)}{d}+a\text{Ci}(dx)\sin(c)+\frac{b\sin(c+dx)}{d^2}+a\cos(c)\text{Si}(dx)
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 54, normalized size = 1.32

$$-\frac{b\cos(dx)(dx\cos(c)-\sin(c))}{d^2}+a\text{Ci}(dx)\sin(c)+\frac{b(\cos(c)+dx\sin(c))\sin(dx)}{d^2}+a\cos(c)\text{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x,x]
[Out] -((b*Cos[d*x]*(d*x*Cos[c] - Sin[c]))/d^2) + a*CosIntegral[d*x]*Sin[c] + (b*
(Cos[c] + d*x*Sin[c])*Sin[d*x])/d^2 + a*Cos[c]*SinIntegral[d*x]
```

Maple [A]
time = 0.05, size = 60, normalized size = 1.46

method	result
derivativedivides	$a(\text{sinIntegral}(dx)\cos(c)+\text{cosineIntegral}(dx)\sin(c))+\frac{2bc\cos(dx+c)}{d^2}+\frac{(c+1)b(\sin(dx+c)-(dx-}{d^2}$
default	$a(\text{sinIntegral}(dx)\cos(c)+\text{cosineIntegral}(dx)\sin(c))+\frac{2bc\cos(dx+c)}{d^2}+\frac{(c+1)b(\sin(dx+c)-(dx-}{d^2}$

meijerg	$\frac{2b\sqrt{\pi}\,\sin(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b\sqrt{\pi}\,\cos(c)\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{a\sqrt{\pi}\,\sin(c)\left(\frac{2\gamma+2}{}\right.}{}$
risch	$-\frac{e^{-ic}\pi\operatorname{csgn}(dx)a}{2}+e^{-ic}\operatorname{sinIntegral}(dx)\,a-\frac{i\operatorname{expIntegral}(1,-idx)e^{-ic}a}{2}+\frac{ia\,e^{ic}\operatorname{expIntegral}(1,-idx)}{2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+2/d^2*b*c*cos(d*x+c)+(c+1)/d^2*b*(sin(d*x
+c)-(d*x+c)*cos(d*x+c))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.50, size = 66, normalized size = 1.61

$$-\frac{2\,bdx\cos\left(dx+c\right)-\left(a(-i\operatorname{Ei}(i\,dx)+i\operatorname{Ei}(-i\,dx))\cos\left(c\right)+a(\operatorname{Ei}(i\,dx)+\operatorname{Ei}(-i\,dx))\sin\left(c\right)\right)d^2-2\,b\sin\left(dx+c\right)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*x*cos(d*x + c) - (a*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a*(E
i(I*d*x) + Ei(-I*d*x))*sin(c))*d^2 - 2*b*sin(d*x + c))/d^2
```

Fricas [A]
time = 0.40, size = 61, normalized size = 1.49

$$\frac{2\,ad^2\cos\left(c\right)\operatorname{Si}\left(dx\right)-2\,bdx\cos\left(dx+c\right)+2\,b\sin\left(dx+c\right)+\left(ad^2\operatorname{Ci}\left(dx\right)+ad^2\operatorname{Ci}\left(-dx\right)\right)\sin\left(c\right)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d^2*cos(c)*sin_integral(d*x) - 2*b*d*x*cos(d*x + c) + 2*b*sin(d*x
+ c) + (a*d^2*cos_integral(d*x) + a*d^2*cos_integral(-d*x))*sin(c))/d^2
```

Sympy [A]
time = 2.60, size = 65, normalized size = 1.59

$$a\sin\left(c\right)\operatorname{Ci}\left(dx\right)+a\cos\left(c\right)\operatorname{Si}\left(dx\right)+bx\left(\begin{cases}x\sin\left(c\right) & \text{for}\: d=0\\-\frac{\cos\left(c+dx\right)}{d} & \text{otherwise}\end{cases}\right)-b\left(\begin{cases}\frac{x^2\sin\left(c\right)}{2} & \text{for}\: d=0\\-\frac{\begin{cases}\frac{\sin\left(c+dx\right)}{d} & \text{for}\: d\neq 0\\x\cos\left(c\right) & \text{otherwise}\end{cases}}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x,x)
```

```
[Out] a*sin(c)*Ci(d*x) + a*cos(c)*Si(d*x) + b*x*Piecewise((x*sin(c), Eq(d, 0)), (
-cos(c + d*x)/d, True)) - b*Piecewise((x**2*sin(c)/2, Eq(d, 0)), (-Piecewis
e((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))/d, True))
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.30, size = 432, normalized size = 10.54

$$-\frac{ad^2\Im(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2 - ad^2\Im(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2 + 2ad^2\,\mathrm{Si}(dx)\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2 - 2ad^2\Re(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right) - 2ad^2\Re(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right) + 2bdx\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2 - ad^2\Im(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}dx\right)^2 + ad^2\Im(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}dx\right)^2 - 2ad^2\,\mathrm{Si}(dx)\tan\left(\frac{1}{2}dx\right)^2 + ad^2\Im(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}c\right)^2 - ad^2\Im(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}c\right)^2 + 2ad^2\,\mathrm{Si}(dx)\tan\left(\frac{1}{2}c\right)^2 - 2bdx\tan\left(\frac{1}{2}dx\right)^2 - 2ad^2\Re(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}c\right) - 2ad^2\Re(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}c\right) - 8bdx\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right) - 2bdx\tan\left(\frac{1}{2}c\right)^2 - ad^2\Im(\mathrm{Ci}(dx)) + ad^2\Im(\mathrm{Ci}(-dx)) - 2ad^2\,\mathrm{Si}(dx) + 4b\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right) + 4b\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)^2 + 2bdx - 4b\tan\left(\frac{1}{2}dx\right) - 4b\tan\left(\frac{1}{2}c\right)}{2\left(d^2\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2 + d^2\tan\left(\frac{1}{2}dx\right)^2 + d^2\tan\left(\frac{1}{2}c\right)^2 + d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] -1/2*(a*d^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^
2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^2*sin_i
ntegral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*real_part(cos_integral(d
*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^2*real_part(cos_integral(-d*x))*tan(
1/2*d*x)^2*tan(1/2*c) + 2*b*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^2*imag_pa
rt(cos_integral(d*x))*tan(1/2*d*x)^2 + a*d^2*imag_part(cos_integral(-d*x))*
tan(1/2*d*x)^2 - 2*a*d^2*sin_integral(d*x)*tan(1/2*d*x)^2 + a*d^2*imag_part
(cos_integral(d*x))*tan(1/2*c)^2 - a*d^2*imag_part(cos_integral(-d*x))*tan(
1/2*c)^2 + 2*a*d^2*sin_integral(d*x)*tan(1/2*c)^2 - 2*b*d*x*tan(1/2*d*x)^2
- 2*a*d^2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^2*real_part(cos_i
ntegral(-d*x))*tan(1/2*c) - 8*b*d*x*tan(1/2*d*x)*tan(1/2*c) - 2*b*d*x*tan(1
/2*c)^2 - a*d^2*imag_part(cos_integral(d*x)) + a*d^2*imag_part(cos_integral
(-d*x)) - 2*a*d^2*sin_integral(d*x) + 4*b*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*t
an(1/2*d*x)*tan(1/2*c)^2 + 2*b*d*x - 4*b*tan(1/2*d*x) - 4*b*tan(1/2*c))/(d^
2*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^2*tan(1/2*d*x)^2 + d^2*tan(1/2*c)^2 + d^2
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$a\,\mathrm{cosint}(d\,x)\,\sin(c) + a\,\mathrm{sinint}(d\,x)\,\cos(c) + \frac{b\,(\sin(c+d\,x) - d\,x\,\cos(c+d\,x))}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^2))/x,x)
```

```
[Out] a*cosint(d*x)*sin(c) + a*sinint(d*x)*cos(c) + (b*(sin(c + d*x) - d*x*cos(c
+ d*x)))/d^2
```

3.45 $\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx$

Optimal. Leaf size=44

$$-\frac{b\cos(c+dx)}{d}+ad\cos(c)\text{Ci}(dx)-\frac{a\sin(c+dx)}{x}-ad\sin(c)\text{Si}(dx)$$

[Out] a*d*Ci(d*x)*cos(c)-b*cos(d*x+c)/d-a*d*Si(d*x)*sin(c)-a*sin(d*x+c)/x

Rubi [A]
time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3420, 2718, 3378, 3384, 3380, 3383}

$$ad\cos(c)\text{CosIntegral}(dx)-ad\sin(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{x}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x^2)*Sin[c + d*x])/x^2,x]

[Out] -((b*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] - (a*Sin[c + d*x])/x - a*d*Sin[c]*SinIntegral[d*x]

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx &= \int \left(b\sin(c+dx)+\frac{a\sin(c+dx)}{x^2}\right)dx\\
&= a\int \frac{\sin(c+dx)}{x^2}\,dx + b\int \sin(c+dx)\,dx\\
&= -\frac{b\cos(c+dx)}{d}-\frac{a\sin(c+dx)}{x}+(ad)\int \frac{\cos(c+dx)}{x}\,dx\\
&= -\frac{b\cos(c+dx)}{d}-\frac{a\sin(c+dx)}{x}+(ad\cos(c))\int \frac{\cos(dx)}{x}\,dx-(ad\sin(c))\int \frac{\sin
\end{aligned}
$$

$$
= -\frac{b\cos(c+dx)}{d}+ad\cos(c)\mathrm{Ci}(dx)-\frac{a\sin(c+dx)}{x}-ad\sin(c)\mathrm{Si}(dx)
$$

Mathematica [A]
time = 0.07, size = 44, normalized size = 1.00

$$-\frac{b\cos(c+dx)}{d}+ad\cos(c)\mathrm{Ci}(dx)-\frac{a\sin(c+dx)}{x}-ad\sin(c)\mathrm{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x^2,x]
[Out] -((b*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] - (a*Sin[c + d*x])/x -
a*d*Sin[c]*SinIntegral[d*x]
```

Maple [A]
time = 0.07, size = 48, normalized size = 1.09

method	result
derivativedivides	$d\left(a\left(-\frac{\sin(dx+c)}{dx}-\mathrm{sinIntegral}\left(dx\right)\sin\left(c\right)+\mathrm{cosineIntegral}\left(dx\right)\cos\left(c\right)\right)-\frac{b\cos(dx+c)}{d^2}\right)$

default	$d\left(a\left(-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}\left(dx\right)\sin\left(c\right)+\text{cosineIntegral}\left(dx\right)\cos\left(c\right)\right)-\frac{b\cos(dx+c)}{d^2}\right)$
risch	$-\frac{d\cos(c)a\,\text{expIntegral}(1,idx)}{2}-\frac{d\cos(c)a\,\text{expIntegral}(1,-idx)}{2}+\frac{id\sin(c)a\,\text{expIntegral}(1,idx)}{2}-\frac{id\sin(c)a\,\text{expInt}}{2}$
meijerg	$\frac{b\sin(c)\sin(dx)}{d}+\frac{b\sqrt{\pi}\cos(c)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}+\frac{a\sqrt{\pi}\sin(c)d^2\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right)}{x\left(d^2\right)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\,\text{sinIntegral}\left(x\sqrt{d^2}\right)}{\sqrt{\pi}}\right.}{4\sqrt{d^2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)

[Out] d*(a*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))-1/d^2*b*cos(d*x+c))

Maxima [C] Result contains complex when optimal does not.
time = 0.46, size = 937, normalized size = 21.30

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)*sin(d*x+c)/x^2,x, algorithm="maxima")

[Out] -1/4*(((I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c)^3
+ (I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c)*sin(c)^
2 + (exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*sin(c)^3 + (I*ex
p_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c) + ((exp_integr
al_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c)^2 + exp_integral_e(2, I*
d*x) + exp_integral_e(2, -I*d*x))*sin(c))*b*c^2/((d*x + c)*(cos(c)^2 + sin(
c)^2)*d^2 - (c*cos(c)^2 + c*sin(c)^2)*d^2) - ((I*exp_integral_e(2, I*d*x) -
 I*exp_integral_e(2, -I*d*x))*cos(c)^3 + (I*exp_integral_e(2, I*d*x) - I*ex
p_integral_e(2, -I*d*x))*cos(c)*sin(c)^2 + (exp_integral_e(2, I*d*x) + exp_
integral_e(2, -I*d*x))*sin(c)^3 + (I*exp_integral_e(2, I*d*x) - I*exp_integ
ral_e(2, -I*d*x))*cos(c) + ((exp_integral_e(2, I*d*x) + exp_integral_e(2, -
I*d*x))*cos(c)^2 + exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*si
n(c))*a/(c*cos(c)^2 + c*sin(c)^2 - (d*x + c)*(cos(c)^2 + sin(c)^2)) + 2*(((
b*cos(c)^2 + b*sin(c)^2)*(d*x + c)^2 - 2*(b*c*cos(c)^2 + b*c*sin(c)^2)*(d*x
 + c))*cos(d*x + c)^3 + (b*c^2*(exp_integral_e(3, I*d*x) + exp_integral_e(3
, -I*d*x))*cos(c)^3 + b*c^2*(exp_integral_e(3, I*d*x) + exp_integral_e(3, -
I*d*x))*cos(c)*sin(c)^2 + b*c^2*(-I*exp_integral_e(3, I*d*x) + I*exp_integr
al_e(3, -I*d*x))*sin(c)^3 + b*c^2*(exp_integral_e(3, I*d*x) + exp_integral_
e(3, -I*d*x))*cos(c) + (b*c^2*(-I*exp_integral_e(3, I*d*x) + I*exp_integral
_e(3, -I*d*x))*cos(c)^2 + b*c^2*(-I*exp_integral_e(3, I*d*x) + I*exp_integr
al_e(3, -I*d*x)))*sin(c))*cos(d*x + c)^2 + (b*c^2*(exp_integral_e(3, I*d*x)
 + exp_integral_e(3, -I*d*x))*cos(c)^3 + b*c^2*(exp_integral_e(3, I*d*x) +
exp_integral_e(3, -I*d*x))*cos(c)*sin(c)^2 + b*c^2*(-I*exp_integral_e(3, I*

```
d*x) + I*exp_integral_e(3, -I*d*x))*sin(c)^3 + b*c^2*(exp_integral_e(3, I*d
*x) + exp_integral_e(3, -I*d*x))*cos(c) + ((b*cos(c)^2 + b*sin(c)^2)*(d*x +
 c)^2 - 2*(b*c*cos(c)^2 + b*c*sin(c)^2)*(d*x + c))*cos(d*x + c) + (b*c^2*(-
I*exp_integral_e(3, I*d*x) + I*exp_integral_e(3, -I*d*x))*cos(c)^2 + b*c^2*
(-I*exp_integral_e(3, I*d*x) + I*exp_integral_e(3, -I*d*x)))*sin(c))*sin(d*
x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*(d*x + c)^2 - 2*(b*c*cos(c)^2 + b*c*s
in(c)^2)*(d*x + c))*cos(d*x + c))/((((d*x + c)^2*(cos(c)^2 + sin(c)^2)*d^2 -
 2*(c*cos(c)^2 + c*sin(c)^2)*(d*x + c)*d^2 + (c^2*cos(c)^2 + c^2*sin(c)^2)*
d^2)*cos(d*x + c)^2 + ((d*x + c)^2*(cos(c)^2 + sin(c)^2)*d^2 - 2*(c*cos(c)^
2 + c*sin(c)^2)*(d*x + c)*d^2 + (c^2*cos(c)^2 + c^2*sin(c)^2)*d^2)*sin(d*x
+ c)^2))*d
```

Fricas [A]
time = 0.39, size = 68, normalized size = 1.55

$$-\frac{2\,ad^2x\sin\left(c\right)\operatorname{Si}\left(dx\right)+2\,bx\cos\left(dx+c\right)+2\,ad\sin\left(dx+c\right)-\left(ad^2x\operatorname{Ci}\left(dx\right)+ad^2x\operatorname{Ci}\left(-dx\right)\right)\cos\left(c\right)}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d^2*x*sin(c)*sin_integral(d*x) + 2*b*x*cos(d*x + c) + 2*a*d*sin(d
*x + c) - (a*d^2*x*cos_integral(d*x) + a*d^2*x*cos_integral(-d*x))*cos(c))/
(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^2\right)\sin\left(c+dx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**2)*sin(c + d*x)/x**2, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.03, size = 411, normalized size = 9.34

$$-\frac{ad^2x\Re(\operatorname{Ci}(dx))\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2+ad^2x\Re(\operatorname{Ci}(-dx))\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2+2\,ad^2x\Im(\operatorname{Ci}(dx))\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)-2\,ad^2x\Im(\operatorname{Ci}(-dx))\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)+4\,ad^2x\operatorname{Si}(dx)\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)-ad^2x\Re(\operatorname{Ci}(dx))\tan\left(\frac{1}{2}dx\right)^2-ad^2x\Re(\operatorname{Ci}(-dx))\tan\left(\frac{1}{2}dx\right)^2+ad^2x\Re(\operatorname{Ci}(dx))\tan\left(\frac{1}{2}c\right)^2+ad^2x\Re(\operatorname{Ci}(-dx))\tan\left(\frac{1}{2}c\right)^2+2\,ad^2x\Im(\operatorname{Ci}(dx))\tan\left(\frac{1}{2}c\right)-2\,ad^2x\Im(\operatorname{Ci}(-dx))\tan\left(\frac{1}{2}c\right)+4\,ad^2x\operatorname{Si}(dx)\tan\left(\frac{1}{2}c\right)+2\,bx\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2-ad^2x\Re(\operatorname{Ci}(dx))-ad^2x\Re(\operatorname{Ci}(-dx))-4\,ad\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)-4\,ad\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)^2-2\,bx\tan\left(\frac{1}{2}dx\right)^2-8\,bx\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)-2\,bx\tan\left(\frac{1}{2}c\right)^2+4\,ad\tan\left(\frac{1}{2}dx\right)+4\,ad\tan\left(\frac{1}{2}c\right)+2\,bx}{2\left(dx\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2+dx\tan\left(\frac{1}{2}dx\right)^2+dx\tan\left(\frac{1}{2}c\right)^2+dx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d^2*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*
d^2*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^2*x
*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^2*x*imag_pa
```

```
rt(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^2*x*sin_integral(d
*x)*tan(1/2*d*x)^2*tan(1/2*c) - a*d^2*x*real_part(cos_integral(d*x))*tan(1/
2*d*x)^2 - a*d^2*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + a*d^2*x*r
eal_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d^2*x*real_part(cos_integral(-
d*x))*tan(1/2*c)^2 + 2*a*d^2*x*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*
a*d^2*x*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a*d^2*x*sin_integral(d
*x)*tan(1/2*c) + 2*b*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^2*x*real_part(cos_
integral(d*x)) - a*d^2*x*real_part(cos_integral(-d*x)) - 4*a*d*tan(1/2*d*x)
^2*tan(1/2*c) - 4*a*d*tan(1/2*d*x)*tan(1/2*c)^2 - 2*b*x*tan(1/2*d*x)^2 - 8*
b*x*tan(1/2*d*x)*tan(1/2*c) - 2*b*x*tan(1/2*c)^2 + 4*a*d*tan(1/2*d*x) + 4*a
*d*tan(1/2*c) + 2*b*x)/(d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*x*tan(1/2*d*x)^
2 + d*x*tan(1/2*c)^2 + d*x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin (c+d\,x)\,\left(b\,x^2+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^2))/x^2,x)
[Out] int((sin(c + d*x)*(a + b*x^2))/x^2, x)
```

3.46 $\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx$

Optimal. Leaf size=74

$$-\frac{ad\cos(c+dx)}{2x}+b\text{Ci}(dx)\sin(c)-\frac{1}{2}ad^2\text{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{2x^2}+b\cos(c)\text{Si}(dx)-\frac{1}{2}ad^2\cos(c)\text{Si}(dx)$$

```
[Out] -1/2*a*d*cos(d*x+c)/x+b*cos(c)*Si(d*x)-1/2*a*d^2*cos(c)*Si(d*x)+b*Ci(d*x)*s
in(c)-1/2*a*d^2*Ci(d*x)*sin(c)-1/2*a*sin(d*x+c)/x^2
```

Rubi [A]
time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3420, 3378, 3384, 3380, 3383}

$$-\frac{1}{2}ad^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}ad^2\cos(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{2x^2}-\frac{ad\cos(c+dx)}{2x}+b\sin(c)\text{CosIntegral}(dx)+b\cos(c)\text{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sin[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a*d*Cos[c + d*x])/x + b*CosIntegral[d*x]*Sin[c] - (a*d^2*CosIntegral[
d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) + b*Cos[c]*SinIntegral[d*x] - (a*
d^2*Cos[c]*SinIntegral[d*x])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^3}+\frac{b\sin(c+dx)}{x}\right)dx\\ &= a\int \frac{\sin(c+dx)}{x^3}\,dx+b\int \frac{\sin(c+dx)}{x}\,dx\\ &= -\frac{a\sin(c+dx)}{2x^2}+\frac{1}{2}(ad)\int \frac{\cos(c+dx)}{x^2}\,dx+(b\cos(c))\int \frac{\sin(dx)}{x}\,dx+(b\sin(\\ &= -\frac{ad\cos(c+dx)}{2x}+b\text{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{2x^2}+b\cos(c)\text{Si}(dx)-\frac{1}{2}(ad^2)\\ &= -\frac{ad\cos(c+dx)}{2x}+b\text{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{2x^2}+b\cos(c)\text{Si}(dx)-\frac{1}{2}(ad^2\text{ c}\\ &= -\frac{ad\cos(c+dx)}{2x}+b\text{Ci}(dx)\sin(c)-\frac{1}{2}ad^2\text{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{2x^2}+b\text{ cc}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 82, normalized size = 1.11

$$b\text{Ci}(dx)\sin(c)-\frac{a\cos(dx)(dx\cos(c)+\sin(c))}{2x^2}+\frac{a(-\cos(c)+dx\sin(c))\sin(dx)}{2x^2}+b\cos(c)\text{Si}(dx)-\frac{1}{2}ad^2(\text{Ci}(dx)\sin(c)+\cos(c)\text{Si}(dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x^3,x]
```

```
[Out] b*CosIntegral[d*x]*Sin[c] - (a*Cos[d*x]*(d*x*Cos[c] + Sin[c]))/(2*x^2) + (a
*(-Cos[c] + d*x*Sin[c])*Sin[d*x])/(2*x^2) + b*Cos[c]*SinIntegral[d*x] - (a*
d^2*(CosIntegral[d*x]*Sin[c] + Cos[c]*SinIntegral[d*x]))/2
```

Maple [A]
time = 0.08, size = 73, normalized size = 0.99

method	result

derivativedivides	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\text{sinIntegral}(dx)\cos(c)}{2}-\frac{\text{cosineIntegral}(dx)\sin(c)}{2}\right)+\frac{b(\text{sinIntegral}(dx)\cos(c)}{}\right.$
default	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\text{sinIntegral}(dx)\cos(c)}{2}-\frac{\text{cosineIntegral}(dx)\sin(c)}{2}\right)+\frac{b(\text{sinIntegral}(dx)\cos(c)}{}\right.$
risch	$-\frac{i\cos(c)\,\text{expIntegral}(1,-idx)a\,d^2}{4}+\frac{i\cos(c)\,\text{expIntegral}(1,idx)a\,d^2}{4}+\frac{i\cos(c)\,\text{expIntegral}(1,-idx)b}{2}-\frac{i\cos(c)\,\text{expIn}}{}$
meijerg	$\frac{b\sqrt{\pi}\,\sin(c)\left(\frac{2\gamma+2\ln(x)+\ln(d^2)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\,\text{cosineIntegral}(dx)}{\sqrt{\pi}}\right)}{2}+b\cos(c)\,\text{sinIntegral}(dx)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(a*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*C
i(d*x)*sin(c))+1/d^2*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.77, size = 123, normalized size = 1.66

$$-\frac{2\,bdx\cos(dx+c)+((a(-i\,\Gamma(-2,i\,dx)+i\,\Gamma(-2,-i\,dx))\cos(c)-a(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin(c))d^4-2\,(b(-i\,\Gamma(-2,i\,dx)+i\,\Gamma(-2,-i\,dx))\cos(c)-b(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin(c))d^2)x^2+2\,b\sin(dx+c)}{2\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*x*cos(d*x + c) + ((a*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x)
)*cos(c) - a*(gamma(-2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^4 - 2*(b*(-I*
gamma(-2, I*d*x) + I*gamma(-2, -I*d*x))*cos(c) - b*(gamma(-2, I*d*x) + gamm
a(-2, -I*d*x))*sin(c))*d^2)*x^2 + 2*b*sin(d*x + c))/(d^2*x^2)
```

Fricas [A]
time = 0.36, size = 85, normalized size = 1.15

$$-\frac{2\,(ad^2-2\,b)x^2\cos(c)\,\text{Si}(dx)+2\,adx\cos(dx+c)+2\,a\sin(dx+c)+((ad^2-2\,b)x^2\,\text{Ci}(dx)+(ad^2-2\,b)x^2\,\text{Ci}(-dx))\sin(c)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a*d^2 - 2*b)*x^2*cos(c)*sin_integral(d*x) + 2*a*d*x*cos(d*x + c) +
 2*a*sin(d*x + c) + ((a*d^2 - 2*b)*x^2*cos_integral(d*x) + (a*d^2 - 2*b)*x^
2*cos_integral(-d*x))*sin(c))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**2)*sin(c + d*x)/x**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.73, size = 766, normalized size = 10.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a
*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^
2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*x^2*real_part
(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^2*x^2*real_part(cos_i
ntegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a*d^2*x^2*imag_part(cos_integral
(d*x))*tan(1/2*d*x)^2 + a*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x
)^2 - 2*a*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + a*d^2*x^2*imag_part(co
s_integral(d*x))*tan(1/2*c)^2 - a*d^2*x^2*imag_part(cos_integral(-d*x))*tan
(1/2*c)^2 + 2*a*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 - 2*b*x^2*imag_part(
cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b*x^2*imag_part(cos_inte
gral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b*x^2*sin_integral(d*x)*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*c)
 - 2*a*d^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 4*b*x^2*real_part(
cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*x^2*real_part(cos_integr
al(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 a*d^2*x^2*imag_part(cos_integral(d*x)) + a*d^2*x^2*imag_part(cos_integral(
-d*x)) - 2*a*d^2*x^2*sin_integral(d*x) + 2*b*x^2*imag_part(cos_integral(d*x
))*tan(1/2*d*x)^2 - 2*b*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 +
4*b*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 2*b*x^2*imag_part(cos_integral(d
*x))*tan(1/2*c)^2 + 2*b*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*
b*x^2*sin_integral(d*x)*tan(1/2*c)^2 + 2*a*d*x*tan(1/2*d*x)^2 + 4*b*x^2*rea
l_part(cos_integral(d*x))*tan(1/2*c) + 4*b*x^2*real_part(cos_integral(-d*x)
)*tan(1/2*c) + 8*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 2*a*d*x*tan(1/2*c)^2 + 2*b
*x^2*imag_part(cos_integral(d*x)) - 2*b*x^2*imag_part(cos_integral(-d*x)) +
 4*b*x^2*sin_integral(d*x) + 4*a*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*tan(1/2*d*
x)*tan(1/2*c)^2 - 2*a*d*x - 4*a*tan(1/2*d*x) - 4*a*tan(1/2*c))/(x^2*tan(1/2
*d*x)^2*tan(1/2*c)^2 + x^2*tan(1/2*d*x)^2 + x^2*tan(1/2*c)^2 + x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^2+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^2))/x^3,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^2))/x^3, x)
```

3.47 $\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx$

Optimal. Leaf size=106

$$-\frac{ad\cos(c+dx)}{6x^2}+bd\cos(c)\text{Ci}(dx)-\frac{1}{6}ad^3\cos(c)\text{Ci}(dx)-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{x}+\frac{ad^2\sin(c+dx)}{6x}-bd\,\text{si}$$

```
[Out] b*d*Ci(d*x)*cos(c)-1/6*a*d^3*Ci(d*x)*cos(c)-1/6*a*d*cos(d*x+c)/x^2-b*d*Si(d
*x)*sin(c)+1/6*a*d^3*Si(d*x)*sin(c)-1/3*a*sin(d*x+c)/x^3-b*sin(d*x+c)/x+1/6
*a*d^2*sin(d*x+c)/x
```

Rubi [A]

time = 0.15, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3420, 3378, 3384, 3380, 3383}

$$-\frac{1}{6}ad^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}ad^3\sin(c)\text{Si}(dx)+\frac{ad^2\sin(c+dx)}{6x}-\frac{a\sin(c+dx)}{3x^3}-\frac{ad\cos(c+dx)}{6x^2}+bd\cos(c)\text{CosIntegral}(dx)-bd\sin(c)\text{Si}(dx)-\frac{b\sin(c+dx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sin[c + d*x])/x^4,x]
```

```
[Out] -1/6*(a*d*Cos[c + d*x])/x^2 + b*d*Cos[c]*CosIntegral[d*x] - (a*d^3*Cos[c]*C
osIntegral[d*x])/6 - (a*Sin[c + d*x])/(3*x^3) - (b*Sin[c + d*x])/x + (a*d^2
*Sin[c + d*x])/(6*x) - b*d*Sin[c]*SinIntegral[d*x] + (a*d^3*Sin[c]*SinInteg
ral[d*x])/6
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^4}+\frac{b\sin(c+dx)}{x^2}\right)dx\\ &= a\int \frac{\sin(c+dx)}{x^4}\,dx+b\int \frac{\sin(c+dx)}{x^2}\,dx\\ &= -\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{x}+\frac{1}{3}(ad)\int \frac{\cos(c+dx)}{x^3}\,dx+(bd)\int \frac{\cos(c+d}{x}\\ &= -\frac{ad\cos(c+dx)}{6x^2}-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{x}-\frac{1}{6}\left(ad^2\right)\int \frac{\sin(c+dx)}{x^2}\,dx-\\ &= -\frac{ad\cos(c+dx)}{6x^2}+bd\cos(c)\text{Ci}(dx)-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{x}+\frac{ad^2\sin(}{6}\\ &= -\frac{ad\cos(c+dx)}{6x^2}+bd\cos(c)\text{Ci}(dx)-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{x}+\frac{ad^2\sin(}{6}\\ &= -\frac{ad\cos(c+dx)}{6x^2}+bd\cos(c)\text{Ci}(dx)-\frac{1}{6}ad^3\cos(c)\text{Ci}(dx)-\frac{a\sin(c+dx)}{3x^3}-\frac{b\,\text{si}}{}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 95, normalized size = 0.90

$$\frac{-adx\cos(c+dx)+d(6b-ad^2)\,x^3\cos(c)\text{Ci}(dx)-2a\sin(c+dx)-6bx^2\sin(c+dx)+ad^2x^2\sin(c+dx)+d(-6b+ad^2)\,x^3\sin(c)\text{Si}(dx)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x^4,x]
```

```
[Out] (-(a*d*x*Cos[c + d*x]) + d*(6*b - a*d^2)*x^3*Cos[c]*CosIntegral[d*x] - 2*a*
Sin[c + d*x] - 6*b*x^2*Sin[c + d*x] + a*d^2*x^2*Sin[c + d*x] + d*(-6*b + a*
d^2)*x^3*Sin[c]*SinIntegral[d*x])/(6*x^3)
```

Maple [A]
time = 0.11, size = 102, normalized size = 0.96

method	result
derivativedivides	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\text{sinIntegral}(dx)\sin(c)}{6}-\frac{\text{cosineIntegral}(dx)\cos(c)}{6}\right)+\frac{b(-}{}\right.$
default	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\text{sinIntegral}(dx)\sin(c)}{6}-\frac{\text{cosineIntegral}(dx)\cos(c)}{6}\right)+\frac{b(-}{}\right.$
risch	$\frac{\text{expIntegral}(1,idx)\cos(c)a\,d^3}{12}+\frac{\text{expIntegral}(1,-idx)\cos(c)a\,d^3}{12}-\frac{\cos(c)\,\text{expIntegral}(1,idx)bd}{2}-\frac{\cos(c)\,\text{expIntegral}}{2}$
meijerg	$\frac{d^2b\sqrt{\pi}\,\sin(c)\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right)}{x\left(d^2\right)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\,\text{sinIntegral}\left(x\sqrt{d^2}\right)}{\sqrt{\pi}}\right)}{4\sqrt{d^2}}+\frac{db\sqrt{\pi}\,\cos(c)\left(\frac{4\gamma-4+4\ln(x)+4\ln(d)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(a*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+1
/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+1/d^2*b*(-sin(d*x+c)/d/x-Si(d*x)*sin(
c)+Ci(d*x)*cos(c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.95, size = 121, normalized size = 1.14

$$-\frac{((a(\Gamma(-3,idx)+\Gamma(-3,-idx))\cos(c)+a(-i\Gamma(-3,idx)+i\Gamma(-3,-idx))\sin(c))d^5-6(b(\Gamma(-3,idx)+\Gamma(-3,-idx))\cos(c)+b(-i\Gamma(-3,idx)+i\Gamma(-3,-idx))\sin(c))d^3)x^3+2bdx\cos(dx+c)+4b\sin(dx+c)}{2d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a*(-I*gamma(-3, I
*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^5 - 6*(b*(gamma(-3, I*d*x) + gamma(-
3, -I*d*x))*cos(c) + b*(-I*gamma(-3, I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*
d^3)*x^3 + 2*b*d*x*cos(d*x + c) + 4*b*sin(d*x + c))/(d^2*x^3)
```

Fricas [A]
time = 0.38, size = 105, normalized size = 0.99

$$\frac{2\,(ad^3-6\,bd)x^3\sin(c)\,\mathrm{Si}(dx)-2\,adx\cos(dx+c)-((ad^3-6\,bd)x^3\,\mathrm{Ci}(dx)+(ad^3-6\,bd)x^3\,\mathrm{Ci}(-dx))\cos(c)+2\,((ad^2-6\,b)x^2-2\,a)\sin(dx+c)}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(a*d^3 - 6*b*d)*x^3*sin(c)*sin_integral(d*x) - 2*a*d*x*cos(d*x + c)
 - ((a*d^3 - 6*b*d)*x^3*cos_integral(d*x) + (a*d^3 - 6*b*d)*x^3*cos_integra
l(-d*x))*cos(c) + 2*((a*d^2 - 6*b)*x^2 - 2*a)*sin(d*x + c))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)\sin{(c+dx)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**2)*sin(c + d*x)/x**4, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.39, size = 834, normalized size = 7.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d
^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^3
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x^3*sin_
integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a*d^3*x^3*real_part(cos_integral(
d*x))*tan(1/2*d*x)^2 - a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)
^2 + a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d^3*x^3*real_p
art(cos_integral(-d*x))*tan(1/2*c)^2 - 6*b*d*x^3*real_part(cos_integral(d*x
))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*b*d*x^3*real_part(cos_integral(-d*x))*ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/
2*c) - 2*a*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a*d^3*x^3*s
in_integral(d*x)*tan(1/2*c) - 12*b*d*x^3*imag_part(cos_integral(d*x))*tan(1
/2*d*x)^2*tan(1/2*c) + 12*b*d*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x
)^2*tan(1/2*c) - 24*b*d*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a
*d^3*x^3*real_part(cos_integral(d*x)) - a*d^3*x^3*real_part(cos_integral(-d
*x)) + 6*b*d*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 6*b*d*x^3*re
al_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 4*a*d^2*x^2*tan(1/2*d*x)^2*tan
(1/2*c) - 6*b*d*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 - 6*b*d*x^3*r
eal_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*a*d^2*x^2*tan(1/2*d*x)*tan(1/
2*c)^2 - 12*b*d*x^3*imag_part(cos_integral(d*x))*tan(1/2*c) + 12*b*d*x^3*im
ag_part(cos_integral(-d*x))*tan(1/2*c) - 24*b*d*x^3*sin_integral(d*x)*tan(1
/2*c) - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*b*d*x^3*real_part(cos_integ
ral(d*x)) + 6*b*d*x^3*real_part(cos_integral(-d*x)) + 4*a*d^2*x^2*tan(1/2*d
*x) + 4*a*d^2*x^2*tan(1/2*c) + 24*b*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 24*b*x^
2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a*d*x*tan(1/2*d*x)^2 + 8*a*d*x*tan(1/2*d*x)
*tan(1/2*c) + 2*a*d*x*tan(1/2*c)^2 - 24*b*x^2*tan(1/2*d*x) - 24*b*x^2*tan(1
```

```
/2*c) + 8*a*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a
*d*x - 8*a*tan(1/2*d*x) - 8*a*tan(1/2*c))/(x^3*tan(1/2*d*x)^2*tan(1/2*c)^2
+ x^3*tan(1/2*d*x)^2 + x^3*tan(1/2*c)^2 + x^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^2+a\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^2))/x^4,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^2))/x^4, x)
```

3.48 $\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx$

Optimal. Leaf size=149

$$-\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{2x}+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{2}bd^2\text{Ci}(dx)\sin(c)+\frac{1}{24}ad^4\text{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{4x^4}-$$

```
[Out] -1/12*a*d*cos(d*x+c)/x^3-1/2*b*d*cos(d*x+c)/x+1/24*a*d^3*cos(d*x+c)/x-1/2*b
*d^2*cos(c)*Si(d*x)+1/24*a*d^4*cos(c)*Si(d*x)-1/2*b*d^2*Ci(d*x)*sin(c)+1/24
*a*d^4*Ci(d*x)*sin(c)-1/4*a*sin(d*x+c)/x^4-1/2*b*sin(d*x+c)/x^2+1/24*a*d^2*
sin(d*x+c)/x^2
```

Rubi [A]
time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3420, 3378, 3384, 3380, 3383}

$$\frac{1}{24}ad^4\sin(c)\text{CosIntegral}(dx)+\frac{1}{24}ad^4\cos(c)\text{Si}(dx)+\frac{ad^3\cos(c+dx)}{24x}+\frac{ad^2\sin(c+dx)}{24x^2}-\frac{a\sin(c+dx)}{4x^4}-\frac{ad\cos(c+dx)}{12x^3}-\frac{1}{2}bd^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}bd^2\cos(c)\text{Si}(dx)-\frac{b\sin(c+dx)}{2x^2}-\frac{bd\cos(c+dx)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sin[c + d*x])/x^5,x]
```

```
[Out] -1/12*(a*d*Cos[c + d*x])/x^3 - (b*d*Cos[c + d*x])/(2*x) + (a*d^3*Cos[c + d*
x])/(24*x) - (b*d^2*CosIntegral[d*x]*Sin[c])/2 + (a*d^4*CosIntegral[d*x]*Si
n[c])/24 - (a*Sin[c + d*x])/(4*x^4) - (b*Sin[c + d*x])/(2*x^2) + (a*d^2*Sin
[c + d*x])/(24*x^2) - (b*d^2*Cos[c]*SinIntegral[d*x])/2 + (a*d^4*Cos[c]*Sin
Integral[d*x])/24
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^5}+\frac{b\sin(c+dx)}{x^3}\right)dx \\
&= a\int \frac{\sin(c+dx)}{x^5}\,dx+b\int \frac{\sin(c+dx)}{x^3}\,dx \\
&= -\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{2x^2}+\frac{1}{4}(ad)\int \frac{\cos(c+dx)}{x^4}\,dx+\frac{1}{2}(bd)\int \frac{\cos(c+\cdots}{x^2\cdots} \\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{2x}-\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{2x^2}-\frac{1}{12}(ad^2\cdots \\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{2x}-\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{2x^2}+\frac{ad^2\sin(\cdots}{24\cdots} \\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{2x}+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{2}bd^2\mathrm{Ci}(dx)\sin(c)-\cdots \\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{2x}+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{2}bd^2\mathrm{Ci}(dx)\sin(c)-\cdots \\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{2x}+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{2}bd^2\mathrm{Ci}(dx)\sin(c)+\cdots
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 125, normalized size = 0.84

$$\frac{-2adx\cos(c+dx)-12bdx^3\cos(c+dx)+ad^3x^3\cos(c+dx)+d^2(-12b+ad^2)\,x^4\mathrm{Ci}(dx)\sin(c)-6a\sin(c+dx)-12bx^2\sin(c+dx)+ad^2x^2\sin(c+dx)+d^2(-12b+ad^2)\,x^4\cos(c)\mathrm{Si}(dx)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x^5,x]
```

```
[Out] (-2*a*d*x*Cos[c + d*x] - 12*b*d*x^3*Cos[c + d*x] + a*d^3*x^3*Cos[c + d*x] +
 d^2*(-12*b + a*d^2)*x^4*CosIntegral[d*x]*Sin[c] - 6*a*Sin[c + d*x] - 12*b*
```

```
x^2*Sin[c + d*x] + a*d^2*x^2*Sin[c + d*x] + d^2*(-12*b + a*d^2)*x^4*Cos[c]*
SinIntegral[d*x])/(24*x^4)
```

Maple [A]
time = 0.11, size = 131, normalized size = 0.88

method	result
derivativedivides	$d^4\left(a\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\text{sinIntegral}(dx)\cos(c)}{24}+\frac{\text{cosineIntegral}(dx)\sin(c}{24}\right.\right.$
default	$d^4\left(a\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\text{sinIntegral}(dx)\cos(c)}{24}+\frac{\text{cosineIntegral}(dx)\sin(c}{24}\right.\right.$
risch	$-\frac{i\cos(c)\,\text{expIntegral}(1,idx)a\,d^4}{48}+\frac{i\cos(c)\,\text{expIntegral}(1,-idx)a\,d^4}{48}+\frac{i\,\text{expIntegral}(1,idx)\cos(c)b\,d^2}{4}-\frac{i\,\text{expIntegra}}{}$
meijerg	$\frac{d^2b\sqrt{\pi}\,\sin(c)\left(-\frac{4}{\sqrt{\pi}\,x^2d^2}-\frac{2\left(2\gamma-3+2\ln(x)+\ln\left(d^2\right)\right)}{\sqrt{\pi}}+\frac{-6d^2x^2+4}{\sqrt{\pi}\,x^2d^2}+\frac{4\gamma}{\sqrt{\pi}}+\frac{4\ln(2)}{\sqrt{\pi}}+\frac{4\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}-\frac{4\cos(dx)}{\sqrt{\pi}\,d^2x^2}+\frac{4\sin(dx)}{\sqrt{\pi}\,d}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] d^4*(a*(-1/4*sin(d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/d^2
/x^2+1/24*cos(d*x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c))+1/d^2*b*(
-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*s
in(c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.89, size = 121, normalized size = 0.81

$$-\frac{((a(i\,\Gamma(-4,i\,dx)-i\,\Gamma(-4,-i\,dx))\cos(c)+a(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\sin(c))d^6-12\,(b(i\,\Gamma(-4,i\,dx)-i\,\Gamma(-4,-i\,dx))\cos(c)+b(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\sin(c))d^4)x^4+2\,bdx\cos(dx+c)+6\,b\sin(dx+c)}{2\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a*(gamma(-4,
I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^6 - 12*(b*(I*gamma(-4, I*d*x) - I*gam
ma(-4, -I*d*x))*cos(c) + b*(gamma(-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d
^4)*x^4 + 2*b*d*x*cos(d*x + c) + 6*b*sin(d*x + c))/(d^2*x^4)
```

Fricas [A]
time = 0.36, size = 127, normalized size = 0.85

$$\frac{2\,(ad^4-12\,bd^2)x^4\cos(c)\,\mathrm{Si}(dx)+2\,((ad^3-12\,bd)x^3-2\,adx)\cos(dx+c)+2\,((ad^2-12\,b)x^2-6\,a)\sin(dx+c)+((ad^4-12\,bd^2)x^4\,\mathrm{Ci}(dx)+(ad^4-12\,bd^2)x^4\,\mathrm{Ci}(-dx))\sin(c)}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(a*d^4 - 12*b*d^2)*x^4*cos(c)*sin_integral(d*x) + 2*((a*d^3 - 12*b*
d)*x^3 - 2*a*d*x)*cos(d*x + c) + 2*((a*d^2 - 12*b)*x^2 - 6*a)*sin(d*x + c)
+ ((a*d^4 - 12*b*d^2)*x^4*cos_integral(d*x) + (a*d^4 - 12*b*d^2)*x^4*cos_in
tegral(-d*x))*sin(c))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)\sin{(c + dx)}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**2)*sin(c + d*x)/x**5, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.90, size = 1086, normalized size = 7.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(a*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*
d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^4*x^4*real_pa
rt(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^4*x^4*real_part(cos
_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a*d^4*x^4*imag_part(cos_integr
al(d*x))*tan(1/2*d*x)^2 + a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d
*x)^2 - 2*a*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 + a*d^4*x^4*imag_part(
cos_integral(d*x))*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integral(-d*x))*t
an(1/2*c)^2 + 2*a*d^4*x^4*sin_integral(d*x)*tan(1/2*c)^2 - 12*b*d^2*x^4*ima
g_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*b*d^2*x^4*imag_p
art(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*b*d^2*x^4*sin_inte
gral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^4*x^4*real_part(cos_integral(
d*x))*tan(1/2*c) - 2*a*d^4*x^4*real_part(cos_integral(-d*x))*tan(1/2*c) + 2
4*b*d^2*x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 24*b*d
^2*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^
3*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integral(d*x)) + a*
d^4*x^4*imag_part(cos_integral(-d*x)) - 2*a*d^4*x^4*sin_integral(d*x) + 12*
b*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 12*b*d^2*x^4*imag_p
art(cos_integral(-d*x))*tan(1/2*d*x)^2 + 24*b*d^2*x^4*sin_integral(d*x)*tan
(1/2*d*x)^2 - 12*b*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 12*b
*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 24*b*d^2*x^4*sin_inte
gral(d*x)*tan(1/2*c)^2 + 2*a*d^3*x^3*tan(1/2*d*x)^2 + 24*b*d^2*x^4*real_par
```

```
t(cos_integral(d*x))*tan(1/2*c) + 24*b*d^2*x^4*real_part(cos_integral(-d*x)
)*tan(1/2*c) + 8*a*d^3*x^3*tan(1/2*d*x)*tan(1/2*c) + 2*a*d^3*x^3*tan(1/2*c)
^2 + 24*b*d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*b*d^2*x^4*imag_part(cos_in
tegral(d*x)) - 12*b*d^2*x^4*imag_part(cos_integral(-d*x)) + 24*b*d^2*x^4*si
n_integral(d*x) + 4*a*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^2*x^2*tan(1
/2*d*x)*tan(1/2*c)^2 - 2*a*d^3*x^3 - 24*b*d*x^3*tan(1/2*d*x)^2 - 96*b*d*x^3
*tan(1/2*d*x)*tan(1/2*c) - 24*b*d*x^3*tan(1/2*c)^2 + 4*a*d*x*tan(1/2*d*x)^2
*tan(1/2*c)^2 - 4*a*d^2*x^2*tan(1/2*d*x) - 4*a*d^2*x^2*tan(1/2*c) - 48*b*x^
2*tan(1/2*d*x)^2*tan(1/2*c) - 48*b*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 24*b*d*x
^3 - 4*a*d*x*tan(1/2*d*x)^2 - 16*a*d*x*tan(1/2*d*x)*tan(1/2*c) - 4*a*d*x*ta
n(1/2*c)^2 + 48*b*x^2*tan(1/2*d*x) + 48*b*x^2*tan(1/2*c) - 24*a*tan(1/2*d*x
)^2*tan(1/2*c) - 24*a*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a*d*x + 24*a*tan(1/2*d*
x) + 24*a*tan(1/2*c))/(x^4*tan(1/2*d*x)^2*tan(1/2*c)^2 + x^4*tan(1/2*d*x)^2
 + x^4*tan(1/2*c)^2 + x^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)\,\left(b\,x^2+a\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^2))/x^5,x)
[Out] int((sin(c + d*x)*(a + b*x^2))/x^5, x)
```

3.49 $\int x^2(a+bx^2)^2\sin(c+dx)\,dx$

Optimal. Leaf size=236

$$\frac{720b^2\cos(c+dx)}{d^7}-\frac{48ab\cos(c+dx)}{d^5}+\frac{2a^2\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+dx)}{d^5}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx)}{d}$$

```
[Out] 720*b^2*cos(d*x+c)/d^7-48*a*b*cos(d*x+c)/d^5+2*a^2*cos(d*x+c)/d^3-360*b^2*x
^2*cos(d*x+c)/d^5+24*a*b*x^2*cos(d*x+c)/d^3-a^2*x^2*cos(d*x+c)/d+30*b^2*x^4
*cos(d*x+c)/d^3-2*a*b*x^4*cos(d*x+c)/d-b^2*x^6*cos(d*x+c)/d+720*b^2*x*sin(d
*x+c)/d^6-48*a*b*x*sin(d*x+c)/d^4+2*a^2*x*sin(d*x+c)/d^2-120*b^2*x^3*sin(d*
x+c)/d^4+8*a*b*x^3*sin(d*x+c)/d^2+6*b^2*x^5*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.22, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3420, 3377, 2718}

$$\frac{2a^2\cos(c+dx)}{d^3}+\frac{2a^2x\sin(c+dx)}{d^2}-\frac{a^2x^2\cos(c+dx)}{d}-\frac{48ab\cos(c+dx)}{d^5}-\frac{48abx\sin(c+dx)}{d^4}+\frac{24abx^2\cos(c+dx)}{d^3}+\frac{8abx^3\sin(c+dx)}{d^2}-\frac{2abx^4\cos(c+dx)}{d}+\frac{720b^2\cos(c+dx)}{d^7}+\frac{720b^2x\sin(c+dx)}{d^6}-\frac{360b^2x^2\cos(c+dx)}{d^5}-\frac{120b^2x^3\sin(c+dx)}{d^4}+\frac{30b^2x^4\cos(c+dx)}{d^3}+\frac{6b^2x^5\sin(c+dx)}{d^2}-\frac{b^2x^6\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (720*b^2*Cos[c + d*x])/d^7 - (48*a*b*Cos[c + d*x])/d^5 + (2*a^2*Cos[c + d*x
])/d^3 - (360*b^2*x^2*Cos[c + d*x])/d^5 + (24*a*b*x^2*Cos[c + d*x])/d^3 - (
a^2*x^2*Cos[c + d*x])/d + (30*b^2*x^4*Cos[c + d*x])/d^3 - (2*a*b*x^4*Cos[c
+ d*x])/d - (b^2*x^6*Cos[c + d*x])/d + (720*b^2*x*Sin[c + d*x])/d^6 - (48*a
*b*x*Sin[c + d*x])/d^4 + (2*a^2*x*Sin[c + d*x])/d^2 - (120*b^2*x^3*Sin[c +
d*x])/d^4 + (8*a*b*x^3*Sin[c + d*x])/d^2 + (6*b^2*x^5*Sin[c + d*x])/d^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2(a+bx^2)^2\sin(c+dx)\,dx &= \int \left(a^2x^2\sin(c+dx)+2abx^4\sin(c+dx)+b^2x^6\sin(c+dx)\right)\,dx\\
&= a^2\int x^2\sin(c+dx)\,dx+(2ab)\int x^4\sin(c+dx)\,dx+b^2\int x^6\sin(c+dx)\,dx\\
&= -\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^4\cos(c+dx)}{d}-\frac{b^2x^6\cos(c+dx)}{d}+\frac{(2a^2)\int x\cos(}{d}\\
&= -\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^4\cos(c+dx)}{d}-\frac{b^2x^6\cos(c+dx)}{d}+\frac{2a^2x\sin(c+}{d^2}\\
&= \frac{2a^2\cos(c+dx)}{d^3}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx)}{d}+\frac{30b^2x^4\cos(c+}{d^3}\\
&= \frac{2a^2\cos(c+dx)}{d^3}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx)}{d}+\frac{30b^2x^4\cos(c+}{d^3}\\
&= -\frac{48ab\cos(c+dx)}{d^5}+\frac{2a^2\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+dx)}{d^5}+\frac{24abx^2\cos(}{d^3}\\
&= -\frac{48ab\cos(c+dx)}{d^5}+\frac{2a^2\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+dx)}{d^5}+\frac{24abx^2\cos(}{d^3}\\
&= \frac{720b^2\cos(c+dx)}{d^7}-\frac{48ab\cos(c+dx)}{d^5}+\frac{2a^2\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+}{d^5}
\end{aligned}
$$

Mathematica [**A**]
time = 0.24, size = 139, normalized size = 0.59

$$\frac{-((a^2d^4(-2+d^2x^2)+2abd^2(24-12d^2x^2+d^4x^4)+b^2(-720+360d^2x^2-30d^4x^4+d^6x^6))\cos(c+dx))+2dx(a^2d^4+4abd^2(-6+d^2x^2)+3b^2(120-20d^2x^2+d^4x^4))\sin(c+dx)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (-((a^2*d^4*(-2 + d^2*x^2) + 2*a*b*d^2*(24 - 12*d^2*x^2 + d^4*x^4) + b^2*(-
720 + 360*d^2*x^2 - 30*d^4*x^4 + d^6*x^6))*Cos[c + d*x]) + 2*d*x*(a^2*d^4 +
 4*a*b*d^2*(-6 + d^2*x^2) + 3*b^2*(120 - 20*d^2*x^2 + d^4*x^4))*Sin[c + d*x
])/d^7
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(236) = 472.
time = 0.07, size = 746, normalized size = 3.16 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(-a^2*c^2*cos(d*x+c)-2*a^2*c*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+a^2*(-(d
*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-2/d^2*a*b*c^4*cos(d*x
```

```
+c)-8/d^2*a*b*c^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+12/d^2*a*b*c^2*(-(d*x+c)^
2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-8/d^2*a*b*c*(-(d*x+c)^3*cos
(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+2/d^2*a*b
*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24*c
os(d*x+c)-24*(d*x+c)*sin(d*x+c))-1/d^4*b^2*c^6*cos(d*x+c)-6/d^4*b^2*c^5*(si
n(d*x+c)-(d*x+c)*cos(d*x+c))+15/d^4*b^2*c^4*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*
x+c)+2*(d*x+c)*sin(d*x+c))-20/d^4*b^2*c^3*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^
2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+15/d^4*b^2*c^2*(-(d*x+c)^4*
cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)-24*
(d*x+c)*sin(d*x+c))-6/d^4*b^2*c*(-(d*x+c)^5*cos(d*x+c)+5*(d*x+c)^4*sin(d*x+
c)+20*(d*x+c)^3*cos(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(d*x+c)-120*(d*x+
c)*cos(d*x+c))+1/d^4*b^2*(-(d*x+c)^6*cos(d*x+c)+6*(d*x+c)^5*sin(d*x+c)+30*(
d*x+c)^4*cos(d*x+c)-120*(d*x+c)^3*sin(d*x+c)-360*(d*x+c)^2*cos(d*x+c)+720*c
os(d*x+c)+720*(d*x+c)*sin(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(236) = 472$.
time = 0.33, size = 612, normalized size = 2.59

$$-\frac{a^2c^2\cos(dx+c) + \frac{b^2c^6\cos(dx+c)}{d^4} + \frac{2abc^4\cos(dx+c)}{d^2} - 2((dx+c)\cos(dx+c)-\sin(dx+c))a^2c - \frac{6((dx+c)\cos(dx+c)-\sin(dx+c))b^2c^5}{d^4} - \frac{8((dx+c)\cos(dx+c)-\sin(dx+c))abc^3}{d^2} + (((dx+c)^2-2)\cos(dx+c) - 2(dx+c)\sin(dx+c))a^2 + \frac{15(((dx+c)^2-2)\cos(dx+c)-2(dx+c)\sin(dx+c))b^2c^4}{d^4} + \frac{12(((dx+c)^2-2)\cos(dx+c)-2(dx+c)\sin(dx+c))abc^2}{d^2} - \frac{20(((dx+c)^3-6dx-6c)\cos(dx+c)-3((dx+c)^2-2)\sin(dx+c))b^2c^3}{d^4} - \frac{8(((dx+c)^3-6dx-6c)\cos(dx+c)-3((dx+c)^2-2)\sin(dx+c))abc}{d^2} + \frac{15(((dx+c)^4-12(dx+c)^2+24)\cos(dx+c)-4((dx+c)^3-6dx-6c)\sin(dx+c))b^2c^2}{d^4} + \frac{2(((dx+c)^4-12(dx+c)^2+24)\cos(dx+c)-4((dx+c)^3-6dx-6c)\sin(dx+c))ab}{d^2} - \frac{6(((dx+c)^5-20(dx+c)^3+120dx+120c)\cos(dx+c)-5((dx+c)^4-12(dx+c)^2+24)\sin(dx+c))b^2c}{d^4} + \frac{(((dx+c)^6-30(dx+c)^4+360(dx+c)^2-720)\cos(dx+c)-6((dx+c)^5-20(dx+c)^3+120dx+120c)\sin(dx+c))b^2}{d^4}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*c^2*cos(d*x + c) + b^2*c^6*cos(d*x + c)/d^4 + 2*a*b*c^4*cos(d*x + c)/
d^2 - 2*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2*c - 6*((d*x + c)*cos(d*
x + c) - sin(d*x + c))*b^2*c^5/d^4 - 8*((d*x + c)*cos(d*x + c) - sin(d*x +
c))*a*b*c^3/d^2 + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c
))*a^2 + 15*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b^2
*c^4/d^4 + 12*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a
*b*c^2/d^2 - 20*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2
- 2)*sin(d*x + c))*b^2*c^3/d^4 - 8*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c
) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b*c/d^2 + 15*(((d*x + c)^4 - 12*(d*
x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b
^2*c^2/d^4 + 2*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x
+ c)^3 - 6*d*x - 6*c)*sin(d*x + c))*a*b/d^2 - 6*(((d*x + c)^5 - 20*(d*x + c
)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 + 24)
*sin(d*x + c))*b^2*c/d^4 + (((d*x + c)^6 - 30*(d*x + c)^4 + 360*(d*x + c)^2
 - 720)*cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 120*c)*s
in(d*x + c))*b^2/d^4)/d^3
```

Fricas [A]
time = 0.38, size = 154, normalized size = 0.65

$$-\frac{(b^2d^6x^6 - 2\,a^2d^4 + 2\,(abd^6 - 15\,b^2d^4)x^4 + 48\,abd^2 + (a^2d^6 - 24\,abd^4 + 360\,b^2d^2)x^2 - 720\,b^2)\cos(dx+c) - 2\,(3\,b^2d^5x^5 + 4\,(abd^5 - 15\,b^2d^3)x^3 + (a^2d^5 - 24\,abd^3 + 360\,b^2d)x)\sin(dx+c)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^6*x^6 - 2*a^2*d^4 + 2*(a*b*d^6 - 15*b^2*d^4)*x^4 + 48*a*b*d^2 + (a
^2*d^6 - 24*a*b*d^4 + 360*b^2*d^2)*x^2 - 720*b^2)*cos(d*x + c) - 2*(3*b^2*d
^5*x^5 + 4*(a*b*d^5 - 15*b^2*d^3)*x^3 + (a^2*d^5 - 24*a*b*d^3 + 360*b^2*d)*
x)*sin(d*x + c))/d^7
```

Sympy [A]
time = 0.68, size = 286, normalized size = 1.21

$$\begin{cases} - \frac{a^{2} x^{2} \cos{\left(c + d x \right)}}{d} + \frac{2 a^{2} x \sin{\left(c + d x \right)}}{d^{2}} + \frac{2 a^{2} \cos{\left(c + d x \right)}}{d^{3}} - \frac{2 a b x^{4} \cos{\left(c + d x \right)}}{d} + \frac{8 a b x^{3} \sin{\left(c + d x \right)}}{d^{2}} + \frac{24 a b x^{2} \cos{\left(c + d x \right)}}{d^{3}} - \frac{48 a b x \sin{\left(c + d x \right)}}{d^{4}} - \frac{48 a b \cos{\left(c + d x \right)}}{d^{5}} - \frac{b^{2} x^{6} \cos{\left(c + d x \right)}}{d} + \frac{6 b^{2} x^{5} \sin{\left(c + d x \right)}}{d^{2}} + \frac{30 b^{2} x^{4} \cos{\left(c + d x \right)}}{d^{3}} - \frac{120 b^{2} x^{3} \sin{\left(c + d x \right)}}{d^{4}} - \frac{360 b^{2} x^{2} \cos{\left(c + d x \right)}}{d^{5}} + \frac{720 b^{2} x \sin{\left(c + d x \right)}}{d^{6}} + \frac{720 b^{2} \cos{\left(c + d x \right)}}{d^{7}} & \text{for}\: d \neq 0 \\\left(\frac{a^{2} x^{3}}{3} + \frac{2 a b x^{5}}{5} + \frac{b^{2} x^{7}}{7}\right) \sin{\left(c \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*x**2*cos(c + d*x)/d + 2*a**2*x*sin(c + d*x)/d**2 + 2*a**2*
cos(c + d*x)/d**3 - 2*a*b*x**4*cos(c + d*x)/d + 8*a*b*x**3*sin(c + d*x)/d**
2 + 24*a*b*x**2*cos(c + d*x)/d**3 - 48*a*b*x*sin(c + d*x)/d**4 - 48*a*b*cos
(c + d*x)/d**5 - b**2*x**6*cos(c + d*x)/d + 6*b**2*x**5*sin(c + d*x)/d**2 +
 30*b**2*x**4*cos(c + d*x)/d**3 - 120*b**2*x**3*sin(c + d*x)/d**4 - 360*b**
2*x**2*cos(c + d*x)/d**5 + 720*b**2*x*sin(c + d*x)/d**6 + 720*b**2*cos(c +
d*x)/d**7, Ne(d, 0)), ((a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7)*sin(c), T
rue))
```

Giac [A]
time = 6.18, size = 162, normalized size = 0.69

$$-\frac{{\left(b^{2} d^{6} x^{6} + 2 \, a b d^{6} x^{4} + a^{2} d^{6} x^{2} - 30 \, b^{2} d^{4} x^{4} - 24 \, a b d^{4} x^{2} - 2 \, a^{2} d^{4} + 360 \, b^{2} d^{2} x^{2} + 48 \, a b d^{2} - 720 \, b^{2}\right)} \cos\left(d x + c\right)}{d^{7}} + \frac{2 \, {\left(3 \, b^{2} d^{5} x^{5} + 4 \, a b d^{5} x^{3} + a^{2} d^{5} x - 60 \, b^{2} d^{3} x^{3} - 24 \, a b d^{3} x + 360 \, b^{2} d x\right)} \sin\left(d x + c\right)}{d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^6*x^6 + 2*a*b*d^6*x^4 + a^2*d^6*x^2 - 30*b^2*d^4*x^4 - 24*a*b*d^4*x
^2 - 2*a^2*d^4 + 360*b^2*d^2*x^2 + 48*a*b*d^2 - 720*b^2)*cos(d*x + c)/d^7 +
 2*(3*b^2*d^5*x^5 + 4*a*b*d^5*x^3 + a^2*d^5*x - 60*b^2*d^3*x^3 - 24*a*b*d^3
*x + 360*b^2*d*x)*sin(d*x + c)/d^7
```

Mupad [B]
time = 0.58, size = 186, normalized size = 0.79

$$\frac{2\,\cos\left(c+d\,x\right)\,\left(a^2\,d^4-24\,a\,b\,d^2+360\,b^2\right)}{d^7}-\frac{b^2\,x^6\,\cos\left(c+d\,x\right)}{d}+\frac{6\,b^2\,x^5\,\sin\left(c+d\,x\right)}{d^2}+\frac{2\,x\,\sin\left(c+d\,x\right)\,\left(a^2\,d^4-24\,a\,b\,d^2+360\,b^2\right)}{d^6}-\frac{x^2\,\cos\left(c+d\,x\right)\,\left(a^2\,d^4-24\,a\,b\,d^2+360\,b^2\right)}{d^5}+\frac{2\,x^4\,\cos\left(c+d\,x\right)\,\left(15\,b^2-a\,b\,d^2\right)}{d^3}-\frac{8\,x^3\,\sin\left(c+d\,x\right)\,\left(15\,b^2-a\,b\,d^2\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(c + d*x)*(a + b*x^2)^2,x)
```

```
[Out] (2*cos(c + d*x)*(360*b^2 + a^2*d^4 - 24*a*b*d^2))/d^7 - (b^2*x^6*cos(c + d*
x))/d + (6*b^2*x^5*sin(c + d*x))/d^2 + (2*x*sin(c + d*x)*(360*b^2 + a^2*d^4
 - 24*a*b*d^2))/d^6 - (x^2*cos(c + d*x)*(360*b^2 + a^2*d^4 - 24*a*b*d^2))/d
^5 + (2*x^4*cos(c + d*x)*(15*b^2 - a*b*d^2))/d^3 - (8*x^3*sin(c + d*x)*(15*
b^2 - a*b*d^2))/d^4
```

3.50 $\int x(a+bx^2)^2 \sin(c+dx)\,dx$

Optimal. Leaf size=185

$$-\frac{120b^2x\cos(c+dx)}{d^5}+\frac{12abx\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^5 \mathrm{c}}{}$$

```
[Out] -120*b^2*x*cos(d*x+c)/d^5+12*a*b*x*cos(d*x+c)/d^3-a^2*x*cos(d*x+c)/d+20*b^2
*x^3*cos(d*x+c)/d^3-2*a*b*x^3*cos(d*x+c)/d-b^2*x^5*cos(d*x+c)/d+120*b^2*sin
(d*x+c)/d^6-12*a*b*sin(d*x+c)/d^4+a^2*sin(d*x+c)/d^2-60*b^2*x^2*sin(d*x+c)/
d^4+6*a*b*x^2*sin(d*x+c)/d^2+5*b^2*x^4*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.17, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3420, 3377, 2717}

$$\frac{a^2\sin(c+dx)}{d^2}-\frac{a^2x\cos(c+dx)}{d}-\frac{12ab\sin(c+dx)}{d^4}+\frac{12abx\cos(c+dx)}{d^3}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{2abx^3\cos(c+dx)}{d}+\frac{120b^2\sin(c+dx)}{d^6}-\frac{120b^2x\cos(c+dx)}{d^5}-\frac{60b^2x^2\sin(c+dx)}{d^4}+\frac{20b^2x^3\cos(c+dx)}{d^3}+\frac{5b^2x^4\sin(c+dx)}{d^2}-\frac{b^2x^5\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (-120*b^2*x*Cos[c + d*x])/d^5 + (12*a*b*x*Cos[c + d*x])/d^3 - (a^2*x*Cos[c
+ d*x])/d + (20*b^2*x^3*Cos[c + d*x])/d^3 - (2*a*b*x^3*Cos[c + d*x])/d - (b
^2*x^5*Cos[c + d*x])/d + (120*b^2*Sin[c + d*x])/d^6 - (12*a*b*Sin[c + d*x])
/d^4 + (a^2*Sin[c + d*x])/d^2 - (60*b^2*x^2*Sin[c + d*x])/d^4 + (6*a*b*x^2*
Sin[c + d*x])/d^2 + (5*b^2*x^4*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x(a+bx^2)^2 \sin(c+dx)\,dx &= \int \left(a^2x\sin(c+dx)+2abx^3\sin(c+dx)+b^2x^5\sin(c+dx)\right)\,dx \\ &= a^2\int x\sin(c+dx)\,dx+(2ab)\int x^3\sin(c+dx)\,dx+b^2\int x^5\sin(c+dx)\,dx \\ &= -\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^5\cos(c+dx)}{d}+\frac{a^2\int\cos(c+dx)}{d} \\ &= -\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^5\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d^2} \\ &= \frac{12abx\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+}{d} \\ &= \frac{12abx\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+}{d} \\ &= -\frac{120b^2x\cos(c+dx)}{d^5}+\frac{12abx\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c}{d^3} \\ &= -\frac{120b^2x\cos(c+dx)}{d^5}+\frac{12abx\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c}{d^3}\end{aligned}$$

Mathematica [**A**]
time = 0.15, size = 113, normalized size = 0.61

$$\frac{-dx(a^2d^4+2abd^2(-6+d^2x^2)+b^2(120-20d^2x^2+d^4x^4))\cos(c+dx)+(a^2d^4+6abd^2(-2+d^2x^2)+5b^2(24-12d^2x^2+d^4x^4))\sin(c+dx)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (-(d*x*(a^2*d^4 + 2*a*b*d^2*(-6 + d^2*x^2) + b^2*(120 - 20*d^2*x^2 + d^4*x^
4))*Cos[c + d*x]) + (a^2*d^4 + 6*a*b*d^2*(-2 + d^2*x^2) + 5*b^2*(24 - 12*d^
2*x^2 + d^4*x^4))*Sin[c + d*x])/d^6
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(185) = 370$.
time = 0.07, size = 514, normalized size = 2.78 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(a^2*c*cos(d*x+c)+a^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+2/d^2*a*b*c^3*c
os(d*x+c)+6/d^2*a*b*c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-6/d^2*a*b*c*(-(d*x+
c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+2/d^2*a*b*(-(d*x+c)^3*co
s(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+1/d^4*b^
2*c^5*cos(d*x+c)+5/d^4*b^2*c^4*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-10/d^4*b^2*c
^3*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+10/d^4*b^2*c^2
```

```
*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d
*x+c))-5/d^4*b^2*c*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)
^2*cos(d*x+c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))+1/d^4*b^2*(-(d*x+c)^5*co
s(d*x+c)+5*(d*x+c)^4*sin(d*x+c)+20*(d*x+c)^3*cos(d*x+c)-60*(d*x+c)^2*sin(d*
x+c)+120*sin(d*x+c)-120*(d*x+c)*cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(185) = 370$.
time = 0.30, size = 438, normalized size = 2.37

$$\frac{a^2c\cos(dx+c) + \frac{b^2c^5\cos(dx+c)}{d^4} + \frac{2abc^3\cos(dx+c)}{d^2} - ((dx+c)\cos(dx+c) - \sin(dx+c))a^2 - \frac{5((dx+c)\cos(dx+c)-\sin(dx+c))b^2c^4}{d^4} - \frac{6((dx+c)\cos(dx+c)-\sin(dx+c))abc^2}{d^2} + \frac{10\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)b^2c^3}{d^4} + \frac{6\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)abc}{d^2} - \frac{10\left(\left((dx+c)^3-6dx-6c\right)\cos(dx+c)-3\left((dx+c)^2-2\right)\sin(dx+c)\right)b^2c^2}{d^4} - \frac{2\left(\left((dx+c)^3-6dx-6c\right)\cos(dx+c)-3\left((dx+c)^2-2\right)\sin(dx+c)\right)ab}{d^2} + \frac{5\left(\left((dx+c)^4-12(dx+c)^2+24\right)\cos(dx+c)-4\left((dx+c)^3-6dx-6c\right)\sin(dx+c)\right)b^2c}{d^4} - \frac{\left(\left((dx+c)^5-20(dx+c)^3+120dx+120c\right)\cos(dx+c)-5\left((dx+c)^4-12(dx+c)^2+24\right)\sin(dx+c)\right)b^2}{d^4}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a^2*c*cos(d*x + c) + b^2*c^5*cos(d*x + c)/d^4 + 2*a*b*c^3*cos(d*x + c)/d^2
 - ((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2 - 5*((d*x + c)*cos(d*x + c)
- sin(d*x + c))*b^2*c^4/d^4 - 6*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a*b
*c^2/d^2 + 10*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b
^2*c^3/d^4 + 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*
a*b*c/d^2 - 10*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 -
 2)*sin(d*x + c))*b^2*c^2/d^4 - 2*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c)
 - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b/d^2 + 5*(((d*x + c)^4 - 12*(d*x +
c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b^2*c
/d^4 - (((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*(
(d*x + c)^4 - 12*(d*x + c)^2 + 24)*sin(d*x + c))*b^2/d^4)/d^2
```

Fricas [A]
time = 0.37, size = 126, normalized size = 0.68

$$-\frac{(b^2d^5x^5 + 2(abd^5 - 10b^2d^3)x^3 + (a^2d^5 - 12abd^3 + 120b^2d)x)\cos(dx+c) - (5b^2d^4x^4 + a^2d^4 - 12abd^2 + 6(abd^4 - 10b^2d^2)x^2 + 120b^2)\sin(dx+c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^5*x^5 + 2*(a*b*d^5 - 10*b^2*d^3)*x^3 + (a^2*d^5 - 12*a*b*d^3 + 120
*b^2*d)*x)*cos(d*x + c) - (5*b^2*d^4*x^4 + a^2*d^4 - 12*a*b*d^2 + 6*(a*b*d^
4 - 10*b^2*d^2)*x^2 + 120*b^2)*sin(d*x + c))/d^6
```

Sympy [A]
time = 0.49, size = 226, normalized size = 1.22

$$\begin{cases} -\frac{a^2x\cos(c+dx)}{d} + \frac{a^2\sin(c+dx)}{d^2} - \frac{2abx^3\cos(c+dx)}{d} + \frac{6abx^2\sin(c+dx)}{d^2} + \frac{12abx\cos(c+dx)}{d^3} - \frac{12ab\sin(c+dx)}{d^4} - \frac{b^2x^5\cos(c+dx)}{d} + \frac{5b^2x^4\sin(c+dx)}{d^2} + \frac{20b^2x^3\cos(c+dx)}{d^3} - \frac{60b^2x^2\sin(c+dx)}{d^4} - \frac{120b^2x\cos(c+dx)}{d^5} + \frac{120b^2\sin(c+dx)}{d^6} & \text{for } d \neq 0 \\ \left(\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*x*cos(c + d*x)/d + a**2*sin(c + d*x)/d**2 - 2*a*b*x**3*cos
(c + d*x)/d + 6*a*b*x**2*sin(c + d*x)/d**2 + 12*a*b*x*cos(c + d*x)/d**3 - 1
2*a*b*sin(c + d*x)/d**4 - b**2*x**5*cos(c + d*x)/d + 5*b**2*x**4*sin(c + d*
x)/d**2 + 20*b**2*x**3*cos(c + d*x)/d**3 - 60*b**2*x**2*sin(c + d*x)/d**4 -
 120*b**2*x*cos(c + d*x)/d**5 + 120*b**2*sin(c + d*x)/d**6, Ne(d, 0)), ((a*
*2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)*sin(c), True))
```

Giac [A]
time = 4.87, size = 129, normalized size = 0.70

$$-\frac{(b^2d^5x^5 + 2\,abd^5x^3 + a^2d^5x - 20\,b^2d^3x^3 - 12\,abd^3x + 120\,b^2dx)\cos(dx + c)}{d^6} + \frac{(5\,b^2d^4x^4 + 6\,abd^4x^2 + a^2d^4 - 60\,b^2d^2x^2 - 12\,abd^2 + 120\,b^2)\sin(dx + c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^5*x^5 + 2*a*b*d^5*x^3 + a^2*d^5*x - 20*b^2*d^3*x^3 - 12*a*b*d^3*x +
 120*b^2*d*x)*cos(d*x + c)/d^6 + (5*b^2*d^4*x^4 + 6*a*b*d^4*x^2 + a^2*d^4 -
 60*b^2*d^2*x^2 - 12*a*b*d^2 + 120*b^2)*sin(d*x + c)/d^6
```

Mupad [B]
time = 4.93, size = 151, normalized size = 0.82

$$\frac{\sin(c+d\,x)\,(a^2\,d^4 - 12\,a\,b\,d^2 + 120\,b^2)}{d^6} - \frac{b^2\,x^5\,\cos(c+d\,x)}{d} + \frac{5\,b^2\,x^4\,\sin(c+d\,x)}{d^2} - \frac{x\,\cos(c+d\,x)\,(a^2\,d^4 - 12\,a\,b\,d^2 + 120\,b^2)}{d^5} + \frac{2\,x^3\,\cos(c+d\,x)\,(10\,b^2 - a\,b\,d^2)}{d^3} - \frac{6\,x^2\,\sin(c+d\,x)\,(10\,b^2 - a\,b\,d^2)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(c + d*x)*(a + b*x^2)^2,x)
```

```
[Out] (sin(c + d*x)*(120*b^2 + a^2*d^4 - 12*a*b*d^2))/d^6 - (b^2*x^5*cos(c + d*x)
)/d + (5*b^2*x^4*sin(c + d*x))/d^2 - (x*cos(c + d*x)*(120*b^2 + a^2*d^4 - 1
2*a*b*d^2))/d^5 + (2*x^3*cos(c + d*x)*(10*b^2 - a*b*d^2))/d^3 - (6*x^2*sin(
c + d*x)*(10*b^2 - a*b*d^2))/d^4
```

3.51 $\int (a+bx^2)^2 \sin(c+dx)\, dx$

Optimal. Leaf size=138

$$-\frac{24b^2\cos(c+dx)}{d^5}+\frac{4ab\cos(c+dx)}{d^3}-\frac{a^2\cos(c+dx)}{d}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}$$

```
[Out] -24*b^2*cos(d*x+c)/d^5+4*a*b*cos(d*x+c)/d^3-a^2*cos(d*x+c)/d+12*b^2*x^2*cos
(d*x+c)/d^3-2*a*b*x^2*cos(d*x+c)/d-b^2*x^4*cos(d*x+c)/d-24*b^2*x*sin(d*x+c)
/d^4+4*a*b*x*sin(d*x+c)/d^2+4*b^2*x^3*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3410, 2718, 3377}

$$-\frac{a^2\cos(c+dx)}{d}+\frac{4ab\cos(c+dx)}{d^3}+\frac{4abx\sin(c+dx)}{d^2}-\frac{2abx^2\cos(c+dx)}{d}-\frac{24b^2\cos(c+dx)}{d^5}-\frac{24b^2x\sin(c+dx)}{d^4}+\frac{12b^2x^2\cos(c+dx)}{d^3}+\frac{4b^2x^3\sin(c+dx)}{d^2}-\frac{b^2x^4\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (-24*b^2*Cos[c + d*x])/d^5 + (4*a*b*Cos[c + d*x])/d^3 - (a^2*Cos[c + d*x])/
d + (12*b^2*x^2*Cos[c + d*x])/d^3 - (2*a*b*x^2*Cos[c + d*x])/d - (b^2*x^4*C
os[c + d*x])/d - (24*b^2*x*Sin[c + d*x])/d^4 + (4*a*b*x*Sin[c + d*x])/d^2 +
 (4*b^2*x^3*Sin[c + d*x])/d^2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3410

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx^2\right)^2 \sin(c+dx)\,dx &= \int \left(a^2\sin(c+dx)+2abx^2\sin(c+dx)+b^2x^4\sin(c+dx)\right)\,dx \\
&= a^2\int \sin(c+dx)\,dx+(2ab)\int x^2\sin(c+dx)\,dx+b^2\int x^4\sin(c+dx)\,dx \\
&= -\frac{a^2\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}+\frac{(4ab)\int x\cos(c+ }{d} \\
&= -\frac{a^2\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}+\frac{4abx\sin(c+dx)}{d^2} \\
&= \frac{4ab\cos(c+dx)}{d^3}-\frac{a^2\cos(c+dx)}{d}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d} \\
&= \frac{4ab\cos(c+dx)}{d^3}-\frac{a^2\cos(c+dx)}{d}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d} \\
&= -\frac{24b^2\cos(c+dx)}{d^5}+\frac{4ab\cos(c+dx)}{d^3}-\frac{a^2\cos(c+dx)}{d}+\frac{12b^2x^2\cos(c+dx)}{d^3}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 86, normalized size = 0.62

$$\frac{-((a^2d^4+2abd^2(-2+d^2x^2)+b^2(24-12d^2x^2+d^4x^4))\cos(c+dx))+4bdx(ad^2+b(-6+d^2x^2))\sin(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (-((a^2*d^4 + 2*a*b*d^2*(-2 + d^2*x^2) + b^2*(24 - 12*d^2*x^2 + d^4*x^4))*C
os[c + d*x]) + 4*b*d*x*(a*d^2 + b*(-6 + d^2*x^2))*Sin[c + d*x])/d^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(138) = 276.
time = 0.06, size = 336, normalized size = 2.43

method	result
risch	$-\frac{\left(b^2x^4d^4+2ab\,d^4x^2+a^2d^4-12d^2x^2b^2-4ab\,d^2+24b^2\right)\cos(dx+c)}{d^5}+\frac{4bx\left(d^2x^2b+d^2a-6b\right)\sin(dx+c)}{d^4}$
norman	$\frac{\frac{b^2x^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^2d^4-8ab\,d^2+48b^2}{d^5}-\frac{b^2x^4}{d}+\frac{8b^2x^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{2b\left(d^2a-6b\right)x^2}{d^3}+\frac{8b\left(d^2a-6b\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{2b\left(d^2a-6\right.}{}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
meijerg	$\frac{16b^2\sqrt{\pi}\,\sin(c)\left(-\frac{x\left(d^2\right)^{\frac{5}{2}}\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{10\sqrt{\pi}\,d^4}+\frac{\left(d^2\right)^{\frac{5}{2}}\left(\frac{5}{8}d^4x^4-\frac{15}{2}d^2x^2+15\right)\sin(dx)}{10\sqrt{\pi}\,d^5}\right)}{d^4\sqrt{d^2}}+\frac{16b^2\sqrt{\pi}\,\cos(c)\left(\frac{3}{2\sqrt{\pi}}\right.}{}$
derivativedivides	$\frac{-a^2\cos(dx+c)-\frac{2ab\,c^2\cos(dx+c)}{d^2}-\frac{4abc(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^2}+\frac{2ab\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c\right.}{d^2}}{}$

default	$\frac{-a^2\cos(dx+c)-\frac{2ab\,c^2\cos(dx+c)}{d^2}-\frac{4abc(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^2}+\frac{2ab\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx-\right.}{d^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*cos(d*x+c)-2/d^2*a*b*c^2*cos(d*x+c)-4/d^2*a*b*c*(sin(d*x+c)-(d*x+
c)*cos(d*x+c))+2/d^2*a*b*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(
d*x+c))-1/d^4*b^2*c^4*cos(d*x+c)-4/d^4*b^2*c^3*(sin(d*x+c)-(d*x+c)*cos(d*x+
c))+6/d^4*b^2*c^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))
-4/d^4*b^2*c*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(
d*x+c)*cos(d*x+c))+1/d^4*b^2*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+
12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(138) = 276$.
time = 0.30, size = 292, normalized size = 2.12

$$-\frac{a^2\cos(dx+c)+\frac{b^2c^4\cos(dx+c)}{d^4}+\frac{2\,abc^2\cos(dx+c)}{d^2}-\frac{4\,((dx+c)\cos(dx+c)-\sin(dx+c))b^2c^3}{d^4}-\frac{4\,((dx+c)\cos(dx+c)-\sin(dx+c))abc}{d^2}+\frac{6\left(\left((dx+c)^2-2\right)\cos(dx+c)-2\,(dx+c)\sin(dx+c)\right)b^2c^2}{d^4}+\frac{2\left(\left((dx+c)^2-2\right)\cos(dx+c)-2\,(dx+c)\sin(dx+c)\right)ab}{d^2}-\frac{4\left(\left((dx+c)^3-6\,dx-6\,c\right)\cos(dx+c)-3\left((dx+c)^2-2\right)\sin(dx+c)\right)b^2c}{d^4}+\frac{\left(\left((dx+c)^4-12\,(dx+c)^2+24\right)\cos(dx+c)-4\left((dx+c)^3-6\,dx-6\,c\right)\sin(dx+c)\right)b^2}{d^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*cos(d*x + c) + b^2*c^4*cos(d*x + c)/d^4 + 2*a*b*c^2*cos(d*x + c)/d^2
- 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b^2*c^3/d^4 - 4*((d*x + c)*cos(
d*x + c) - sin(d*x + c))*a*b*c/d^2 + 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*
(d*x + c)*sin(d*x + c))*b^2*c^2/d^4 + 2*(((d*x + c)^2 - 2)*cos(d*x + c) - 2
*(d*x + c)*sin(d*x + c))*a*b/d^2 - 4*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x +
 c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b^2*c/d^4 + (((d*x + c)^4 - 12*(d*x
 + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b^
2/d^4)/d
```

Fricas [A]
time = 0.37, size = 97, normalized size = 0.70

$$-\frac{\left(b^2d^4x^4+a^2d^4-4\,abd^2+2\,(abd^4-6\,b^2d^2)x^2+24\,b^2\right)\cos(dx+c)-4\left(b^2d^3x^3+(abd^3-6\,b^2d)x\right)\sin(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^4*x^4 + a^2*d^4 - 4*a*b*d^2 + 2*(a*b*d^4 - 6*b^2*d^2)*x^2 + 24*b^2
)*cos(d*x + c) - 4*(b^2*d^3*x^3 + (a*b*d^3 - 6*b^2*d)*x)*sin(d*x + c))/d^5
```

Sympy [A]
time = 0.34, size = 172, normalized size = 1.25

$$\begin{cases} -\frac{a^2\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}+\frac{4abx\sin(c+dx)}{d^2}+\frac{4ab\cos(c+dx)}{d^3}-\frac{b^2x^4\cos(c+dx)}{d}+\frac{4b^2x^3\sin(c+dx)}{d^2}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{24b^2x\sin(c+dx)}{d^4}-\frac{24b^2\cos(c+dx)}{d^5} & \text{for } d\neq 0 \\ \left(a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)/d - 2*a*b*x**2*cos(c + d*x)/d + 4*a*b*x*sin(c
 + d*x)/d**2 + 4*a*b*cos(c + d*x)/d**3 - b**2*x**4*cos(c + d*x)/d + 4*b**2*
x**3*sin(c + d*x)/d**2 + 12*b**2*x**2*cos(c + d*x)/d**3 - 24*b**2*x*sin(c +
 d*x)/d**4 - 24*b**2*cos(c + d*x)/d**5, Ne(d, 0)), ((a**2*x + 2*a*b*x**3/3
+ b**2*x**5/5)*sin(c), True))
```

Giac [A]
time = 4.08, size = 99, normalized size = 0.72

$$-\frac{(b^2d^4x^4+2\,abd^4x^2+a^2d^4-12\,b^2d^2x^2-4\,abd^2+24\,b^2)\cos{(dx+c)}}{d^5}+\frac{4\,(b^2d^3x^3+abd^3x-6\,b^2dx)\sin{(dx+c)}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^4*x^4 + 2*a*b*d^4*x^2 + a^2*d^4 - 12*b^2*d^2*x^2 - 4*a*b*d^2 + 24*b
^2)*cos(d*x + c)/d^5 + 4*(b^2*d^3*x^3 + a*b*d^3*x - 6*b^2*d*x)*sin(d*x + c)
/d^5
```

Mupad [B]
time = 4.83, size = 118, normalized size = 0.86

$$\frac{4\,b^2\,x^3\,\sin\left(c+d\,x\right)}{d^2}-\frac{b^2\,x^4\,\cos\left(c+d\,x\right)}{d}-\frac{\cos\left(c+d\,x\right)\,\left(a^2\,d^4-4\,a\,b\,d^2+24\,b^2\right)}{d^5}-\frac{4\,x\,\sin\left(c+d\,x\right)\,\left(6\,b^2-a\,b\,d^2\right)}{d^4}+\frac{2\,x^2\,\cos\left(c+d\,x\right)\,\left(6\,b^2-a\,b\,d^2\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*x^2)^2,x)
```

```
[Out] (4*b^2*x^3*sin(c + d*x))/d^2 - (b^2*x^4*cos(c + d*x))/d - (cos(c + d*x)*(24
*b^2 + a^2*d^4 - 4*a*b*d^2))/d^5 - (4*x*sin(c + d*x)*(6*b^2 - a*b*d^2))/d^4
 + (2*x^2*cos(c + d*x)*(6*b^2 - a*b*d^2))/d^3
```

3.52 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x} dx$

Optimal. Leaf size=111

$$\frac{6b^2x\cos(c+dx)}{d^3}-\frac{2abx\cos(c+dx)}{d}-\frac{b^2x^3\cos(c+dx)}{d}+a^2\text{Ci}(dx)\sin(c)-\frac{6b^2\sin(c+dx)}{d^4}+\frac{2ab\sin(c+dx)}{d^2}+$$

```
[Out] 6*b^2*x*cos(d*x+c)/d^3-2*a*b*x*cos(d*x+c)/d-b^2*x^3*cos(d*x+c)/d+a^2*cos(c)
*Si(d*x)+a^2*Ci(d*x)*sin(c)-6*b^2*sin(d*x+c)/d^4+2*a*b*sin(d*x+c)/d^2+3*b^2
*x^2*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3420, 3384, 3380, 3383, 3377, 2717}

$$a^2\sin(c)\text{CosIntegral}(dx)+a^2\cos(c)\text{Si}(dx)+\frac{2ab\sin(c+dx)}{d^2}-\frac{2abx\cos(c+dx)}{d}-\frac{6b^2\sin(c+dx)}{d^4}+\frac{6b^2x\cos(c+dx)}{d^3}+\frac{3b^2x^2\sin(c+dx)}{d^2}-\frac{b^2x^3\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x,x]
```

```
[Out] (6*b^2*x*Cos[c + d*x])/d^3 - (2*a*b*x*Cos[c + d*x])/d - (b^2*x^3*Cos[c + d*
x])/d + a^2*CosIntegral[d*x]*Sin[c] - (6*b^2*Sin[c + d*x])/d^4 + (2*a*b*Sin
[c + d*x])/d^2 + (3*b^2*x^2*Sin[c + d*x])/d^2 + a^2*Cos[c]*SinIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x}\,dx = \int \left(\frac{a^2\sin(c+dx)}{x} + 2abx\sin(c+dx) + b^2x^3\sin(c+dx)\right)dx$$
$$= a^2\int \frac{\sin(c+dx)}{x}\,dx + (2ab)\int x\sin(c+dx)\,dx + b^2\int x^3\sin(c+dx)\,dx$$
$$= -\frac{2abx\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} + \frac{(2ab)\int\cos(c+dx)\,dx}{d} + \frac{(3b^2)\int x^2$$
$$= -\frac{2abx\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} + a^2\mathrm{Ci}(dx)\sin(c) + \frac{2ab\sin(c+dx)}{d^2} +$$
$$= \frac{6b^2x\cos(c+dx)}{d^3} - \frac{2abx\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} + a^2\mathrm{Ci}(dx)\sin(c) +$$
$$= \frac{6b^2x\cos(c+dx)}{d^3} - \frac{2abx\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} + a^2\mathrm{Ci}(dx)\sin(c) -$$

Mathematica [A]
time = 0.25, size = 82, normalized size = 0.74

$$-\frac{bx(2ad^2+b(-6+d^2x^2))\cos(c+dx)}{d^3} + a^2\mathrm{Ci}(dx)\sin(c) + \frac{b(2ad^2+3b(-2+d^2x^2))\sin(c+dx)}{d^4} + a^2\cos(c)\mathrm{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x,x]
```

```
[Out] -((b*x*(2*a*d^2 + b*(-6 + d^2*x^2))*Cos[c + d*x])/d^3) + a^2*CosIntegral[d*
x]*Sin[c] + (b*(2*a*d^2 + 3*b*(-2 + d^2*x^2))*Sin[c + d*x])/d^4 + a^2*Cos[c
]*SinIntegral[d*x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(111) = 222$.
time = 0.14, size = 236, normalized size = 2.13

method	result
derivativedivides	$a^2(\text{sinIntegral}(dx)\cos(c)+\text{cosineIntegral}(dx)\sin(c))+\frac{4abc\cos(dx+c)}{d^2}+\frac{2(c+1)ab(\sin(dx+c}{}$
default	$a^2(\text{sinIntegral}(dx)\cos(c)+\text{cosineIntegral}(dx)\sin(c))+\frac{4abc\cos(dx+c)}{d^2}+\frac{2(c+1)ab(\sin(dx+c}{}$
meijerg	$\frac{8b^2\sqrt{\pi}\,\sin(c)\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3d^2x^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}}-\frac{dx\left(-\frac{d^2x^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4}+\frac{8b^2\sqrt{\pi}\,\cos(c)\left(\frac{xd\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx}{20\sqrt{\pi}}\right.}{d^4}$
risch	$-\frac{e^{-ic}\pi\,\text{csgn}(dx)a^2}{2}+e^{-ic}\text{sinIntegral}(dx)\,a^2+\frac{ia^2e^{ic}\,\text{expIntegral}(1,-idx)}{2}-\frac{ie^{-ic}\,\text{expIntegral}(1,-idx)a}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+4/d^2*a*b*c*cos(d*x+c)+2*(c+1)/d^2*a*b*
(sin(d*x+c)-(d*x+c)*cos(d*x+c))+4/d^4*b^2*c^3*cos(d*x+c)+6*(c+1)/d^4*b^2*c^
2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-4*b^2*c*(c^2+c+1)/d^4*(-(d*x+c)^2*cos(d*x
+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+(c^3+c^2+c+1)/d^4*b^2*(-(d*x+c)^3*co
s(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))
```

Maxima [C] Result contains complex when optimal does not.
time = 1.73, size = 116, normalized size = 1.05

$$\frac{(a^2(-i\,\text{Ei}(i\,dx)+i\,\text{Ei}(-i\,dx))\cos(c)+a^2(\text{Ei}(i\,dx)+\text{Ei}(-i\,dx))\sin(c))d^4-2\,(b^2d^3x^3+2\,(abd^3-3\,b^2d)x)\cos(dx+c)+2\,(3\,b^2d^2x^2+2\,abd^2-6\,b^2)\sin(dx+c)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a^2*(Ei(I*d*x) + Ei(-I*d*x
))*sin(c))*d^4 - 2*(b^2*d^3*x^3 + 2*(a*b*d^3 - 3*b^2*d)*x)*cos(d*x + c) + 2
*(3*b^2*d^2*x^2 + 2*a*b*d^2 - 6*b^2)*sin(d*x + c))/d^4
```

Fricas [A]
time = 0.37, size = 114, normalized size = 1.03

$$\frac{2\,a^2d^4\cos(c)\,\text{Si}(dx)-2\,(b^2d^3x^3+2\,(abd^3-3\,b^2d)x)\cos(dx+c)+2\,(3\,b^2d^2x^2+2\,abd^2-6\,b^2)\sin(dx+c)+(a^2d^4\,\text{Ci}(dx)+a^2d^4\,\text{Ci}(-dx))\sin(c)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*d^4*cos(c)*sin_integral(d*x) - 2*(b^2*d^3*x^3 + 2*(a*b*d^3 - 3*b
^2*d)*x)*cos(d*x + c) + 2*(3*b^2*d^2*x^2 + 2*a*b*d^2 - 6*b^2)*sin(d*x + c)
+ (a^2*d^4*cos_integral(d*x) + a^2*d^4*cos_integral(-d*x))*sin(c))/d^4
```

Sympy [A]
time = 4.35, size = 160, normalized size = 1.44

$$a^2 \sin(c) \operatorname{Ci}(dx) + a^2 \cos(c) \operatorname{Si}(dx) + 2abx \left(\begin{cases} x \sin(c) & \text{for } d = 0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise} \end{cases} \right) - 2ab \left(\begin{cases} \frac{x^2 \sin(c)}{2} & \text{for } d = 0 \\ -\frac{\begin{cases} \frac{\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x \cos(c) & \text{otherwise} \end{cases}}{d} & \text{otherwise} \end{cases} \right) + b^2 x^3 \left(\begin{cases} x \sin(c) & \text{for } d = 0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise} \end{cases} \right) - 3b^2 \left(\begin{cases} \frac{x^4 \sin(c)}{4} & \text{for } d = 0 \\ -\frac{\begin{cases} \frac{x^2 \sin(c+dx)}{d} + \frac{2x \cos(c+dx)}{d^2} - \frac{2 \sin(c+dx)}{d^3} & \text{for } d \neq 0 \\ \frac{x^3 \cos(c)}{3} & \text{otherwise} \end{cases}}{d} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*sin(d*x+c)/x,x)
```

```
[Out] a**2*sin(c)*Ci(d*x) + a**2*cos(c)*Si(d*x) + 2*a*b*x*Piecewise((x*sin(c), Eq
(d, 0)), (-cos(c + d*x)/d, True)) - 2*a*b*Piecewise((x**2*sin(c)/2, Eq(d, 0
)), (-Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))/d, True)) + b
**2*x**3*Piecewise((x*sin(c), Eq(d, 0)), (-cos(c + d*x)/d, True)) - 3*b**2*
Piecewise((x**4*sin(c)/4, Eq(d, 0)), (-Piecewise((x**2*sin(c + d*x)/d + 2*x
*cos(c + d*x)/d**2 - 2*sin(c + d*x)/d**3, Ne(d, 0)), (x**3*cos(c)/3, True))
/d, True))
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.53, size = 725, normalized size = 6.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*d^3*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - a^2*d^4*imag_part(
cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + a^2*d^4*imag_part(
cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^2*d^4*sin_int
egral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 2*b^2*d^3*x^3*tan(1/2*d*x
+ 1/2*c)^2 + 2*a^2*d^4*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*
tan(1/2*c) + 2*a^2*d^4*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*c) - 2*b^2*d^3*x^3*tan(1/2*c)^2 + 4*a*b*d^3*x*tan(1/2*d*x + 1/2*c)
^2*tan(1/2*c)^2 + a^2*d^4*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)
^2 - a^2*d^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2 + 2*a^2*d
^4*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2 - a^2*d^4*imag_part(cos_integra
l(d*x))*tan(1/2*c)^2 + a^2*d^4*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 -
 2*a^2*d^4*sin_integral(d*x)*tan(1/2*c)^2 + 12*b^2*d^2*x^2*tan(1/2*d*x + 1/
2*c)*tan(1/2*c)^2 - 2*b^2*d^3*x^3 + 4*a*b*d^3*x*tan(1/2*d*x + 1/2*c)^2 + 2*
a^2*d^4*real_part(cos_integral(d*x))*tan(1/2*c) + 2*a^2*d^4*real_part(cos_i
ntegral(-d*x))*tan(1/2*c) - 4*a*b*d^3*x*tan(1/2*c)^2 - 12*b^2*d*x*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*c)^2 + a^2*d^4*imag_part(cos_integral(d*x)) - a^2*d^4
*imag_part(cos_integral(-d*x)) + 2*a^2*d^4*sin_integral(d*x) + 12*b^2*d^2*x
^2*tan(1/2*d*x + 1/2*c) + 8*a*b*d^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 - 4*a
*b*d^3*x - 12*b^2*d*x*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*d*x*tan(1/2*c)^2 + 8*
a*b*d^2*tan(1/2*d*x + 1/2*c) - 24*b^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 + 1
```

```
2*b^2*d*x - 24*b^2*tan(1/2*d*x + 1/2*c))/(d^4*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*c)^2 + d^4*tan(1/2*d*x + 1/2*c)^2 + d^4*tan(1/2*c)^2 + d^4)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)\,{\left(b\,x^2+a\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^2)^2)/x,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^2)^2)/x, x)
```

3.53 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2} \, dx$

Optimal. Leaf size=97

$$\frac{2b^2\cos(c+dx)}{d^3}-\frac{2ab\cos(c+dx)}{d}-\frac{b^2x^2\cos(c+dx)}{d}+a^2d\cos(c)\text{Ci}(dx)-\frac{a^2\sin(c+dx)}{x}+\frac{2b^2x\sin(c+dx)}{d^2}-a^2$$

```
[Out] a^2*d*Ci(d*x)*cos(c)+2*b^2*cos(d*x+c)/d^3-2*a*b*cos(d*x+c)/d-b^2*x^2*cos(d*
x+c)/d-a^2*d*Si(d*x)*sin(c)-a^2*sin(d*x+c)/x+2*b^2*x*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3420, 2718, 3378, 3384, 3380, 3383, 3377}

$$a^2d\cos(c)\text{CosIntegral}(dx)-a^2d\sin(c)\text{Si}(dx)-\frac{a^2\sin(c+dx)}{x}-\frac{2ab\cos(c+dx)}{d}+\frac{2b^2\cos(c+dx)}{d^3}+\frac{2b^2x\sin(c+dx)}{d^2}-\frac{b^2x^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] (2*b^2*Cos[c + d*x])/d^3 - (2*a*b*Cos[c + d*x])/d - (b^2*x^2*Cos[c + d*x])/
d + a^2*d*Cos[c]*CosIntegral[d*x] - (a^2*Sin[c + d*x])/x + (2*b^2*x*Sin[c +
 d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2\sin(c+dx)}{x^2}\,dx &= \int \left(2ab\sin(c+dx)+\frac{a^2\sin(c+dx)}{x^2}+b^2x^2\sin(c+dx)\right)dx \\
&= a^2\int \frac{\sin(c+dx)}{x^2}\,dx+(2ab)\int \sin(c+dx)\,dx+b^2\int x^2\sin(c+dx)\,dx \\
&= -\frac{2ab\cos(c+dx)}{d}-\frac{b^2x^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{x}+\frac{(2b^2)\int x\cos(c+d
\end{aligned}$$

$$= -\frac{2ab\cos(c+dx)}{d}-\frac{b^2x^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{x}+\frac{2b^2x\sin(c+dx)}{d^2}-$$

$$= \frac{2b^2\cos(c+dx)}{d^3}-\frac{2ab\cos(c+dx)}{d}-\frac{b^2x^2\cos(c+dx)}{d}+a^2d\cos(c)\text{Ci}(dx)-$$

Mathematica [A]
time = 0.17, size = 97, normalized size = 1.00

$$\frac{2b^2\cos(c+dx)}{d^3}-\frac{2ab\cos(c+dx)}{d}-\frac{b^2x^2\cos(c+dx)}{d}+a^2d\cos(c)\text{Ci}(dx)-\frac{a^2\sin(c+dx)}{x}+\frac{2b^2x\sin(c+dx)}{d^2}-a^2d\sin(c)\text{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^2,x]

[Out] (2*b^2*Cos[c + d*x])/d^3 - (2*a*b*Cos[c + d*x])/d - (b^2*x^2*Cos[c + d*x])/
d + a^2*d*Cos[c]*CosIntegral[d*x] - (a^2*Sin[c + d*x])/x + (2*b^2*x*Sin[c +
 d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Maple [A]
time = 0.15, size = 156, normalized size = 1.61

method	result
derivativedivides	$d\left(a^2\left(-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}\left(dx\right)\sin\left(c\right)+\text{cosineIntegral}\left(dx\right)\cos\left(c\right)\right)-\frac{2ab\cos(dx+c)}{d^2}-\right.$
default	$d\left(a^2\left(-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}\left(dx\right)\sin\left(c\right)+\text{cosineIntegral}\left(dx\right)\cos\left(c\right)\right)-\frac{2ab\cos(dx+c)}{d^2}-\right.$
risch	$-\frac{d\cos(c)a^2\,\text{expIntegral}(1,idx)}{2}-\frac{d\cos(c)a^2\,\text{expIntegral}(1,-idx)}{2}+\frac{id\sin(c)a^2\,\text{expIntegral}(1,idx)}{2}-\frac{id\sin(c)a^2\,\text{expI}}{}$
meijerg	$\frac{4b^2\sqrt{\pi}\,\sin(c)\left(\frac{x\left(d^2\right)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\,d^2}-\frac{\left(d^2\right)^{\frac{3}{2}}\left(-\frac{3d^2x^2}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\,d^3}\right)}{d^2\sqrt{d^2}}+\frac{4b^2\sqrt{\pi}\,\cos(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{d^2x^2}{2}+1\right)\cos(dx)}{2\sqrt{\pi}}+\frac{d}{}\right.}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(a^2*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))-2/d^2*a*b*cos(d*x+c)
-6/d^4*b^2*c^2*cos(d*x+c)-4*c*b^2*(2*c+1)/d^4*(sin(d*x+c)-(d*x+c)*cos(d*x+c
))+(3*c^2+2*c+1)/d^4*b^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(
d*x+c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 1.44, size = 97, normalized size = 1.00

$$\frac{(a^2(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos(c)+a^2(-i\,\Gamma(-1,i\,dx)+i\,\Gamma(-1,-i\,dx))\sin(c))d^4+4\,b^2dx\sin(dx+c)-2\,(b^2d^2x^2+2\,abd^2-2\,b^2)\cos(dx+c)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) + a^2*(-I*gamma(-1,
 I*d*x) + I*gamma(-1, -I*d*x))*sin(c))*d^4 + 4*b^2*d*x*sin(d*x + c) - 2*(b^
2*d^2*x^2 + 2*a*b*d^2 - 2*b^2)*cos(d*x + c))/d^3
```

Fricas [A]
time = 0.35, size = 113, normalized size = 1.16

$$-\frac{2\,a^2d^4x\sin(c)\,\text{Si}\,(dx)+2\,(b^2d^2x^3+2\,(abd^2-b^2)x)\cos(dx+c)-(a^2d^4x\,\text{Ci}\,(dx)+a^2d^4x\,\text{Ci}\,(-dx))\cos(c)+2\,(a^2d^3-2\,b^2dx^2)\sin(dx+c)}{2\,d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d^4*x*sin(c)*sin_integral(d*x) + 2*(b^2*d^2*x^3 + 2*(a*b*d^2 -
b^2)*x)*cos(d*x + c) - (a^2*d^4*x*cos_integral(d*x) + a^2*d^4*x*cos_integra
l(-d*x))*cos(c) + 2*(a^2*d^3 - 2*b^2*d*x^2)*sin(d*x + c))/(d^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**2)**2*sin(c + d*x)/x**2, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.56, size = 1638, normalized size = 16.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*d^4*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2
*d*x)^2*tan(1/2*c)^2 + a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x
+ 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^4*x*imag_part(cos_integral
(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x*imag_
part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) +
 4*a^2*d^4*x*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/
2*c) - 2*b^2*d^2*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - a
^2*d^4*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2
 - a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d
*x)^2 + a^2*d^4*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1
/2*c)^2 + a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*c)^2 + a^2*d^4*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*
c)^2 + a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*b^2*d^2*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2 + 2*a^2*d^4*x*imag_pa
rt(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*a^2*d^4*x*imag_
part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) + 4*a^2*d^4*x*si
n_integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) + 2*a^2*d^4*x*imag_part(c
os_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x*imag_part(cos_int
egral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^4*x*sin_integral(d*x)*tan(
1/2*d*x)^2*tan(1/2*c) - 2*b^2*d^2*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 +
 2*b^2*d^2*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*d^2*x*tan(1/2*d*x + 1/2*
c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x*real_part(cos_integral(d*x))*t
an(1/2*d*x + 1/2*c)^2 - a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x
 + 1/2*c)^2 - a^2*d^4*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 - a^2*d
^4*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 4*a^2*d^3*tan(1/2*d*x +
 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^4*x*real_part(cos_integral(d*x)
```

```
)*tan(1/2*c)^2 + a^2*d^4*x*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*a
^2*d^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)*tan(1/2*c)^2 - 8*b^2*d*x^2*tan(1
/2*d*x + 1/2*c)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*d^2*x^3*tan(1/2*d*x + 1
/2*c)^2 + 2*b^2*d^2*x^3*tan(1/2*d*x)^2 - 4*a*b*d^2*x*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x)^2 + 2*a^2*d^4*x*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*a
^2*d^4*x*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a^2*d^4*x*sin_integra
l(d*x)*tan(1/2*c) + 2*b^2*d^2*x^3*tan(1/2*c)^2 - 4*a*b*d^2*x*tan(1/2*d*x +
1/2*c)^2*tan(1/2*c)^2 + 4*a*b*d^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*b^2*x*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x*real_part(cos
_integral(d*x)) - a^2*d^4*x*real_part(cos_integral(-d*x)) + 4*a^2*d^3*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x) - 8*b^2*d*x^2*tan(1/2*d*x + 1/2*c)*tan(1/2*d
*x)^2 + 4*a^2*d^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 4*a^2*d^3*tan(1/2*d*x
)^2*tan(1/2*c) - 8*b^2*d*x^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 - 4*a^2*d^3*
tan(1/2*d*x)*tan(1/2*c)^2 + 2*b^2*d^2*x^3 - 4*a*b*d^2*x*tan(1/2*d*x + 1/2*c
)^2 + 4*a*b*d^2*x*tan(1/2*d*x)^2 + 4*b^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d
*x)^2 + 4*a*b*d^2*x*tan(1/2*c)^2 + 4*b^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c
)^2 - 4*b^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 8*b^2*d*x^2*tan(1/2*d*x + 1/2*c
) + 4*a^2*d^3*tan(1/2*d*x) + 4*a^2*d^3*tan(1/2*c) + 4*a*b*d^2*x + 4*b^2*x*t
an(1/2*d*x + 1/2*c)^2 - 4*b^2*x*tan(1/2*d*x)^2 - 4*b^2*x*tan(1/2*c)^2 - 4*b
^2*x)/(d^3*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^3*x*tan
(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2 + d^3*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c
)^2 + d^3*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^3*x*tan(1/2*d*x + 1/2*c)^2 + d^
3*x*tan(1/2*d*x)^2 + d^3*x*tan(1/2*c)^2 + d^3*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)\,{\left(b\,x^2+a\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^2)^2)/x^2,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^2)^2)/x^2, x)
```

3.54 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3} \, dx$

Optimal. Leaf size=114

$$-\frac{a^2 d\cos(c+dx)}{2x}-\frac{b^2 x\cos(c+dx)}{d}+2ab\text{Ci}(dx)\sin(c)-\frac{1}{2}a^2d^2\text{Ci}(dx)\sin(c)+\frac{b^2\sin(c+dx)}{d^2}-\frac{a^2\sin(c+dx)}{2x^2}+$$

```
[Out] -1/2*a^2*d*cos(d*x+c)/x-b^2*x*cos(d*x+c)/d+2*a*b*cos(c)*Si(d*x)-1/2*a^2*d^2
*cos(c)*Si(d*x)+2*a*b*Ci(d*x)*sin(c)-1/2*a^2*d^2*Ci(d*x)*sin(c)+b^2*sin(d*x
+c)/d^2-1/2*a^2*sin(d*x+c)/x^2
```

Rubi [A]

time = 0.14, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3420, 3378, 3384, 3380, 3383, 3377, 2717}

$$-\frac{1}{2}a^2d^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)-\frac{a^2\sin(c+dx)}{2x^2}-\frac{a^2d\cos(c+dx)}{2x}+2ab\sin(c)\text{CosIntegral}(dx)+2ab\cos(c)\text{Si}(dx)+\frac{b^2\sin(c+dx)}{d^2}-\frac{b^2x\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] -1/2*(a^2*d*Cos[c + d*x])/x - (b^2*x*Cos[c + d*x])/d + 2*a*b*CosIntegral[d*
x]*Sin[c] - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 + (b^2*Sin[c + d*x])/d^2 -
(a^2*Sin[c + d*x])/(2*x^2) + 2*a*b*Cos[c]*SinIntegral[d*x] - (a^2*d^2*Cos[c
]*SinIntegral[d*x])/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)^2\sin(c+dx)}{x^3}\,dx &= \int \left(\frac{a^2\sin(c+dx)}{x^3}+\frac{2ab\sin(c+dx)}{x}+b^2x\sin(c+dx)\right)dx\\
&= a^2\int\frac{\sin(c+dx)}{x^3}\,dx+(2ab)\int\frac{\sin(c+dx)}{x}\,dx+b^2\int x\sin(c+dx)\,dx\\
&= -\frac{b^2x\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{2x^2}+\frac{b^2\int\cos(c+dx)\,dx}{d}+\frac{1}{2}(a^2d)\int\frac{\cos(c+\ldots}{x^2\ldots}\\
&= -\frac{a^2d\cos(c+dx)}{2x}-\frac{b^2x\cos(c+dx)}{d}+2ab\text{Ci}(dx)\sin(c)+\frac{b^2\sin(c+dx)}{d^2}-\frac{a^2\ldots}{\ldots}\\
&= -\frac{a^2d\cos(c+dx)}{2x}-\frac{b^2x\cos(c+dx)}{d}+2ab\text{Ci}(dx)\sin(c)+\frac{b^2\sin(c+dx)}{d^2}-\frac{a^2\ldots}{\ldots}\\
&= -\frac{a^2d\cos(c+dx)}{2x}-\frac{b^2x\cos(c+dx)}{d}+2ab\text{Ci}(dx)\sin(c)-\frac{1}{2}a^2d^2\text{Ci}(dx)\sin(c)\ldots
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 99, normalized size = 0.87

$$\frac{1}{2}\left(-\frac{a^2d\cos(c+dx)}{x}-\frac{2b^2x\cos(c+dx)}{d}+a\left(4b-ad^2\right)\text{Ci}(dx)\sin(c)+\frac{2b^2\sin(c+dx)}{d^2}-\frac{a^2\sin(c+dx)}{x^2}+a\left(4b-ad^2\right)\cos(c)\text{Si}(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] (-((a^2*d*Cos[c + d*x])/x) - (2*b^2*x*Cos[c + d*x])/d + a*(4*b - a*d^2)*Cos
Integral[d*x]*Sin[c] + (2*b^2*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/x^2 +
a*(4*b - a*d^2)*Cos[c]*SinIntegral[d*x])/2
```

Maple [A]
time = 0.15, size = 124, normalized size = 1.09

method	result
derivativedivides	$d^2\left(a^2\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\text{sinIntegral}(dx)\cos(c)}{2}-\frac{\text{cosineIntegral}(dx)\sin(c)}{2}\right)+\frac{2ab(\text{sinIntegral}(dx\right.$
default	$d^2\left(a^2\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\text{sinIntegral}(dx)\cos(c)}{2}-\frac{\text{cosineIntegral}(dx)\sin(c)}{2}\right)+\frac{2ab(\text{sinIntegral}(dx\right.$
risch	$\frac{i\cos(c)\,\text{expIntegral}(1,idx)a^2d^2}{4}-\frac{i\cos(c)\,\text{expIntegral}(1,-idx)a^2d^2}{4}-i\cos(c)\,\text{expIntegral}(1,idx)\,ab+$
meijerg	$\frac{2b^2\sqrt{\pi}\,\sin(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b^2\sqrt{\pi}\,\cos(c)\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+ab\sqrt{\pi}\,\sin(c)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(a^2*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2
*Ci(d*x)*sin(c))+2/d^2*a*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+4/d^4*b^2*c*cos(
d*x+c)+(3*c+1)/d^4*b^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 2.38, size = 150, normalized size = 1.32

$$\frac{\left(\left(a^2(i\,\Gamma(-2,i\,dx)-i\,\Gamma(-2,-i\,dx))\cos(c)+a^2(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin(c)\right)d^4-4\left(ab(i\,\Gamma(-2,i\,dx)-i\,\Gamma(-2,-i\,dx))\cos(c)+ab(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin(c)\right)d^2\right)x^2-2\left(b^2dx^3+2\,abdx\right)\cos(dx+c)+2\left(b^2x^2-2\,ab\right)\sin(dx+c)}{2\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(((a^2*(I*gamma(-2, I*d*x) - I*gamma(-2, -I*d*x))*cos(c) + a^2*(gamma(-
2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^4 - 4*(a*b*(I*gamma(-2, I*d*x) - I
*gamma(-2, -I*d*x))*cos(c) + a*b*(gamma(-2, I*d*x) + gamma(-2, -I*d*x))*sin
(c))*d^2)*x^2 - 2*(b^2*d*x^3 + 2*a*b*d*x)*cos(d*x + c) + 2*(b^2*x^2 - 2*a*b
)*sin(d*x + c))/(d^2*x^2)
```

Fricas [A]
time = 0.37, size = 136, normalized size = 1.19

$$-\frac{2\left(a^2d^4-4\,abd^2\right)x^2\cos(c)\,\mathrm{Si}(dx)+2\left(a^2d^3x+2\,b^2dx^3\right)\cos(dx+c)+2\left(a^2d^2-2\,b^2x^2\right)\sin(dx+c)+\left(\left(a^2d^4-4\,abd^2\right)x^2\,\mathrm{Ci}(dx)+\left(a^2d^4-4\,abd^2\right)x^2\,\mathrm{Ci}(-dx)\right)\sin(c)}{4\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^3,x, algorithm="fricas")

[Out] -1/4*(2*(a^2*d^4 - 4*a*b*d^2)*x^2*cos(c)*sin_integral(d*x) + 2*(a^2*d^3*x + 2*b^2*d*x^3)*cos(d*x + c) + 2*(a^2*d^2 - 2*b^2*x^2)*sin(d*x + c) + ((a^2*d^4 - 4*a*b*d^2)*x^2*cos_integral(d*x) + (a^2*d^4 - 4*a*b*d^2)*x^2*cos_integral(-d*x))*sin(c))/(d^2*x^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**2*sin(d*x+c)/x**3,x)

[Out] Integral((a + b*x**2)**2*sin(c + d*x)/x**3, x)

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.14, size = 1058, normalized size = 9.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^3,x, algorithm="giac")

[Out] 1/4*(a^2*d^4*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^4*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^4*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^4*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^4*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a^2*d^4*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + a^2*d^4*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^4*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^4*x^2*sin_integral(d*x)*tan(1/2*c)^2 - 4*a*b*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 8*a*b*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^4*x^2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^4*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a*b*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*b*d^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^3*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^2*d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x^2*imag_part(cos_integral(d*x)) + a^2*d^4*x^2*imag_part(cos_integral(-d*x)) - 2*a^2*d^4*x^2*sin_integral(d*x) + 4

```
*a*b*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 4*a*b*d^2*x^2*im
ag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 8*a*b*d^2*x^2*sin_integral(d*x
)*tan(1/2*d*x)^2 - 4*a*b*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2
+ 4*a*b*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 8*a*b*d^2*x^2*
sin_integral(d*x)*tan(1/2*c)^2 + 2*a^2*d^3*x*tan(1/2*d*x)^2 + 4*b^2*d*x^3*t
an(1/2*d*x)^2 + 8*a*b*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*c) + 8*a
*b*d^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a^2*d^3*x*tan(1/2*d
*x)*tan(1/2*c) + 16*b^2*d*x^3*tan(1/2*d*x)*tan(1/2*c) + 2*a^2*d^3*x*tan(1/2
*c)^2 + 4*b^2*d*x^3*tan(1/2*c)^2 + 4*a*b*d^2*x^2*imag_part(cos_integral(d*x
)) - 4*a*b*d^2*x^2*imag_part(cos_integral(-d*x)) + 8*a*b*d^2*x^2*sin_integr
al(d*x) + 4*a^2*d^2*tan(1/2*d*x)^2*tan(1/2*c) - 8*b^2*x^2*tan(1/2*d*x)^2*ta
n(1/2*c) + 4*a^2*d^2*tan(1/2*d*x)*tan(1/2*c)^2 - 8*b^2*x^2*tan(1/2*d*x)*tan
(1/2*c)^2 - 2*a^2*d^3*x - 4*b^2*d*x^3 - 4*a^2*d^2*tan(1/2*d*x) + 8*b^2*x^2*
tan(1/2*d*x) - 4*a^2*d^2*tan(1/2*c) + 8*b^2*x^2*tan(1/2*c))/(d^2*x^2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + d^2*x^2*tan(1/2*d*x)^2 + d^2*x^2*tan(1/2*c)^2 + d^2
*x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (c+d\,x)\,\left(b\,x^2+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^2)^2)/x^3,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^2)^2)/x^3, x)
```

3.55 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4} \, dx$

Optimal. Leaf size=134

$$-\frac{b^2 \cos(c+dx)}{d}-\frac{a^2 d \cos(c+dx)}{6x^2}+2abd\cos(c)\text{Ci}(dx)-\frac{1}{6}a^2d^3\cos(c)\text{Ci}(dx)-\frac{a^2\sin(c+dx)}{3x^3}-\frac{2ab\sin(c+dx)}{x}-$$

```
[Out] 2*a*b*d*Ci(d*x)*cos(c)-1/6*a^2*d^3*Ci(d*x)*cos(c)-b^2*cos(d*x+c)/d-1/6*a^2*
d*cos(d*x+c)/x^2-2*a*b*d*Si(d*x)*sin(c)+1/6*a^2*d^3*Si(d*x)*sin(c)-1/3*a^2*
sin(d*x+c)/x^3-2*a*b*sin(d*x+c)/x+1/6*a^2*d^2*sin(d*x+c)/x
```

Rubi [A]
time = 0.16, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3420, 2718, 3378, 3384, 3380, 3383}

$$-\frac{1}{6}a^2d^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{6x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{a^2d\cos(c+dx)}{6x^2}+2abd\cos(c)\text{CosIntegral}(dx)-2abd\sin(c)\text{Si}(dx)-\frac{2ab\sin(c+dx)}{x}-\frac{b^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] -((b^2*Cos[c + d*x])/d) - (a^2*d*Cos[c + d*x])/(6*x^2) + 2*a*b*d*Cos[c]*Cos
Integral[d*x] - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 - (a^2*Sin[c + d*x])/(3
*x^3) - (2*a*b*Sin[c + d*x])/x + (a^2*d^2*Sin[c + d*x])/(6*x) - 2*a*b*d*Sin
[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)^2\sin(c+dx)}{x^4}\,dx &= \int \left(b^2\sin(c+dx)+\frac{a^2\sin(c+dx)}{x^4}+\frac{2ab\sin(c+dx)}{x^2}\right)dx \\
&= a^2\int \frac{\sin(c+dx)}{x^4}\,dx+(2ab)\int \frac{\sin(c+dx)}{x^2}\,dx+b^2\int \sin(c+dx)\,dx \\
&= -\frac{b^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{2ab\sin(c+dx)}{x}+\frac{1}{3}\left(a^2d\right)\int \frac{\cos(c+dx}{x^3} \\
&= -\frac{b^2\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{6x^2}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{2ab\sin(c+dx)}{x}-\frac{1}{6}(\\
&= -\frac{b^2\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{6x^2}+2abd\cos(c)\text{Ci}(dx)-\frac{a^2\sin(c+dx)}{3x^3}- \\
&= -\frac{b^2\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{6x^2}+2abd\cos(c)\text{Ci}(dx)-\frac{a^2\sin(c+dx)}{3x^3}- \\
&= -\frac{b^2\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{6x^2}+2abd\cos(c)\text{Ci}(dx)-\frac{1}{6}a^2d^3\cos(c)\text{Ci}(d
\end{aligned}
$$

Mathematica [A]
time = 0.27, size = 114, normalized size = 0.85

$$\frac{1}{6}\left(-\frac{6b^2\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{x^2}-ad\left(-12b+ad^2\right)\cos(c)\text{Ci}(dx)-\frac{2a^2\sin(c+dx)}{x^3}-\frac{12ab\sin(c+dx)}{x}+\frac{a^2d^2\sin(c+dx)}{x}+ad\left(-12b+ad^2\right)\sin(c)\text{Si}(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] ((-6*b^2*Cos[c + d*x])/d - (a^2*d*Cos[c + d*x])/x^2 - a*d*(-12*b + a*d^2)*C
os[c]*CosIntegral[d*x] - (2*a^2*Sin[c + d*x])/x^3 - (12*a*b*Sin[c + d*x])/x
```

```
 + (a^2*d^2*Sin[c + d*x])/x + a*d*(-12*b + a*d^2)*Sin[c]*SinIntegral[d*x])/
6
```

Maple [A]
time = 0.16, size = 120, normalized size = 0.90

method	result
derivativedivides	$d^3\left(a^2\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{sinIntegral}(dx)\sin(c)}{6}-\frac{\mathrm{cosineIntegral}(dx)\cos(c)}{6}\right)+\frac{2ab\left(-\right.}{}\right.$
default	$d^3\left(a^2\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{sinIntegral}(dx)\sin(c)}{6}-\frac{\mathrm{cosineIntegral}(dx)\cos(c)}{6}\right)+\frac{2ab\left(-\right.}{}\right.$
risch	$\frac{\mathrm{expIntegral}(1,-idx)\cos(c)a^2d^3}{12}+\frac{\mathrm{expIntegral}(1,idx)\cos(c)a^2d^3}{12}-\cos(c)\,\mathrm{expIntegral}(1,-idx)\,abd-\mathrm{co}$
meijerg	$\frac{b^2\sin(c)\sin(dx)}{d}+\frac{b^2\sqrt{\pi}\,\cos(c)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}+\frac{d^2ab\sqrt{\pi}\,\sin(c)\left(-\frac{4d^2\cos\left(x\sqrt{d^2}\right)}{x\left(d^2\right)^{\frac{3}{2}}\sqrt{\pi}}-\frac{4\,\mathrm{sinIntegral}\left(x\sqrt{d^2}\right.}{\sqrt{\pi}}\right.}{2\sqrt{d^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(a^2*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x
+1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+2/d^2*a*b*(-sin(d*x+c)/d/x-Si(d*x)*
sin(c)+Ci(d*x)*cos(c))-1/d^4*b^2*cos(d*x+c))
```

Maxima [C] Result contains complex when optimal does not.
time = 1.82, size = 140, normalized size = 1.04

$$-\frac{\left(\left(a^2(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos(c)+a^2(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin(c)\right)d^5-12\left(ab(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos(c)+ab(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin(c)\right)d^3\right)x^3+8\,ab\sin(dx+c)+2\left(b^2dx^3+2\,abdx\right)\cos(dx+c)}{2\,d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a^2*(-I*gamma(-
3, I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^5 - 12*(a*b*(gamma(-3, I*d*x) +
gamma(-3, -I*d*x))*cos(c) + a*b*(-I*gamma(-3, I*d*x) + I*gamma(-3, -I*d*x))
*sin(c))*d^3)*x^3 + 8*a*b*sin(d*x + c) + 2*(b^2*d*x^3 + 2*a*b*d*x)*cos(d*x
+ c))/(d^2*x^3)
```

Fricas [A]
time = 0.36, size = 145, normalized size = 1.08

$$\frac{2\left(a^2d^4-12\,abd^2\right)x^3\sin(c)\,\mathrm{Si}(dx)-2\left(a^2d^2x+6\,b^2x^3\right)\cos(dx+c)-\left(\left(a^2d^4-12\,abd^2\right)x^3\,\mathrm{Ci}(dx)+\left(a^2d^4-12\,abd^2\right)x^3\,\mathrm{Ci}(-dx)\right)\cos(c)-2\left(2\,a^2d-\left(a^2d^3-12\,abd\right)x^2\right)\sin(dx+c)}{12\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(a^2*d^4 - 12*a*b*d^2)*x^3*sin(c)*sin_integral(d*x) - 2*(a^2*d^2*x
+ 6*b^2*x^3)*cos(d*x + c) - ((a^2*d^4 - 12*a*b*d^2)*x^3*cos_integral(d*x) +
 (a^2*d^4 - 12*a*b*d^2)*x^3*cos_integral(-d*x))*cos(c) - 2*(2*a^2*d - (a^2*
d^3 - 12*a*b*d)*x^2)*sin(d*x + c))/(d*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{2}\right)^{2}\sin\left(c+dx\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**2)**2*sin(c + d*x)/x**4, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.82, size = 1032, normalized size = 7.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a^2*d^4*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ a^2*d^4*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*a^2*d^4*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2
*d^4*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^
4*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^4*x^3*real_part(c
os_integral(d*x))*tan(1/2*d*x)^2 - a^2*d^4*x^3*real_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2 + a^2*d^4*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 +
a^2*d^4*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 12*a*b*d^2*x^3*rea
l_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*a*b*d^2*x^3*real
_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^4*x^3*imag_
part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^4*x^3*imag_part(cos_integral(-
d*x))*tan(1/2*c) + 4*a^2*d^4*x^3*sin_integral(d*x)*tan(1/2*c) - 24*a*b*d^2*
x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 24*a*b*d^2*x^3
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 48*a*b*d^2*x^3*s
in_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^4*x^3*real_part(cos_inte
gral(d*x)) - a^2*d^4*x^3*real_part(cos_integral(-d*x)) + 12*a*b*d^2*x^3*rea
l_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 12*a*b*d^2*x^3*real_part(cos_int
egral(-d*x))*tan(1/2*d*x)^2 - 4*a^2*d^3*x^2*tan(1/2*d*x)^2*tan(1/2*c) - 12*
a*b*d^2*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 - 12*a*b*d^2*x^3*real
_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*a^2*d^3*x^2*tan(1/2*d*x)*tan(1/2
```

```
*c)^2 - 24*a*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*c) + 24*a*b*d^2
*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) - 48*a*b*d^2*x^3*sin_integral
(d*x)*tan(1/2*c) - 2*a^2*d^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*b^2*x^3*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 12*a*b*d^2*x^3*real_part(cos_integral(d*x)) + 12
*a*b*d^2*x^3*real_part(cos_integral(-d*x)) + 4*a^2*d^3*x^2*tan(1/2*d*x) + 4
*a^2*d^3*x^2*tan(1/2*c) + 48*a*b*d*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 48*a*b*d
*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a^2*d^2*x*tan(1/2*d*x)^2 + 12*b^2*x^3*ta
n(1/2*d*x)^2 + 8*a^2*d^2*x*tan(1/2*d*x)*tan(1/2*c) + 48*b^2*x^3*tan(1/2*d*x
)*tan(1/2*c) + 2*a^2*d^2*x*tan(1/2*c)^2 + 12*b^2*x^3*tan(1/2*c)^2 - 48*a*b*
d*x^2*tan(1/2*d*x) - 48*a*b*d*x^2*tan(1/2*c) + 8*a^2*d*tan(1/2*d*x)^2*tan(1
/2*c) + 8*a^2*d*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a^2*d^2*x - 12*b^2*x^3 - 8*a^
2*d*tan(1/2*d*x) - 8*a^2*d*tan(1/2*c))/(d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 d*x^3*tan(1/2*d*x)^2 + d*x^3*tan(1/2*c)^2 + d*x^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac {\sin \left (c+d\,x\right)\,{\left (b\,x^2+a\right)}^2}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^2)^2)/x^4,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^2)^2)/x^4, x)
```

3.56 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^5} \, dx$

Optimal. Leaf size=177

$$-\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{x}+\frac{a^2d^3\cos(c+dx)}{24x}+b^2\text{Ci}(dx)\sin(c)-abd^2\text{Ci}(dx)\sin(c)+\frac{1}{24}a^2d^4\text{Ci}(dx)\text{si}$$

```
[Out] -1/12*a^2*d*cos(d*x+c)/x^3-a*b*d*cos(d*x+c)/x+1/24*a^2*d^3*cos(d*x+c)/x+b^2
*cos(c)*Si(d*x)-a*b*d^2*cos(c)*Si(d*x)+1/24*a^2*d^4*cos(c)*Si(d*x)+b^2*Ci(d
*x)*sin(c)-a*b*d^2*Ci(d*x)*sin(c)+1/24*a^2*d^4*Ci(d*x)*sin(c)-1/4*a^2*sin(d
*x+c)/x^4-a*b*sin(d*x+c)/x^2+1/24*a^2*d^2*sin(d*x+c)/x^2
```

Rubi [A]
time = 0.22, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3420, 3378, 3384, 3380, 3383}

$$\frac{1}{24}a^2d^4\sin(c)\text{CosIntegral}(dx)+\frac{1}{24}a^2d^4\cos(c)\text{Si}(dx)+\frac{a^2d^3\cos(c+dx)}{24x}+\frac{a^2d^2\sin(c+dx)}{24x^2}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{a^2d\cos(c+dx)}{12x^3}-abd^2\sin(c)\text{CosIntegral}(dx)-abd^2\cos(c)\text{Si}(dx)-\frac{ab\sin(c+dx)}{x^2}-\frac{abd\cos(c+dx)}{x}+b^2\sin(c)\text{CosIntegral}(dx)+b^2\cos(c)\text{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] -1/12*(a^2*d*Cos[c + d*x])/x^3 - (a*b*d*Cos[c + d*x])/x + (a^2*d^3*Cos[c +
d*x])/(24*x) + b^2*CosIntegral[d*x]*Sin[c] - a*b*d^2*CosIntegral[d*x]*Sin[c
] + (a^2*d^4*CosIntegral[d*x]*Sin[c])/24 - (a^2*Sin[c + d*x])/(4*x^4) - (a*
b*Sin[c + d*x])/x^2 + (a^2*d^2*Sin[c + d*x])/(24*x^2) + b^2*Cos[c]*SinInteg
ral[d*x] - a*b*d^2*Cos[c]*SinIntegral[d*x] + (a^2*d^4*Cos[c]*SinIntegral[d*
x])/24
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^5}\,dx = \int \left(\frac{a^2\sin(c+dx)}{x^5} + \frac{2ab\sin(c+dx)}{x^3} + \frac{b^2\sin(c+dx)}{x}\right)dx$$
$$= a^2\int \frac{\sin(c+dx)}{x^5}\,dx + (2ab)\int \frac{\sin(c+dx)}{x^3}\,dx + b^2\int \frac{\sin(c+dx)}{x}\,dx$$
$$= -\frac{a^2\sin(c+dx)}{4x^4} - \frac{ab\sin(c+dx)}{x^2} + \frac{1}{4}(a^2d)\int \frac{\cos(c+dx)}{x^4}\,dx + (abd)\int \frac{\cos(}{}$$
$$= -\frac{a^2d\cos(c+dx)}{12x^3} - \frac{abd\cos(c+dx)}{x} + b^2\text{Ci}(dx)\sin(c) - \frac{a^2\sin(c+dx)}{4x^4} - \frac{ab}{}$$
$$= -\frac{a^2d\cos(c+dx)}{12x^3} - \frac{abd\cos(c+dx)}{x} + b^2\text{Ci}(dx)\sin(c) - \frac{a^2\sin(c+dx)}{4x^4} - \frac{ab}{}$$
$$= -\frac{a^2d\cos(c+dx)}{12x^3} - \frac{abd\cos(c+dx)}{x} + \frac{a^2d^3\cos(c+dx)}{24x} + b^2\text{Ci}(dx)\sin(c) - a$$
$$= -\frac{a^2d\cos(c+dx)}{12x^3} - \frac{abd\cos(c+dx)}{x} + \frac{a^2d^3\cos(c+dx)}{24x} + b^2\text{Ci}(dx)\sin(c) - a$$
$$= -\frac{a^2d\cos(c+dx)}{12x^3} - \frac{abd\cos(c+dx)}{x} + \frac{a^2d^3\cos(c+dx)}{24x} + b^2\text{Ci}(dx)\sin(c) - a$$

Mathematica [A]
time = 0.26, size = 122, normalized size = 0.69

$$\frac{adx(-24bx^2+a(-2+d^2x^2))\cos(c+dx)+(24b^2-24abd^2+a^2d^4)\,x^4\text{Ci}(dx)\sin(c)+a(-24bx^2+a(-6+d^2x^2))\sin(c+dx)+(24b^2-24abd^2+a^2d^4)\,x^4\cos(c)\text{Si}(dx)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] (a*d*x*(-24*b*x^2 + a*(-2 + d^2*x^2))*Cos[c + d*x] + (24*b^2 - 24*a*b*d^2 +
 a^2*d^4)*x^4*CosIntegral[d*x]*Sin[c] + a*(-24*b*x^2 + a*(-6 + d^2*x^2))*Si
```

```
n[c + d*x] + (24*b^2 - 24*a*b*d^2 + a^2*d^4)*x^4*Cos[c]*SinIntegral[d*x])/(
24*x^4)
```

Maple [A]
time = 0.23, size = 157, normalized size = 0.89 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] d^4*(a^2*(-1/4*sin(d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/d
^2/x^2+1/24*cos(d*x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c))+2/d^2*a
*b*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*
x)*sin(c))+1/d^4*b^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 9.93, size = 222, normalized size = 1.25

$$-\frac{\left(a^2\left(i\,\Gamma\left(-4,i\,dx\right)-i\,\Gamma\left(-4,-i\,dx\right)\right)\cos\left(c\right)+a^2\left(\Gamma\left(-4,i\,dx\right)+\Gamma\left(-4,-i\,dx\right)\right)\sin\left(c\right)\right)d^8-24\left(ab\left(i\,\Gamma\left(-4,i\,dx\right)-i\,\Gamma\left(-4,-i\,dx\right)\right)\cos\left(c\right)+ab\left(\Gamma\left(-4,i\,dx\right)+\Gamma\left(-4,-i\,dx\right)\right)\sin\left(c\right)\right)d^6-24\left(b^2\left(-i\,\Gamma\left(-4,i\,dx\right)+i\,\Gamma\left(-4,-i\,dx\right)\right)\cos\left(c\right)-b^2\left(\Gamma\left(-4,i\,dx\right)+\Gamma\left(-4,-i\,dx\right)\right)\sin\left(c\right)\right)d^4\right)x^4+2\left(b^2d^3x^3+2\left(abd^3-b^2d\right)x\right)\cos\left(dx+c\right)+2\left(b^2d^2x^2+6\,abd^2-6\,b^2\right)\sin\left(dx+c\right)}{2\,d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a^2*(gamma(
-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^8 - 24*(a*b*(I*gamma(-4, I*d*x) -
 I*gamma(-4, -I*d*x))*cos(c) + a*b*(gamma(-4, I*d*x) + gamma(-4, -I*d*x))*s
in(c))*d^6 - 24*(b^2*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x))*cos(c) - b
^2*(gamma(-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^4)*x^4 + 2*(b^2*d^3*x^3
 + 2*(a*b*d^3 - b^2*d)*x)*cos(d*x + c) + 2*(b^2*d^2*x^2 + 6*a*b*d^2 - 6*b^2
)*sin(d*x + c))/(d^4*x^4)
```

Fricas [A]
time = 0.37, size = 162, normalized size = 0.92

$$\frac{2\left(a^2d^4-24\,abd^2+24\,b^2\right)x^4\cos\left(c\right)\operatorname{Si}\left(dx\right)-2\left(2\,a^2dx-\left(a^2d^3-24\,abd\right)x^3\right)\cos\left(dx+c\right)+2\left(\left(a^2d^2-24\,ab\right)x^2-6\,a^2\right)\sin\left(dx+c\right)+\left(\left(a^2d^4-24\,abd^2+24\,b^2\right)x^4\operatorname{Ci}\left(dx\right)+\left(a^2d^4-24\,abd^2+24\,b^2\right)x^4\operatorname{Ci}\left(-dx\right)\right)\sin\left(c\right)}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(a^2*d^4 - 24*a*b*d^2 + 24*b^2)*x^4*cos(c)*sin_integral(d*x) - 2*(2
*a^2*d*x - (a^2*d^3 - 24*a*b*d)*x^3)*cos(d*x + c) + 2*((a^2*d^2 - 24*a*b)*x
^2 - 6*a^2)*sin(d*x + c) + ((a^2*d^4 - 24*a*b*d^2 + 24*b^2)*x^4*cos_integra
l(d*x) + (a^2*d^4 - 24*a*b*d^2 + 24*b^2)*x^4*cos_integral(-d*x))*sin(c))/x^
4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\sin\left(c+dx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)**2*sin(d*x+c)/x**5,x)

[Out] Integral((a + b*x**2)**2*sin(c + d*x)/x**5, x)

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.49, size = 1497, normalized size = 8.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^5,x, algorithm="giac")

```
[Out] -1/48*(a^2*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
 - a^2*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^4*x^4
*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x^4*rea
l_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^4*x^4*imag_par
t(cos_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^4*x^4*imag_part(cos_integral(-d
*x))*tan(1/2*d*x)^2 - 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 + a^2*
d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^4*x^4*imag_part(c
os_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*c
)^2 - 24*a*b*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)
^2 + 24*a*b*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)
^2 - 48*a*b*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d
^4*x^4*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^4*x^4*real_part(co
s_integral(-d*x))*tan(1/2*c) + 48*a*b*d^2*x^4*real_part(cos_integral(d*x))*
tan(1/2*d*x)^2*tan(1/2*c) + 48*a*b*d^2*x^4*real_part(cos_integral(-d*x))*ta
n(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d
^4*x^4*imag_part(cos_integral(d*x)) + a^2*d^4*x^4*imag_part(cos_integral(-d
*x)) - 2*a^2*d^4*x^4*sin_integral(d*x) + 24*a*b*d^2*x^4*imag_part(cos_integ
ral(d*x))*tan(1/2*d*x)^2 - 24*a*b*d^2*x^4*imag_part(cos_integral(-d*x))*tan
(1/2*d*x)^2 + 48*a*b*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 - 24*a*b*d^2*
x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 24*a*b*d^2*x^4*imag_part(co
s_integral(-d*x))*tan(1/2*c)^2 - 48*a*b*d^2*x^4*sin_integral(d*x)*tan(1/2*c
)^2 + 24*b^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 24*b^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 48*
b^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^3*x^3*tan(1
/2*d*x)^2 + 48*a*b*d^2*x^4*real_part(cos_integral(d*x))*tan(1/2*c) + 48*a*b
*d^2*x^4*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a^2*d^3*x^3*tan(1/2*d
*x)*tan(1/2*c) - 48*b^2*x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan
(1/2*c) - 48*b^2*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c
) + 2*a^2*d^3*x^3*tan(1/2*c)^2 + 48*a*b*d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 24*a*b*d^2*x^4*imag_part(cos_integral(d*x)) - 24*a*b*d^2*x^4*imag_part(cos
_integral(-d*x)) + 48*a*b*d^2*x^4*sin_integral(d*x) - 24*b^2*x^4*imag_part(
```

```
cos_integral(d*x))*tan(1/2*d*x)^2 + 24*b^2*x^4*imag_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2 - 48*b^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 + 4*a^2*d^2*
x^2*tan(1/2*d*x)^2*tan(1/2*c) + 24*b^2*x^4*imag_part(cos_integral(d*x))*tan
(1/2*c)^2 - 24*b^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 48*b^2*
x^4*sin_integral(d*x)*tan(1/2*c)^2 + 4*a^2*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^
2 - 2*a^2*d^3*x^3 - 48*a*b*d*x^3*tan(1/2*d*x)^2 - 48*b^2*x^4*real_part(cos_
integral(d*x))*tan(1/2*c) - 48*b^2*x^4*real_part(cos_integral(-d*x))*tan(1/
2*c) - 192*a*b*d*x^3*tan(1/2*d*x)*tan(1/2*c) - 48*a*b*d*x^3*tan(1/2*c)^2 +
4*a^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*b^2*x^4*imag_part(cos_integral(d
*x)) + 24*b^2*x^4*imag_part(cos_integral(-d*x)) - 48*b^2*x^4*sin_integral(d
*x) - 4*a^2*d^2*x^2*tan(1/2*d*x) - 4*a^2*d^2*x^2*tan(1/2*c) - 96*a*b*x^2*ta
n(1/2*d*x)^2*tan(1/2*c) - 96*a*b*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 48*a*b*d*x
^3 - 4*a^2*d*x*tan(1/2*d*x)^2 - 16*a^2*d*x*tan(1/2*d*x)*tan(1/2*c) - 4*a^2*
d*x*tan(1/2*c)^2 + 96*a*b*x^2*tan(1/2*d*x) + 96*a*b*x^2*tan(1/2*c) - 24*a^2
*tan(1/2*d*x)^2*tan(1/2*c) - 24*a^2*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a^2*d*x +
 24*a^2*tan(1/2*d*x) + 24*a^2*tan(1/2*c))/(x^4*tan(1/2*d*x)^2*tan(1/2*c)^2
+ x^4*tan(1/2*d*x)^2 + x^4*tan(1/2*c)^2 + x^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)\,\left(b\,x^2+a\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^2)^2)/x^5,x)
[Out] int((sin(c + d*x)*(a + b*x^2)^2)/x^5, x)
```

3.57 $\int \frac{x^4 \sin(c+dx)}{a+bx^2} \, dx$

Optimal. Leaf size=273

$$\frac{2\cos(c+dx)}{bd^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{x^2\cos(c+dx)}{bd}-\frac{(-a)^{3/2}\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^{5/2}}+\frac{(-a)^{3/2}\mathrm{Ci}\left(\sqrt{\right.}}{}$$

[Out] 2*cos(d*x+c)/b/d^3+a*cos(d*x+c)/b^2/d-x^2*cos(d*x+c)/b/d+1/2*(-a)^(3/2)*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/b^(5/2)-1/2*(-a)^(3/2)*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/b^(5/2)+2*x*sin(d*x+c)/b/d^2-1/2*(-a)^(3/2)*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/b^(5/2)+1/2*(-a)^(3/2)*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/b^(5/2)

Rubi [A]
time = 0.54, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3426, 2718, 3377, 3414, 3384, 3380, 3383}

$$-\frac{(-a)^{3/2}\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2b^{5/2}}+\frac{(-a)^{3/2}\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^{5/2}}+\frac{a\cos(c+dx)}{b^2d}+\frac{2\cos(c+dx)}{bd^3}+\frac{2x\sin(c+dx)}{bd^2}-\frac{x^2\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[(x^4*Sin[c + d*x])/(a + b*x^2),x]

[Out] (2*Cos[c + d*x])/(b*d^3) + (a*Cos[c + d*x])/(b^2*d) - (x^2*Cos[c + d*x])/(b*d) - ((-a)^(3/2)*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(5/2)) + ((-a)^(3/2)*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(5/2)) + (2*x*Sin[c + d*x])/(b*d^2) - ((-a)^(3/2)*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5/2))

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4\sin(c+dx)}{a+bx^2}\,dx = \int \left(-\frac{a\sin(c+dx)}{b^2}+\frac{x^2\sin(c+dx)}{b}+\frac{a^2\sin(c+dx)}{b^2(a+bx^2)}\right)dx$$

$$= -\frac{a\int \sin(c+dx)\,dx}{b^2}+\frac{a^2\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{b^2}+\frac{\int x^2\sin(c+dx)\,dx}{b}$$

$$= \frac{a\cos(c+dx)}{b^2d}-\frac{x^2\cos(c+dx)}{bd}+\frac{a^2\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{b^2}+$$

$$= \frac{a\cos(c+dx)}{b^2d}-\frac{x^2\cos(c+dx)}{bd}+\frac{2x\sin(c+dx)}{bd^2}-\frac{(-a)^{3/2}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^2}-\frac{(-a)}{}$$

$$= \frac{2\cos(c+dx)}{bd^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{x^2\cos(c+dx)}{bd}+\frac{2x\sin(c+dx)}{bd^2}-\frac{\left((-a)^{3/2}\cos\left(c\right.\right.}{}$$

$$= \frac{2\cos(c+dx)}{bd^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{x^2\cos(c+dx)}{bd}-\frac{(-a)^{3/2}\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\right.}{2b^{5/2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.31, size = 275, normalized size = 1.01

$$\frac{4b^{3/2}\cos(c+dx)+2a\sqrt{b}d^2\cos(c+dx)-2b^{3/2}d^2x^2\cos(c+dx)+ia^{3/2}d^3\text{Ci}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)-ia^{3/2}d^3\text{Ci}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)+4b^{3/2}dx\sin(c+dx)+ia^{3/2}d^3\cos\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+ia^{3/2}d^3\cos\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] (4*b^(3/2)*Cos[c + d*x] + 2*a*Sqrt[b]*d^2*Cos[c + d*x] - 2*b^(3/2)*d^2*x^2*
Cos[c + d*x] + I*a^(3/2)*d^3*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin[c
 - (I*Sqrt[a]*d)/Sqrt[b]] - I*a^(3/2)*d^3*CosIntegral[d*(((-I)*Sqrt[a])/Sqr
t[b] + x)]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]] + 4*b^(3/2)*d*x*Sin[c + d*x] + I*
a^(3/2)*d^3*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[
b] + x)] + I*a^(3/2)*d^3*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt
[a]*d)/Sqrt[b] - d*x])/(2*b^(5/2)*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1629 vs. 2(217) = 434.
time = 0.30, size = 1630, normalized size = 5.97

method	result

risch	$-\frac{a\,\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}\,\mathrm{expIntegral}\left(1,\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)\sqrt{ab}}{4b^3}+\frac{a\,\mathrm{e}^{\frac{ibc-d\sqrt{ab}}{b}}\,\mathrm{expIntegral}\left(1,-\frac{-ibc+d\sqrt{ab}+b(}{b}\right.}{4b^3}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/d^5*(d^2*c^4*(-1/2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2
)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin(
(d*(-a*b)^(1/2)+c*b)/b))-1/2/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*
b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/
b)*sin((d*(-a*b)^(1/2)-c*b)/b)))+2*d^2*c^3*(d*(-a*b)^(1/2)+c*b)/b^2/(-(d*(-
a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+
c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-2*d^2
c^3(d*(-a*b)^(1/2)-c*b)/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b
)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b
)*sin((d*(-a*b)^(1/2)-c*b)/b))-6*d^2*c^2/b*cos(d*x+c)+3*d^2*c^2*(d^2*a+b*c^
2-2*c*(d*(-a*b)^(1/2)+c*b))/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-
a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b
)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+3*d^2*c^2*(d^2*a+b*c^2+2*c*(d*(-a*b)^(1/2
)-c*b))/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*co
s((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1
/2)-c*b)/b))+(8*cos(d*x+c)*d^2*c^2-4*d^2*c*(sin(d*x+c)-(d*x+c)*cos(d*x+c)))
/b-2*d^2*c*(2*a*c*d^2+a*d^2*(d*(-a*b)^(1/2)+c*b)/b+2*b*c^3-3*c^2*(d*(-a*b)^
(1/2)+c*b))/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/
b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*
b)^(1/2)+c*b)/b))-2*d^2*c*(2*a*c*d^2-a*d^2*(d*(-a*b)^(1/2)-c*b)/b+2*b*c^3+3
c^2(d*(-a*b)^(1/2)-c*b))/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*
b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/
b)*sin((d*(-a*b)^(1/2)-c*b)/b))+(a*d^4*cos(d*x+c)-3*b*c^2*d^2*cos(d*x+c)+2*
b*c*d^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+d^2*b*(-(d*x+c)^2*cos(d*x+c)+2*cos(
d*x+c)+2*(d*x+c)*sin(d*x+c)))/b^2-1/2*d^2*(a^2*d^4-2*a*b*c^2*d^2-4*a*c*d^2*
(d*(-a*b)^(1/2)+c*b)-3*b^2*c^4+4*b*c^3*(d*(-a*b)^(1/2)+c*b))/b^3/(-(d*(-a*b
)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b
)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/2*d^2*
(a^2*d^4-2*a*b*c^2*d^2+4*a*c*d^2*(d*(-a*b)^(1/2)-c*b)-3*b^2*c^4-4*b*c^3*(d*
(-a*b)^(1/2)-c*b))/b^3/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)
-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((
d*(-a*b)^(1/2)-c*b)/b)))

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(((b*d^2*x^4*cos(c) + 2*b*d*x^3*sin(c) - 2*b*x^2*cos(c) + 2*a*d*x*sin(
c))*cos(d*x + c)^2 + (b*d^2*x^4*cos(c) + 2*b*d*x^3*sin(c) - 2*b*x^2*cos(c)
+ 2*a*d*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((b*cos(c)^2 + b*sin(c)^
2)*d^2*x^4 - 2*(b*cos(c)^2 + b*sin(c)^2)*x^2)*cos(d*x + c) + 2*(((b^2*cos(c
)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(c)^2 + a*b*sin(c)^2)*d^3)*cos(d*x +
c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(c)^2 + a*b*sin(c)^
2)*d^3)*sin(d*x + c)^2)*integrate(((a^2*d^2 + 2*a*b)*x*cos(d*x + c) + (a*b*
d*x^2 + a^2*d)*sin(d*x + c))/(b^3*d^3*x^4 + 2*a*b^2*d^3*x^2 + a^2*b*d^3), x
) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(c)^2 + a*b*sin(c)^
2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(
c)^2 + a*b*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(((a^2*d^2 + 2*a*b)*x*co
s(d*x + c) + (a*b*d*x^2 + a^2*d)*sin(d*x + c))/((b^3*d^3*x^4 + 2*a*b^2*d^3*
x^2 + a^2*b*d^3)*cos(d*x + c)^2 + (b^3*d^3*x^4 + 2*a*b^2*d^3*x^2 + a^2*b*d^
3)*sin(d*x + c)^2), x) + ((b*d^2*x^4*sin(c) - 2*b*d*x^3*cos(c) - 2*a*d*x*co
s(c) - 2*b*x^2*sin(c))*cos(d*x + c)^2 + (b*d^2*x^4*sin(c) - 2*b*d*x^3*cos(c
) - 2*a*d*x*cos(c) - 2*b*x^2*sin(c))*sin(d*x + c)^2)*sin(d*x + 2*c) - 2*((b
*cos(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d*x)*sin(d*x + c)
)/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(c)^2 + a*b*sin(c)^2)*d
^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^2 + (a*b*cos(c)^2
 + a*b*sin(c)^2)*d^3)*sin(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 240, normalized size = 0.88

$$\frac{\sqrt{\frac{ad^2}{b}}\,ad^2\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\,ad^2\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}\,ad^2\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\,ad^2\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c-\sqrt{\frac{ad^2}{b}}\right)}+8\,bdx\sin\left(dx+c\right)-4\left(bd^2x^2-ad^2-2\,b\right)\cos\left(dx+c\right)}{4\,b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a*d^2/b)*a*d^2*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b))
- sqrt(a*d^2/b)*a*d^2*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + s
qrt(a*d^2/b)*a*d^2*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - sq
rt(a*d^2/b)*a*d^2*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 8*b
*d*x*sin(d*x + c) - 4*(b*d^2*x^2 - a*d^2 - 2*b)*cos(d*x + c))/(b^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sin\left(c + dx\right)}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sin(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*sin(c + d*x)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^4*sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sin\left(c+d\,x\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*sin(c + d*x))/(a + b*x^2),x)
```

```
[Out] int((x^4*sin(c + d*x))/(a + b*x^2), x)
```

3.58 $\int \frac{x^3 \sin(c+dx)}{a+bx^2} \, dx$

Optimal. Leaf size=209

$$-\frac{x\cos(c+dx)}{bd}-\frac{a\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^2}-\frac{a\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^2}+\frac{\sin(c+d}{bd^2}$$

```
[Out] -x*cos(d*x+c)/b/d-1/2*a*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(
1/2))/b^2-1/2*a*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/b^
2+sin(d*x+c)/b/d^2-1/2*a*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^
(1/2))/b^2-1/2*a*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/
b^2
```

Rubi [A]
time = 0.25, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3426, 3377, 2717, 3384, 3380, 3383}

$$-\frac{a\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2b^2}-\frac{a\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2}+\frac{a\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^2}+\frac{\sin(c+dx)}{bd^2}-\frac{x\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] -((x*Cos[c + d*x])/(b*d)) - (a*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[
c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) - (a*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) + Sin[c + d*x]/(b*d^2) + (a*Cos
[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2)
 - (a*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]
)/(2*b^2)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sin(c+dx)}{a+bx^2}\,dx &= \int \left(\frac{x\sin(c+dx)}{b}-\frac{ax\sin(c+dx)}{b\left(a+bx^2\right)}\right)dx \\
&= \frac{\int x\sin(c+dx)\,dx}{b}-\frac{a\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{b} \\
&= -\frac{x\cos(c+dx)}{bd}-\frac{a\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{b}+\frac{\int \cos(
\\
&= -\frac{x\cos(c+dx)}{bd}+\frac{\sin(c+dx)}{bd^2}+\frac{a\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2b^{3/2}}-\frac{a\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2b^{3/2}} \\
&= -\frac{x\cos(c+dx)}{bd}+\frac{\sin(c+dx)}{bd^2}-\frac{\left(a\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2b^{3/2}}-\frac{\left(a
\\
&= -\frac{x\cos(c+dx)}{bd}-\frac{a\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^2}-\frac{a\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)\sin
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.26, size = 202, normalized size = 0.97

$$-\frac{2bdx\cos(c+dx)+ad^2\mathrm{Ci}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)+ad^2\mathrm{Ci}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)-2b\sin(c+dx)+ad^2\cos\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-ad^2\cos\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{i\sqrt{a}\,d}{\sqrt{b}}-dx\right)}{2b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] -1/2*(2*b*d*x*Cos[c + d*x] + a*d^2*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]
*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] + a*d^2*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[
b] + x)]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]] - 2*b*Sin[c + d*x] + a*d^2*Cos[c -
(I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - a*d^2*Cos
[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(b^2*
d^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. 2(169) = 338.
time = 0.10, size = 1164, normalized size = 5.57 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-d^2*c^3*(-1/2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/
2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin
((d*(-a*b)^(1/2)+c*b)/b))-1/2/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a
*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)
/b)*sin((d*(-a*b)^(1/2)-c*b)/b)))-3/2*d^2*c^2*(d*(-a*b)^(1/2)+c*b)/b^2/(-(d
*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/
2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+3/
2*d^2*c^2*(d*(-a*b)^(1/2)-c*b)/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*
(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c
*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))+3*d^2*c/b*cos(d*x+c)-3/2*d^2*c*(d^2*a+b
*c^2-2*c*(d*(-a*b)^(1/2)+c*b))/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d
*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-3/2*d^2*c*(d^2*a+b*c^2+2*c*(d*(-a*b)^(
1/2)-c*b))/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)
*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)
^(1/2)-c*b)/b))+(-2*cos(d*x+c)*d^2*c+d^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c)))/b
+1/2*d^2*(2*a*c*d^2+a*d^2*(d*(-a*b)^(1/2)+c*b)/b+2*b*c^3-3*c^2*(d*(-a*b)^(1
/2)+c*b))/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)
*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)
^(1/2)+c*b)/b))+1/2*d^2*(2*a*c*d^2-a*d^2*(d*(-a*b)^(1/2)-c*b)/b+2*b*c^3+3*c
^2*(d*(-a*b)^(1/2)-c*b))/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)
^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)
*sin((d*(-a*b)^(1/2)-c*b)/b)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^3*cos(d*x + c) - (cos(c)^2 + sin(c)^2)*x^2*
sin(d*x + c) + ((d*x^3*cos(c) + x^2*sin(c))*cos(d*x + c)^2 + (d*x^3*cos(c)
+ x^2*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c)^2
)*d^2*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 +
b*sin(c)^2)*d^2*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integr
ate(-(a*d*x^2*cos(d*x + c) - a*x*sin(d*x + c))/(b^2*d^2*x^4 + 2*a*b*d^2*x^2
 + a^2*d^2), x) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^2 + (a*cos(c)^2 + a*s
in(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d^2*x^2 + (a*cos(
c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(a*d*x^2*cos(d*x + c) -
a*x*sin(d*x + c))/((b^2*d^2*x^4 + 2*a*b*d^2*x^2 + a^2*d^2)*cos(d*x + c)^2 +
 (b^2*d^2*x^4 + 2*a*b*d^2*x^2 + a^2*d^2)*sin(d*x + c)^2), x) + ((d*x^3*sin(
c) - x^2*cos(c))*cos(d*x + c)^2 + (d*x^3*sin(c) - x^2*cos(c))*sin(d*x + c)^
2)*sin(d*x + 2*c))/(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^2 + (a*cos(c)^2 + a*si
n(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d^2*x^2 + (a*cos(c
)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.40, size = 185, normalized size = 0.89

$$\frac{i\,ad^2\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic+\sqrt{\frac{ad^2}{b}}\right)}+i\,ad^2\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic-\sqrt{\frac{ad^2}{b}}\right)}-i\,ad^2\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-ic+\sqrt{\frac{ad^2}{b}}\right)}-i\,ad^2\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-ic-\sqrt{\frac{ad^2}{b}}\right)}-4\,bdx\cos\left(dx+c\right)+4\,b\sin\left(dx+c\right)}{4\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(I*a*d^2*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + I*a*d^2*Ei
(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) - I*a*d^2*Ei(-I*d*x - sqrt(
a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - I*a*d^2*Ei(-I*d*x + sqrt(a*d^2/b))*e^(
-I*c - sqrt(a*d^2/b)) - 4*b*d*x*cos(d*x + c) + 4*b*sin(d*x + c))/(b^2*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sin\left(c+dx\right)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sin\left(c+d\,x\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sin(c + d*x))/(a + b*x^2),x)
```

```
[Out] int((x^3*sin(c + d*x))/(a + b*x^2), x)
```

3.59 $\int \frac{x^2 \sin(c+dx)}{a+bx^2} \, dx$

Optimal. Leaf size=227

$$-\frac{\cos(c+dx)}{bd}-\frac{\sqrt{-a}\,\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{\sqrt{-a}\,\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^{3/2}}-$$

```
[Out] -cos(d*x+c)/b/d+1/2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2)
)*(-a)^(1/2)/b^(3/2)-1/2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^
(1/2))*(-a)^(1/2)/b^(3/2)-1/2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/
2)/b^(1/2))*(-a)^(1/2)/b^(3/2)+1/2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-
a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(3/2)
```

Rubi [A]

time = 0.30, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3426, 2718, 3414, 3384, 3380, 3383}

$$-\frac{\sqrt{-a}\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2b^{3/2}}+\frac{\sqrt{-a}\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] -(Cos[c + d*x]/(b*d)) - (Sqrt[-a]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*S
in[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(3/2)) + (Sqrt[-a]*CosIntegral[(Sqrt[-a]
*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(3/2)) - (Sqrt[-a]*C
os[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^
(3/2)) - (Sqrt[-a]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/S
qrt[b] + d*x])/(2*b^(3/2))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sin(c+dx)}{a+bx^2}\,dx &= \int \left(\frac{\sin(c+dx)}{b}-\frac{a\sin(c+dx)}{b\,(a+bx^2)}\right)dx \\
&= \frac{\int \sin(c+dx)\,dx}{b}-\frac{a\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{b} \\
&= -\frac{\cos(c+dx)}{bd}-\frac{a\int \left(\frac{\sqrt{-a}\,\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sqrt{-a}\,\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{b} \\
&= -\frac{\cos(c+dx)}{bd}-\frac{\sqrt{-a}\,\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2b}-\frac{\sqrt{-a}\,\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2b} \\
&= -\frac{\cos(c+dx)}{bd}-\frac{\left(\sqrt{-a}\,\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2b}+\frac{\left(\sqrt{-a}\,\cos\left(c\cdots\right.\right.}{\cdots} \\
&= -\frac{\cos(c+dx)}{bd}-\frac{\sqrt{-a}\,\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{\sqrt{-a}\,\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\cdots\right.}{2b\cdots}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.23, size = 216, normalized size = 0.95

$$-\frac{2\sqrt{b}\,\cos(c+dx)+i\sqrt{a}\,d\,\mathrm{Ci}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)-i\sqrt{a}\,d\,\mathrm{Ci}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)+i\sqrt{a}\,d\cos\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+i\sqrt{a}\,d\cos\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{i\sqrt{a}\,d}{\sqrt{b}}-dx\right)}{2b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] -1/2*(2*Sqrt[b]*Cos[c + d*x] + I*Sqrt[a]*d*CosIntegral[d*((I*Sqrt[a])/Sqrt[
b] + x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] - I*Sqrt[a]*d*CosIntegral[d*(((-I)*
Sqrt[a])/Sqrt[b] + x)]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]] + I*Sqrt[a]*d*Cos[c -
 (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*Sqrt[a
]*d*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x]
)/(b^(3/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(171) = 342$.
time = 0.07, size = 782, normalized size = 3.44

method	result
risch	$-\frac{e^{\frac{ibc-d\sqrt{ab}}{b}}\,\text{expIntegral}\left(1,-\frac{-ibc+d\sqrt{ab}+b(idx+ic)}{b}\right)\sqrt{ab}}{4b^2}+\frac{e^{\frac{ibc+d\sqrt{ab}}{b}}\,\text{expIntegral}\left(1,\frac{ibc+d\sqrt{ab}-b(idx-}{b}\right.}{4b^2}$
derivativedivides	$d^2c^2\left(-\frac{\text{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\text{cosineIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+}{b}\right.}{2b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}\right.$
default	$d^2c^2\left(-\frac{\text{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\text{cosineIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+}{b}\right.}{2b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(d^2*c^2*(-1/2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2
)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin(
(d*(-a*b)^(1/2)+c*b)/b))-1/2/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*
b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/
b)*sin((d*(-a*b)^(1/2)-c*b)/b)))+d^2*c*(d*(-a*b)^(1/2)+c*b)/b^2/(-(d*(-a*b)
^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)
/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-d^2*c*(d*
(-a*b)^(1/2)-c*b)/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-
c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d
```

```
*(-a*b)^(1/2)-c*b)/b))-d^2/b*cos(d*x+c)+1/2*d^2*(d^2*a+b*c^2-2*c*(d*(-a*b)^
(1/2)+c*b))/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/
b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*
b)^(1/2)+c*b)/b))+1/2*d^2*(d^2*a+b*c^2+2*c*(d*(-a*b)^(1/2)-c*b))/b^2/((d*(-
a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-
c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x^2*cos(d*x + c) + (x^2*cos(d*x + c)^2*cos(c) +
 x^2*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) - 2*(((a*b*cos(c)^2 + a*b*sin(c)
^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((a*b*cos(c)^
2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)*
integrate(x*cos(d*x + c)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) - 2*(((a*b*c
os(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x +
c)^2 + ((a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)
*d)*sin(d*x + c)^2)*integrate(x*cos(d*x + c)/((b^2*d*x^4 + 2*a*b*d*x^2 + a^
2*d)*cos(d*x + c)^2 + (b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d)*sin(d*x + c)^2), x)
 + (x^2*cos(d*x + c)^2*sin(c) + x^2*sin(d*x + c)^2*sin(c))*sin(d*x + 2*c))/
(((b*cos(c)^2 + b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x +
c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(
d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.37, size = 195, normalized size = 0.86

$$-\frac{\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic-\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-ic+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-ic-\sqrt{\frac{ad^2}{b}}\right)}+4\cos\left(dx+c\right)}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(a*d^2/b)*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - sqr
t(a*d^2/b)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + sqrt(a*d^2/b
)*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - sqrt(a*d^2/b)*Ei(-I
*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 4*cos(d*x + c))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sin\left(c+dx\right)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sin\left(c+d\,x\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sin(c + d*x))/(a + b*x^2),x)
```

```
[Out] int((x^2*sin(c + d*x))/(a + b*x^2), x)
```

3.60 $\int \frac{x \sin(c+dx)}{a+bx^2} \, dx$

Optimal. Leaf size=177

$$\frac{\text{Ci}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right)}{2b}+\frac{\text{Ci}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right)}{2b}-\frac{\cos\left(c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}\right.}{2b}$$

[Out] 1/2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/b+1/2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/b+1/2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/b+1/2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/b

Rubi [A]
time = 0.18, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3426, 3384, 3380, 3383}

$$\frac{\sin\left(c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}+dx\right)}{2b}+\frac{\sin\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}-dx\right)}{2b}-\frac{\cos\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cos\left(c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(x*Sin[c + d*x])/(a + b*x^2),x]

[Out] (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b) + (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b)

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(c+dx)}{a+bx^2}\,dx &= \int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx\\ &= -\frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2\sqrt{b}}+\frac{\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2\sqrt{b}}\\ &= \frac{\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2\sqrt{b}}+\frac{\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2\sqrt{b}}+\\ &= \frac{\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b}+\frac{\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b}-\frac{\cos}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 163, normalized size = 0.92

$$\frac{\text{Ci}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)+\text{Ci}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)+\cos\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{Si}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\cos\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{Si}\left(\frac{i\sqrt{a}\,d}{\sqrt{b}}-dx\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] (CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] +
CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]
+ Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] -
 Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(
2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 485 vs. 2(137) = 274.
time = 0.06, size = 486, normalized size = 2.75

method	result
risch	$\frac{i e^{\frac{ibc+d\sqrt{ab}}{b}} \operatorname{expIntegral}\left(1, \frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4b} + \frac{i e^{\frac{ibc-d\sqrt{ab}}{b}} \operatorname{expIntegral}\left(1, -\frac{-ibc+d\sqrt{ab}+b(idx+ic)}{b}\right)}{4b} -$
derivativedivides	$-\frac{d^2\left(d\sqrt{-ab}+cb\right)\left(\operatorname{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\operatorname{cosineIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\right.\right.}{2b^2\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}$
default	$-\frac{d^2\left(d\sqrt{-ab}+cb\right)\left(\operatorname{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\operatorname{cosineIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\right.\right.}{2b^2\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(-1/2*d^2*(d*(-a*b)^(1/2)+c*b)/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*
x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^
(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/2*d^2*(d*(-a*b)^(1/2)-c*b)/b^2
/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)
^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)
)-d^2*c*(-1/2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/
b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*
b)^(1/2)+c*b)/b))-1/2/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2
)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin(
(d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*c
os(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d*x^2
+ (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*
d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(b*x^2 -
 a)*cos(d*x + c)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) + 2*(((b*cos(c)^2 +
b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b*cos(c
)^2 + b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^2)*inte
grate(1/2*(b*x^2 - a)*cos(d*x + c)/((b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d)*cos(d
```

```
*x + c)^2 + (b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d)*sin(d*x + c)^2), x) + (x*cos(
d*x + c)^2*sin(c) + x*sin(d*x + c)^2*sin(c))*sin(d*x + 2*c))/(((b*cos(c)^2
+ b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b*cos
(c)^2 + b*sin(c)^2)*d*x^2 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.36, size = 146, normalized size = 0.82

$$\frac{-i\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-i\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+i\,\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}+i\,\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c-\sqrt{\frac{ad^2}{b}}\right)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(-I*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - I*Ei(I*d*x + sq
rt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + I*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c
 + sqrt(a*d^2/b)) + I*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)))/
b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin{(c+dx)}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x*sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin\left(c+d\,x\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(c + d*x))/(a + b*x^2),x)
```

```
[Out] int((x*sin(c + d*x))/(a + b*x^2), x)
```

3.61 $\int \frac{\sin(c+dx)}{a+bx^2}\,dx$

Optimal. Leaf size=213

$$-\frac{\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2\sqrt{-a}\,\sqrt{b}}+\frac{\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2\sqrt{-a}\,\sqrt{b}}-\frac{\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}}{\sqrt{b}}\right.}{2\sqrt{-a}\,\sqrt{b}}$$

```
[Out] 1/2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/(-a)^(1/2)/b^(
1/2)-1/2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(1/2
)/b^(1/2)-1/2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/(-a)
^(1/2)/b^(1/2)+1/2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2)
)/(-a)^(1/2)/b^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3414, 3384, 3380, 3383}

$$-\frac{\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\,\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\,\sqrt{b}}-\frac{\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\,\sqrt{b}}-\frac{\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2\sqrt{-a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x^2),x]
```

```
[Out] -1/2*(CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]
)/(Sqrt[-a]*Sqrt[b]) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sq
rt[-a]*d)/Sqrt[b]])/(2*Sqrt[-a]*Sqrt[b]) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*S
inIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Cos[c - (Sq
rt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqr
t[b])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a+bx^2}\,dx &= \int \left(\frac{\sqrt{-a}\,\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sqrt{-a}\,\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx\\
&= -\frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2\sqrt{-a}}-\frac{\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2\sqrt{-a}}\\
&= -\frac{\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2\sqrt{-a}}+\frac{\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2\sqrt{-a}}-\\
&= -\frac{\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2\sqrt{-a}\,\sqrt{b}}+\frac{\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2\sqrt{-a}\,\sqrt{b}}-\frac{\cos}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 172, normalized size = 0.81

$$\frac{i\left(\mathrm{Ci}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)-\mathrm{Ci}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)+\cos\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\cos\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{i\sqrt{a}\,d}{\sqrt{b}}-dx\right)\right)}{2\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*x^2),x]
```

```
[Out] ((I/2)*(CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt
[b]] - CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sin[c + (I*Sqrt[a]*d)/Sq
rt[b]] + Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b]
+ x)] + Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] -
d*x]))/(Sqrt[a]*Sqrt[b])
```

Maple [A]
time = 0.04, size = 225, normalized size = 1.06

method	result
derivativedivides	$d\left(-\frac{\operatorname{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\operatorname{cosineIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-a}}{b}\right.}{2b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}\right.$
default	$d\left(-\frac{\operatorname{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\operatorname{cosineIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-a}}{b}\right.}{2b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}\right.$
risch	$\frac{e^{\frac{ibc-d\sqrt{ab}}{b}}\operatorname{expIntegral}\left(1,-\frac{-ibc+d\sqrt{ab}+b(idx+ic)}{b}\right)\sqrt{ab}}{4ab}-\frac{e^{\frac{ibc+d\sqrt{ab}}{b}}\operatorname{expIntegral}\left(1,\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4ab}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos
((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/
2)+c*b)/b))-1/2/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)
/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a
*b)^(1/2)-c*b)/b)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*x^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 187, normalized size = 0.88

$$\frac{\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic-\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-ic+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-ic-\sqrt{\frac{ad^2}{b}}\right)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a*d^2/b)*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - sqrt
(a*d^2/b)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + sqrt(a*d^2/b)
*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - sqrt(a*d^2/b)*Ei(-I*
d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*x^2),x)
```

```
[Out] int(sin(c + d*x)/(a + b*x^2), x)
```

3.62 $\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx$

Optimal. Leaf size=197

$$\frac{\text{Ci}(dx)\sin(c)}{a}-\frac{\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a}-\frac{\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a}+\frac{\cos(c)\text{Si}(dx)}{a}+$$

```
[Out] cos(c)*Si(d*x)/a-1/2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2
))/a-1/2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/a+Ci(d*x)
*sin(c)/a-1/2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a-1/
2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/a
```

Rubi [A]
time = 0.27, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3426, 3384, 3380, 3383}

$$-\frac{\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2a}-\frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a}+\frac{\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a}-\frac{\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a}+\frac{\sin(c)\text{CosIntegral}(dx)}{a}+\frac{\cos(c)\text{Si}(dx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^2)),x]
```

```
[Out] (CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[
c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*a) + (Cos[c]*SinIntegral[d*x])/a + (Cos[
c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a) -
(Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*
a)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx &= \int \left(\frac{\sin(c+dx)}{ax} - \frac{bx\sin(c+dx)}{a(a+bx^2)}\right) dx \\
&= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{a} \\
&= -\frac{b\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right) dx}{a} + \frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a} + \frac{\sin(c
\\
&= \frac{\mathrm{Ci}(dx)\sin(c)}{a} + \frac{\cos(c)\mathrm{Si}(dx)}{a} + \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2a} - \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2a} \\
&= \frac{\mathrm{Ci}(dx)\sin(c)}{a} + \frac{\cos(c)\mathrm{Si}(dx)}{a} - \frac{\left(\sqrt{b}\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2a} - \frac{\left(\sqrt{
\\
&= \frac{\mathrm{Ci}(dx)\sin(c)}{a} - \frac{\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a} - \frac{\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.23, size = 179, normalized size = 0.91

$$
-\frac{-2\mathrm{Ci}(dx)\sin(c) + \mathrm{Ci}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right) + \mathrm{Ci}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right) - 2\cos(c)\mathrm{Si}(dx) + \cos\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right) - \cos\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{i\sqrt{a}\,d}{\sqrt{b}}-dx\right)}{2a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^2)),x]
```

```
[Out] -1/2*(-2*CosIntegral[d*x]*Sin[c] + CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]
*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] + CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x
)]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]] - 2*Cos[c]*SinIntegral[d*x] + Cos[c - (I*
```

```
Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - Cos[c + (I*S
qrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/a
```

Maple [A]
time = 0.09, size = 200, normalized size = 1.02

method	result
derivativedivides	$-\frac{\text{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\text{cosineIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+c}{b}\right.}{2a}$
default	$-\frac{\text{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\text{cosineIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\sin\left(\frac{d\sqrt{-ab}+c}{b}\right.}{2a}$
risch	$-\frac{i\,e^{\frac{ibc+d\sqrt{ab}}{b}}\,\text{expIntegral}\left(1,\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4a}-\frac{i\,e^{\frac{ibc-d\sqrt{ab}}{b}}\,\text{expIntegral}\left(1,\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)}{4a}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x
+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/2/a*(Si(d*x+c+(d*
(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c
*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))+1/a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 168, normalized size = 0.85

$$\frac{-2i\,\mathrm{Ei}(i\,dx)\,e^{(i\,c)}+2i\,\mathrm{Ei}(-i\,dx)\,e^{(-i\,c)}+i\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+i\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}-i\,\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}-i\,\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c-\sqrt{\frac{ad^2}{b}}\right)}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(-2*I*Ei(I*d*x)*e^(I*c) + 2*I*Ei(-I*d*x)*e^(-I*c) + I*Ei(I*d*x - sqrt(a
*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + I*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqr
```

```
t(a*d^2/b)) - I*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - I*Ei(
-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin (c+dx)}{x\left(a+bx^2\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x**2+a),x)
[Out] Integral(sin(c + d*x)/(x*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (c+d\,x)}{x\,\left(b\,x^2+a\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x*(a + b*x^2)),x)
[Out] int(sin(c + d*x)/(x*(a + b*x^2)), x)
```

3.63 $\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=250

$$\frac{d\cos(c)\text{Ci}(dx)}{a}-\frac{\sqrt{b}\,\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2(-a)^{3/2}}+\frac{\sqrt{b}\,\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2(-a)^{3/2}}-\frac{\sin(c}{c}$$

```
[Out] d*Ci(d*x)*cos(c)/a-d*Si(d*x)*sin(c)/a-sin(d*x+c)/a/x+1/2*cos(c+d*(-a)^(1/2)
/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(3/2)-1/2*cos(c-d*(-a)^
(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(3/2)-1/2*Ci(d*x+d
*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(3/2)+1/2*Ci(
-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(3/2)
```

Rubi [A]
time = 0.40, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3426, 3378, 3384, 3380, 3383, 3414}

$$-\frac{\sqrt{b}\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2(-a)^{3/2}}+\frac{\sqrt{b}\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}-\frac{\sqrt{b}\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}-\frac{\sqrt{b}\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2(-a)^{3/2}}+\frac{d\cos(c)\text{CosIntegral}(dx)}{a}-\frac{d\sin(c)\text{Si}(dx)}{a}-\frac{\sin(c+dx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^2)),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a - (Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*(-a)^(3/2)) + (Sqrt[b]*CosIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*(-a)^(3/2)) -
 Sin[c + d*x]/(a*x) - (d*Sin[c]*SinIntegral[d*x])/a - (Sqrt[b]*Cos[c + (Sqr
t[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*(-a)^(3/2)) -
 (Sqrt[b]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] +
d*x])/(2*(-a)^(3/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx = \int \left(\frac{\sin(c+dx)}{ax^2} - \frac{b\sin(c+dx)}{a(a+bx^2)}\right) dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{a}$$

$$= -\frac{\sin(c+dx)}{ax} - \frac{b\int \left(\frac{\sqrt{-a}\,\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{-a}\,\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right) dx}{a} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{a}$$

$$= -\frac{\sin(c+dx)}{ax} - \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2(-a)^{3/2}} - \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2(-a)^{3/2}} + \frac{(d\cos(c))\int \frac{\cos(dx)}{x}\,dx}{a} - \frac{(d$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a} - \frac{\sin(c+dx)}{ax} - \frac{d\sin(c)\text{Si}(dx)}{a} - \frac{\left(b\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+\right.}{\sqrt{-a}+\sqrt{b}}}{2(-a)^{3/2}}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a} - \frac{\sqrt{b}\,\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2(-a)^{3/2}} + \frac{\sqrt{b}\,\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)\text{si}}{2(-a)^{3/2}}$$

__Mathematica__ [C] Result contains complex when optimal does not.
time = 0.34, size = 238, normalized size = 0.95

$$\frac{d\cos(c)\text{Ci}(dx)}{a} - \frac{i\left(\sqrt{b}\,x\text{Ci}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right) - \sqrt{b}\,x\text{Ci}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right) - 2i\sqrt{a}\sin(c+dx) - 2i\sqrt{a}\,dx\sin(c)\text{Si}(dx) + \sqrt{b}\,x\cos\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{Si}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right) + \sqrt{b}\,x\cos\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{Si}\left(\frac{i\sqrt{a}\,d}{\sqrt{b}}-dx\right)\right)}{2a^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^2)),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a - ((I/2)*(Sqrt[b]*x*CosIntegral[d*((I*Sqrt[a]
)/Sqrt[b] + x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] - Sqrt[b]*x*CosIntegral[d*((
(-I)*Sqrt[a])/Sqrt[b] + x)]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]] - (2*I)*Sqrt[a]*
Sin[c + d*x] - (2*I)*Sqrt[a]*d*x*Sin[c]*SinIntegral[d*x] + Sqrt[b]*x*Cos[c
- (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + Sqrt[b]
*x*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])
)/(a^(3/2)*x)
```

__Maple__ [A]
time = 0.12, size = 266, normalized size = 1.06

method	result

derivativedivides	$d\left(\frac{-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}(dx)\sin(c)+\text{cosineIntegral}(dx)\cos(c)}{a}-\frac{b\left(-\frac{\text{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{\cdots}}{\cdots}\right)\cdots}{\cdots}\right.}{\cdots}\right.\cdots$
default	$d\left(\frac{-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}(dx)\sin(c)+\text{cosineIntegral}(dx)\cos(c)}{a}-\frac{b\left(-\frac{\text{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{\cdots}}{\cdots}\right)\cdots}{\cdots}\right.}{\cdots}\right.\cdots$
risch	$\frac{\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}\,\text{expIntegral}\left(1,\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)\sqrt{ab}}{4a^2}-\frac{\mathrm{e}^{\frac{ibc-d\sqrt{ab}}{b}}\,\text{expIntegral}\left(1,-\frac{-ibc+d\sqrt{ab}+b(idx+ic\cdots}{b}\right.\cdots}{4a^2}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] d*(1/a*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))-1/a*b*(-1/2/b/(-(d*(
-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)
+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/2/
b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b
)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b
))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x^2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 240, normalized size = 0.96

$$\frac{2\,ad^2x\mathrm{Ei}(i\,dx)\,e^{(i\,c)}+2\,ad^2x\mathrm{Ei}(-i\,dx)\,e^{(-i\,c)}-\sqrt{\frac{ad^2}{b}}\,bx\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}\,bx\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\,bx\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}\,bx\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c-\sqrt{\frac{ad^2}{b}}\right)}-4\,ad\sin\left(dx+c\right)}{4\,a^2dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d^2*x*Ei(I*d*x)*e^(I*c) + 2*a*d^2*x*Ei(-I*d*x)*e^(-I*c) - sqrt(a*d
^2/b)*b*x*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + sqrt(a*d^2/b)
*b*x*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) - sqrt(a*d^2/b)*b*x*
Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + sqrt(a*d^2/b)*b*x*Ei(
-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*a*d*sin(d*x + c))/(a^2
*d*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x**2+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x**2*(a + b*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)}{x^2\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x^2*(a + b*x^2)),x)
```

```
[Out] int(sin(c + d*x)/(x^2*(a + b*x^2)), x)
```

3.64 $\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=270

$$-\frac{d\cos(c+dx)}{2ax}-\frac{b\text{Ci}(dx)\sin(c)}{a^2}-\frac{d^2\text{Ci}(dx)\sin(c)}{2a}+\frac{b\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a^2}+\frac{b\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-\right.}{}$$

```
[Out] -1/2*d*cos(d*x+c)/a/x-b*cos(c)*Si(d*x)/a^2-1/2*d^2*cos(c)*Si(d*x)/a+1/2*b*c
os(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/a^2+1/2*b*cos(c-d*(
-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/a^2-b*Ci(d*x)*sin(c)/a^2-1/
2*d^2*Ci(d*x)*sin(c)/a-1/2*sin(d*x+c)/a/x^2+1/2*b*Ci(d*x+d*(-a)^(1/2)/b^(1/
2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a^2+1/2*b*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin
(c+d*(-a)^(1/2)/b^(1/2))/a^2
```

Rubi [A]
time = 0.35, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3426, 3378, 3384, 3380, 3383}

$$-\frac{b\sin(c)\text{CosIntegral}(dx)}{a^2}+\frac{b\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2a^2}+\frac{b\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{b\cos(c)\text{Si}(dx)}{a^2}-\frac{b\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2}+\frac{b\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a^2}-\frac{d^2\sin(c)\text{CosIntegral}(dx)}{2a}-\frac{d^2\cos(c)\text{Si}(dx)}{2a}-\frac{\sin(c+dx)}{2ax^2}-\frac{d\cos(c+dx)}{2ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(a*x) - (b*CosIntegral[d*x]*Sin[c])/a^2 - (d^2*CosInt
egral[d*x]*Sin[c])/(2*a) + (b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c
 - (Sqrt[-a]*d)/Sqrt[b]])/(2*a^2) + (b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d
*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*a^2) - Sin[c + d*x]/(2*a*x^2) - (b*Co
s[c]*SinIntegral[d*x])/a^2 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a) - (b*Cos[c
 + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) +
 (b*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(2*a^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^2\right)}\,dx = \int \left(\frac{\sin(c+dx)}{ax^3}-\frac{b\sin(c+dx)}{a^2x}+\frac{b^2x\sin(c+dx)}{a^2\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a}-\frac{b\int \frac{\sin(c+dx)}{x}\,dx}{a^2}+\frac{b^2\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{2ax^2}+\frac{b^2\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{a^2}+\frac{d\int \frac{\cos(c+dx}{x^2}}{2a}$$

$$= -\frac{d\cos(c+dx)}{2ax}-\frac{b\text{Ci}(dx)\sin(c)}{a^2}-\frac{\sin(c+dx)}{2ax^2}-\frac{b\cos(c)\text{Si}(dx)}{a^2}-\frac{b^{3/2}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}}{2a^2}$$

$$= -\frac{d\cos(c+dx)}{2ax}-\frac{b\text{Ci}(dx)\sin(c)}{a^2}-\frac{\sin(c+dx)}{2ax^2}-\frac{b\cos(c)\text{Si}(dx)}{a^2}-\frac{\left(d^2\cos(c)\right)\int \frac{\sin(dx)}{x}}{2a}$$

$$= -\frac{d\cos(c+dx)}{2ax}-\frac{b\text{Ci}(dx)\sin(c)}{a^2}-\frac{d^2\text{Ci}(dx)\sin(c)}{2a}+\frac{b\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-}}{\sqrt{}}\right.}{2a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.43, size = 247, normalized size = 0.91

$$-\frac{adx\cos(c+dx)+\left(2b+ad^2\right)x^2\text{Ci}(dx)\sin(c)-bx^2\text{Ci}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)-bx^2\text{Ci}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\sin\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)+a\sin(c+dx)+2bx^2\cos(c)\text{Si}(dx)+ad^2x^2\cos(c)\text{Si}(dx)-bx^2\cos\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{Si}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+bx^2\cos\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\text{Si}\left(\frac{i\sqrt{a}\,d}{\sqrt{b}}-dx\right)}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*(a*d*x*Cos[c + d*x] + (2*b + a*d^2)*x^2*CosIntegral[d*x]*Sin[c] - b*x^
2*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] -
 b*x^2*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sin[c + (I*Sqrt[a]*d)/Sq
rt[b]] + a*Sin[c + d*x] + 2*b*x^2*Cos[c]*SinIntegral[d*x] + a*d^2*x^2*Cos[c
]*SinIntegral[d*x] - b*x^2*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I
*Sqrt[a])/Sqrt[b] + x)] + b*x^2*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[
(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(a^2*x^2)
```

Maple [A]
time = 0.15, size = 259, normalized size = 0.96

method	result
derivativedivides	$d^{2}\left(-\frac{\sin(dx+c)}{2a\,d^{2}x^{2}}-\frac{\cos(dx+c)}{2adx}+\frac{b\left(\text{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\text{cosineIntegral}\left(d\right.\right.}{2a^{2}d^{2}}\right.$
default	$d^{2}\left(-\frac{\sin(dx+c)}{2a\,d^{2}x^{2}}-\frac{\cos(dx+c)}{2adx}+\frac{b\left(\text{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\text{cosineIntegral}\left(d\right.\right.}{2a^{2}d^{2}}\right.$
risch	$\frac{ib\,\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}\,\text{expIntegral}\left(1,\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{4a^{2}}+\frac{ib\,\mathrm{e}^{\frac{ibc-d\sqrt{ab}}{b}}\,\text{expIntegral}\left(1,-\frac{-ibc+d\sqrt{ab}+b(idx+ic)}{b}\right)}{4a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/2*sin(d*x+c)/a/d^2/x^2-1/2*cos(d*x+c)/a/d/x+1/2*b/a^2/d^2*(Si(d*x+c
-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/
2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/2*b/a^2/d^2*(Si(d*x+c+(d*(-a*b)^(
1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*s
in((d*(-a*b)^(1/2)-c*b)/b))-1/2/a^2*(a*d^2+2*b)/d^2*(Si(d*x)*cos(c)+Ci(d*x)
*sin(c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x^3), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.39, size = 231, normalized size = 0.86

$$\frac{i\,(ad^2+2\,b)x^2\mathrm{Ei}(i\,dx)\,e^{(i\,c)}-i\,(ad^2+2\,b)x^2\mathrm{Ei}(-i\,dx)\,e^{(-i\,c)}-i\,bx^2\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-i\,bx^2\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+i\,bx^2\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}+i\,bx^2\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c-\sqrt{\frac{ad^2}{b}}\right)}-2\,adx\cos\left(dx+c\right)-2\,a\sin\left(dx+c\right)}{4\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(I*(a*d^2 + 2*b)*x^2*Ei(I*d*x)*e^(I*c) - I*(a*d^2 + 2*b)*x^2*Ei(-I*d*x)
*e^(-I*c) - I*b*x^2*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - I*b
*x^2*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + I*b*x^2*Ei(-I*d*x
- sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + I*b*x^2*Ei(-I*d*x + sqrt(a*d^2/
b))*e^(-I*c - sqrt(a*d^2/b)) - 2*a*d*x*cos(d*x + c) - 2*a*sin(d*x + c))/(a^
2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c+dx\right)}{x^3\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**3/(b*x**2+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x**3*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)}{x^3\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x^3*(a + b*x^2)),x)
```

```
[Out] int(sin(c + d*x)/(x^3*(a + b*x^2)), x)
```

3.65 $\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=450

$$-\frac{\cos(c+dx)}{b^2d}-\frac{ad\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^3}-\frac{ad\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4b^3}-\frac{3\sqrt{-a}}{}$$

```
[Out] -cos(d*x+c)/b^2/d-1/4*a*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b
^(1/2))/b^3-1/4*a*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2
))/b^3+1/2*x*sin(d*x+c)/b^2-1/2*x^3*sin(d*x+c)/b/(b*x^2+a)+1/4*a*d*Si(d*x+d
*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/b^3+1/4*a*d*Si(d*x-d*(-a)^
(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/b^3+3/4*cos(c+d*(-a)^(1/2)/b^(1/
2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)-3/4*cos(c-d*(-a)^(1/2)/
b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)-3/4*Ci(d*x+d*(-a)^
(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)+3/4*Ci(-d*x+d
*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)
```

Rubi [A]
time = 0.61, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3424, 3426, 2718, 3414, 3384, 3380, 3383, 3427, 3377}

$$-\frac{3\sqrt{-a}\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4b^{5/2}}+\frac{3\sqrt{-a}\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^{5/2}}-\frac{3\sqrt{-a}\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\operatorname{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^{5/2}}-\frac{3\sqrt{-a}\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4b^{5/2}}-\frac{ad\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^3}-\frac{ad\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4b^3}-\frac{ad\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\operatorname{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^3}+\frac{ad\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4b^3}-\frac{x^3\sin(c+dx)}{2b\left(a+bx^2\right)}+\frac{x\sin(c+dx)}{2b^2}-\frac{\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] -(Cos[c + d*x]/(b^2*d)) - (a*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(S
qrt[-a]*d)/Sqrt[b] - d*x])/(4*b^3) - (a*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*Cos
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^3) - (3*Sqrt[-a]*CosIntegral[(Sq
rt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*b^(5/2)) + (3*Sq
rt[-a]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]
])/(4*b^(5/2)) + (x*Sin[c + d*x])/(2*b^2) - (x^3*Sin[c + d*x])/(2*b*(a + b*
x^2)) - (3*Sqrt[-a]*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/(4*b^(5/2)) - (a*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^3) - (3*Sqrt[-a]*Cos[c - (Sqrt[-a]*d)/S
qrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) + (a*d*Sin[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^3)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2}\,dx = -\frac{x^3\sin(c+dx)}{2b(a+bx^2)} + \frac{3\int \frac{x^2\sin(c+dx)}{a+bx^2}\,dx}{2b} + \frac{d\int \frac{x^3\cos(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^3\sin(c+dx)}{2b(a+bx^2)} + \frac{3\int \left(\frac{\sin(c+dx)}{b} - \frac{a\sin(c+dx)}{b(a+bx^2)}\right)dx}{2b} + \frac{d\int \left(\frac{x\cos(c+dx)}{b} - \frac{ax\cos(c+dx)}{b(a+bx^2)}\right)dx}{2b}$$

$$= -\frac{x^3\sin(c+dx)}{2b(a+bx^2)} + \frac{3\int \sin(c+dx)\,dx}{2b^2} - \frac{(3a)\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{2b^2} + \frac{d\int x\cos(c+dx)\,dx}{2b^2} -$$

$$= -\frac{3\cos(c+dx)}{2b^2d} + \frac{x\sin(c+dx)}{2b^2} - \frac{x^3\sin(c+dx)}{2b(a+bx^2)} - \frac{\int \sin(c+dx)\,dx}{2b^2} - \frac{(3a)\int \Bigg(\frac{\sqrt{\ }}{2a(}}{}$$

$$= -\frac{\cos(c+dx)}{b^2d} + \frac{x\sin(c+dx)}{2b^2} - \frac{x^3\sin(c+dx)}{2b(a+bx^2)} - \frac{\left(3\sqrt{-a}\right)\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4b^2} - \frac{(3}{}$$

$$= -\frac{\cos(c+dx)}{b^2d} + \frac{x\sin(c+dx)}{2b^2} - \frac{x^3\sin(c+dx)}{2b(a+bx^2)} - \frac{\left(3\sqrt{-a}\cos\left(c - \frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right)\int}{4b^2}$$

$$= -\frac{\cos(c+dx)}{b^2d} - \frac{ad\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}} - dx\right)}{4b^3} - \frac{ad\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{C}}{4b^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.70, size = 632, normalized size = 1.40

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] -1/4*(4*a*b*Cos[c + d*x] + 4*b^2*x^2*Cos[c + d*x] + Sqrt[a]*d*(a + b*x^2)*C
osIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c - (I*Sqrt[a]*d)/Sq
```

```
rt[b]] + (3*I)*Sqrt[b]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]) + Sqrt[a]*d*(a + b*x
^2)*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c + (I*Sqrt[
a]*d)/Sqrt[b]] - (3*I)*Sqrt[b]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]) - 2*a*b*d*x*
Sin[c + d*x] + (3*I)*a^(3/2)*Sqrt[b]*d*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIn
tegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + (3*I)*Sqrt[a]*b^(3/2)*d*x^2*Cos[c - (
I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - a^2*d^2*Si
n[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - a*b
*d^2*x^2*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b]
+ x)] + (3*I)*a^(3/2)*Sqrt[b]*d*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[
(I*Sqrt[a]*d)/Sqrt[b] - d*x] + (3*I)*Sqrt[a]*b^(3/2)*d*x^2*Cos[c + (I*Sqrt[
a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + a^2*d^2*Sin[c + (
I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + a*b*d^2*x^
2*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/
(b^3*d*(a + b*x^2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3410 vs. $2(350) = 700$.

time = 0.57, size = 3411, normalized size = 7.58

method	result
risch	$\frac{da\,\mathrm{e}^{\frac{ibc-d\sqrt{ab}}{b}}\,\mathrm{expIntegral}\left(1,\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)}{8b^3}+\frac{da\,\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}\,\mathrm{expIntegral}\left(1,\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{8b^3}-$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^5*(d^4*c^4*(sin(d*x+c)*(1/2/a/d^2*(d*x+c)-1/2*c/a/d^2)/(d^2*a+b*c^2-2*b
*c*(d*x+c)+b*(d*x+c)^2)-1/4/a/d^2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(
d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)
+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/d^2/b/((d*(-a*b)^(1/2)-c*b)/b+c
)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d
*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4/a/b/d^2*(-Si(d*x+c-(
d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)
+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/b/d^2*(Si(d*x+c+(d*(-a*b)^(1/2)
-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((
d*(-a*b)^(1/2)-c*b)/b)))+sin(d*x+c)*(-2*c^4*d^2/a*(d*x+c)+2*c^3*d^2*(a*d^2+
b*c^2)/a/b)/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)+c^4*d^2/a/b/(-(d*(-a*b)
^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)
/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+c^4*d^2/a
/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*
b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/
b))+c^3*d^2*(d^2*a+b*c^2-c*(d*(-a*b)^(1/2)+c*b))/a/b^2/(-(d*(-a*b)^(1/2)+c*
```

b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d
x+c-(d(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))+c^3*d^2*(d^2*a+b*
c^2+c*(d*(-a*b)^(1/2)-c*b))/a/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(
-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*
b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))+sin(d*x+c)*(-3*d^2*c^2*(a*d^2-b*c^2)/a/b
*(d*x+c)-3*c^3*d^2*(a*d^2+b*c^2)/a/b)/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^
2)-3/2*d^2*c^2*(a*d^2+b*c^2)/a/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d
*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-3/2*d^2*c^2*(a*d^2+b*c^2)/a/b^2/((d*(-
a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-
c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-3/2*d
^2*c^2*(a*c*d^2+a*d^2*(d*(-a*b)^(1/2)+c*b)/b+b*c^3-c^2*(d*(-a*b)^(1/2)+c*b)
)/a/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin(
(d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2
)+c*b)/b))-3/2*d^2*c^2*(a*c*d^2-a*d^2*(d*(-a*b)^(1/2)-c*b)/b+b*c^3+c^2*(d*(
-a*b)^(1/2)-c*b))/a/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2
)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos(
(d*(-a*b)^(1/2)-c*b)/b))+sin(d*x+c)*(2*d^2*c^2*(3*a*d^2-b*c^2)/a/b*(d*x+c)-
2*d^2*c*(a^2*d^4-b^2*c^4)/a/b^2)/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)+d^
2*c*(a*c*d^2+2*a*d^2*(d*(-a*b)^(1/2)+c*b)/b+b*c^3)/a/b^2/(-(d*(-a*b)^(1/2)+
c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(
d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+d^2*c*(a*c*d^2-2
*a*d^2*(d*(-a*b)^(1/2)-c*b)/b+b*c^3)/a/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d
x+c+(d(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)
^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-d^2*c*(a^2*d^4-3*a*c*d^2*(d*(-a
*b)^(1/2)+c*b)-b^2*c^4+b*c^3*(d*(-a*b)^(1/2)+c*b))/a/b^3/(-(d*(-a*b)^(1/2)+
c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci
(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-d^2*c*(a^2*d^4+
3*a*c*d^2*(d*(-a*b)^(1/2)-c*b)-b^2*c^4-b*c^3*(d*(-a*b)^(1/2)-c*b))/a/b^3/((
d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1
/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))-d
^4/b^2*cos(d*x+c)+sin(d*x+c)*(1/2*d^2*(a^2*d^4-6*a*b*c^2*d^2+b^2*c^4)/a*(d*
x+c)+1/2*c*d^2*(3*a^2*d^4+2*a*b*c^2*d^2-b^2*c^4)/a)/b^2/(d^2*a+b*c^2-2*b*c*
(d*x+c)+b*(d*x+c)^2)+1/4*d^2*(3*a^2*d^4+2*a*b*c^2*d^2-8*a*c*d^2*(d*(-a*b)^(
1/2)+c*b)-b^2*c^4)/a/b^3/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1
/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*si
n((d*(-a*b)^(1/2)+c*b)/b))+1/4*d^2*(3*a^2*d^4+2*a*b*c^2*d^2+8*a*c*d^2*(d*(-
a*b)^(1/2)-c*b)-b^2*c^4)/a/b^3/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*
b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/
b)*sin((d*(-a*b)^(1/2)-c*b)/b))+1/4*d^2*(3*a^2*c*d^4+a^2*d^4*(d*(-a*b)^(1/2
)+c*b)/b+2*a*b*c^3*d^2-6*a*c^2*d^2*(d*(-a*b)^(1/2)+c*b)-b^2*c^5+b*c^4*(d*(-
a*b)^(1/2)+c*b))/a/b^3/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/
2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos
((d*(-a*b)^(1/2)+c*b)/b))+1/4*d^2*(3*a^2*c*d^4-a^2*d^4*(d*(-a*b)^(1/2)-c*b)
/b+2*a*b*c^3*d^2+6*a*c^2*d^2*(d*(-a*b)^(1/2)-c*b)-b^2*c^5-b*c^4*(d*(-a*b)^(

```
1/2)-c*b))/a/b^3/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/
b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*
b)^(1/2)-c*b)/b)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b*cos(c)^2 + b*sin(c)^2)*d*x^4*cos(d*x + c) - 4*(a*cos(c)^2 + a*sin(
c)^2)*x*sin(d*x + c) + ((b*d*x^4*cos(c) + 4*a*x*sin(c))*cos(d*x + c)^2 + (b
*d*x^4*cos(c) + 4*a*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) - 2*(((b^3*cos
(c)^2 + b^3*sin(c)^2)*d^2*x^4 + 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^2
 + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^3*cos(c)^2 +
 b^3*sin(c)^2)*d^2*x^4 + 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^2 + (a^2
*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-2*(a^2*d*x*co
s(d*x + c) - (3*a*b*x^2 - a^2)*sin(d*x + c))/(b^4*d^2*x^6 + 3*a*b^3*d^2*x^4
 + 3*a^2*b^2*d^2*x^2 + a^3*b*d^2), x) - 2*(((b^3*cos(c)^2 + b^3*sin(c)^2)*d
^2*x^4 + 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^2 + (a^2*b*cos(c)^2 + a^
2*b*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^4
+ 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^2 + (a^2*b*cos(c)^2 + a^2*b*sin
(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-2*(a^2*d*x*cos(d*x + c) - (3*a*b*x^2
 - a^2)*sin(d*x + c))/((b^4*d^2*x^6 + 3*a*b^3*d^2*x^4 + 3*a^2*b^2*d^2*x^2 +
 a^3*b*d^2)*cos(d*x + c)^2 + (b^4*d^2*x^6 + 3*a*b^3*d^2*x^4 + 3*a^2*b^2*d^2
*x^2 + a^3*b*d^2)*sin(d*x + c)^2), x) + ((b*d*x^4*sin(c) - 4*a*x*cos(c))*co
s(d*x + c)^2 + (b*d*x^4*sin(c) - 4*a*x*cos(c))*sin(d*x + c)^2)*sin(d*x + 2*
c))/(((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^4 + 2*(a*b^2*cos(c)^2 + a*b^2*sin
(c)^2)*d^2*x^2 + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((
b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^4 + 2*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*
d^2*x^2 + (a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.37, size = 351, normalized size = 0.78

$$\frac{4\,abdx\sin\left(dx+c\right)-\left(abd^2x^2+a^2d^2+3\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\left(abd^2x^2+a^2d^2-3\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}-\left(abd^2x^2+a^2d^2+3\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\left(abd^2x^2+a^2d^2-3\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c-\sqrt{\frac{ad^2}{b}}\right)}-8\left(b^2x^2+ab\right)\cos\left(dx+c\right)}{8\left(b^4dx^2+ab^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*b*d*x*sin(d*x + c) - (a*b*d^2*x^2 + a^2*d^2 + 3*(b^2*x^2 + a*b)*sq
rt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - (a*b*d^2*x
^2 + a^2*d^2 - 3*(b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e
^(I*c - sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*d^2 + 3*(b^2*x^2 + a*b)*sqrt(a*
```

```
d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^2
+ a^2*d^2 - 3*(b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(
-I*c - sqrt(a*d^2/b)) - 8*(b^2*x^2 + a*b)*cos(d*x + c))/(b^4*d*x^2 + a*b^3*
d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sin{(c + dx)}}{(a + bx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sin(d*x+c)/(b*x**2+a)**2,x)
[Out] Integral(x**4*sin(c + d*x)/(a + b*x**2)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(x^4*sin(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sin\left(c+d\,x\right)}{{\left(b\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*sin(c + d*x))/(a + b*x^2)^2,x)
[Out] int((x^4*sin(c + d*x))/(a + b*x^2)^2, x)
```

3.66 $\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=431

$$\frac{\sqrt{-a}\, d \cos\left(c+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right) \text{Ci}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}-dx\right)}{4b^{5/2}}-\frac{\sqrt{-a}\, d \cos\left(c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right) \text{Ci}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}+dx\right)}{4b^{5/2}}+\frac{\text{Ci}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}+\right.}{}$$

```
[Out] 1/2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/b^2+1/2*cos(c-
d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/b^2+1/2*sin(d*x+c)/b^2-1
/2*x^2*sin(d*x+c)/b/(b*x^2+a)+1/2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)
^(1/2)/b^(1/2))/b^2+1/2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^
(1/2))/b^2-1/4*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))*(
-a)^(1/2)/b^(5/2)+1/4*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^
(1/2))*(-a)^(1/2)/b^(5/2)+1/4*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(
1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)-1/4*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(c+d
*(-a)^(1/2)/b^(1/2))*(-a)^(1/2)/b^(5/2)
```

Rubi [A]

time = 0.52, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3424, 3426, 3384, 3380, 3383, 3427, 2717, 3415}

$$\frac{\sqrt{-a}\, d \cos\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}+c\right) \text{CosIntegral}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}-dx\right)}{4b^{5/2}}-\frac{\sqrt{-a}\, d \cos\left(c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}+dx\right)}{4b^{5/2}}+\frac{\sqrt{-a}\, d \sin\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}+c\right) \text{Si}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}-dx\right)}{4b^{5/2}}+\frac{\sqrt{-a}\, d \sin\left(c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right) \text{Si}\left(xd+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right)}{4b^{5/2}}+\frac{\sin\left(c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}+dx\right)}{2b^2}+\frac{\sin\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}+c\right) \text{CosIntegral}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{\cos\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}+c\right) \text{Si}\left(\frac{\sqrt{-a}\, d}{\sqrt{b}}-dx\right)}{2b^2}+\frac{\cos\left(c-\frac{\sqrt{-a}\, d}{\sqrt{b}}\right) \text{Si}\left(xd+\frac{\sqrt{-a}\, d}{\sqrt{b}}\right)}{2b^2}-\frac{x^2 \sin(c+dx)}{2b\left(a+bx^2\right)}+\frac{\sin(c+dx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[-a]*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x])/(4*b^(5/2)) - (Sqrt[-a]*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral
[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[
b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) + (CosIntegral[(Sqrt[-a]*d
)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) + Sin[c + d*x]/(2*b
^2) - (x^2*Sin[c + d*x])/(2*b*(a + b*x^2)) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]
*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) + (Sqrt[-a]*d*Sin[c + (Sq
rt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^(5/2)) + (
Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b
^2) + (Sqrt[-a]*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] + d*x])/(4*b^(5/2))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2}\,dx = -\frac{x^2\sin(c+dx)}{2b(a+bx^2)} + \frac{\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{b} + \frac{d\int \frac{x^2\cos(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^2\sin(c+dx)}{2b(a+bx^2)} + \frac{\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{b} + \frac{d\int \left(\frac{\cos(\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{x^2\sin(c+dx)}{2b(a+bx^2)} - \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2b^{3/2}} + \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2b^{3/2}} + \frac{d\int \cos(c+dx)\,dx}{2b^2} - \frac{(a\ldots}{\ldots}$$

$$= \frac{\sin(c+dx)}{2b^2} - \frac{x^2\sin(c+dx)}{2b(a+bx^2)} - \frac{(ad)\int \left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{2b^2} + \ldots$$

$$= \frac{\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^2} + \frac{\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^2} + \frac{\sin(\ldots}{\ldots}$$

$$= \frac{\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^2} + \frac{\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^2} + \frac{\sin(\ldots}{\ldots}$$

$$= \frac{\sqrt{-a}\,d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^{5/2}} - \frac{\sqrt{-a}\,d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}\ldots}{\sqrt{b}}\right.}{4b^{5/2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.51, size = 583, normalized size = 1.35

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] ((a + b*x^2)*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*((-I)*Sqrt[a]*d*Cos[c
 - (I*Sqrt[a]*d)/Sqrt[b]] + 2*Sqrt[b]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]) + (a
+ b*x^2)*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*(I*Sqrt[a]*d*Cos[c + (
I*Sqrt[a]*d)/Sqrt[b]] + 2*Sqrt[b]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]) + 2*a*Sqr
t[b]*Sin[c + d*x] + 2*a*Sqrt[b]*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[
d*((I*Sqrt[a])/Sqrt[b] + x)] + 2*b^(3/2)*x^2*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]
```

```
*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*a^(3/2)*d*Sin[c - (I*Sqrt[a]*
d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*Sqrt[a]*b*d*x^2*Si
n[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - 2*a
*Sqrt[b]*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] -
 d*x] - 2*b^(3/2)*x^2*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]
*d)/Sqrt[b] - d*x] + I*a^(3/2)*d*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral
[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + I*Sqrt[a]*b*d*x^2*Sin[c + (I*Sqrt[a]*d)/Sqr
t[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(4*b^(5/2)*(a + b*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2530 vs. $2(331) = 662$.

time = 0.16, size = 2531, normalized size = 5.87

method	result
risch	$-\frac{id\sqrt{ab}\, e^{\frac{ibc-d\sqrt{ab}}{b}}\, \text{expIntegral}\left(1, \frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)}{8b^3}+\frac{id\, e^{\frac{ibc+d\sqrt{ab}}{b}}\, \text{expIntegral}\left(1, \frac{ibc+d\sqrt{ab}-b(id}{b}\right.}{8b^3}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-d^4*c^3*(sin(d*x+c)*(1/2/a/d^2*(d*x+c)-1/2*c/a/d^2)/(d^2*a+b*c^2-2*
b*c*(d*x+c)+b*(d*x+c)^2)-1/4/a/d^2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-
(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2
)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/d^2/b/((d*(-a*b)^(1/2)-c*b)/b+
c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(
d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4/a/b/d^2*(-Si(d*x+c-
(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2
)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/b/d^2*(Si(d*x+c+(d*(-a*b)^(1/2
)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos(
(d*(-a*b)^(1/2)-c*b)/b)))+sin(d*x+c)*(3/2*c^3*d^2/a*(d*x+c)-3/2*d^2*c^2*(a*
d^2+b*c^2)/a/b)/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)-3/4*c^3*d^2/a/b/(-(
d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1
/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-3
/4*c^3*d^2/a/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)
*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)
^(1/2)-c*b)/b))-3/4*d^2*c^2*(d^2*a+b*c^2-c*(d*(-a*b)^(1/2)+c*b))/a/b^2/(-(d
*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1
/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-3
/4*d^2*c^2*(d^2*a+b*c^2+c*(d*(-a*b)^(1/2)-c*b))/a/b^2/((d*(-a*b)^(1/2)-c*b)
/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+
c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))+sin(d*x+c)*(3/2*d^2*
c*(a*d^2-b*c^2)/a/b*(d*x+c)+3/2*d^2*c^2*(a*d^2+b*c^2)/a/b)/(d^2*a+b*c^2-2*b
```

```
*c*(d*x+c)+b*(d*x+c)^2)+3/4*d^2*c*(a*d^2+b*c^2)/a/b^2/(-(d*(-a*b)^(1/2)+c*b
)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x
+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+3/4*d^2*c*(a*d^2+b*
c^2)/a/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos
((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/
2)-c*b)/b))+3/4*d^2*c*(a*c*d^2+a*d^2*(d*(-a*b)^(1/2)+c*b)/b+b*c^3-c^2*(d*(-
a*b)^(1/2)+c*b))/a/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/
2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos
((d*(-a*b)^(1/2)+c*b)/b))+3/4*d^2*c*(a*c*d^2-a*d^2*(d*(-a*b)^(1/2)-c*b)/b+b
*c^3+c^2*(d*(-a*b)^(1/2)-c*b))/a/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(
d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)
-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))+sin(d*x+c)*(-1/2*d^2*c*(3*a*d^2-b*c^2
)/a/b*(d*x+c)+1/2*d^2*(a^2*d^4-b^2*c^4)/a/b^2)/(d^2*a+b*c^2-2*b*c*(d*x+c)+b
*(d*x+c)^2)-1/4*d^2*(a*c*d^2+2*a*d^2*(d*(-a*b)^(1/2)+c*b)/b+b*c^3)/a/b^2/(-
(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(
1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-
1/4*d^2*(a*c*d^2-2*a*d^2*(d*(-a*b)^(1/2)-c*b)/b+b*c^3)/a/b^2/((d*(-a*b)^(1/
2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-
Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))+1/4*d^2*(a^2*
d^4-3*a*c*d^2*(d*(-a*b)^(1/2)+c*b)-b^2*c^4+b*c^3*(d*(-a*b)^(1/2)+c*b))/a/b^
3/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a
*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)
/b))+1/4*d^2*(a^2*d^4+3*a*c*d^2*(d*(-a*b)^(1/2)-c*b)-b^2*c^4-b*c^3*(d*(-a*b
)^(1/2)-c*b))/a/b^3/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*
b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(
-a*b)^(1/2)-c*b)/b)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^2*sin(d*x + c) + ((d^2*x^3*cos(c) - d*x^2*s
in(c) - 2*x*cos(c))*cos(d*x + c)^2 + (d^2*x^3*cos(c) - d*x^2*sin(c) - 2*x*c
os(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((cos(c)^2 + sin(c)^2)*d^2*x^3 - 2*
(cos(c)^2 + sin(c)^2)*x)*cos(d*x + c) - 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d
^3*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^2 + (a^2*cos(c)^2 + a^2*sin(
c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^4 + 2*(a*b
*cos(c)^2 + a*b*sin(c)^2)*d^3*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*sin(
d*x + c)^2)*integrate((2*a*d*x*sin(d*x + c) + ((2*a*d^2 + 3*b)*x^2 - a)*cos
(d*x + c))/(b^3*d^3*x^6 + 3*a*b^2*d^3*x^4 + 3*a^2*b*d^3*x^2 + a^3*d^3), x)
- 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^
```

```
2)*d^3*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(
c)^2 + b^2*sin(c)^2)*d^3*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^2 + (a
^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate((2*a*d*x*sin(d*x
 + c) + ((2*a*d^2 + 3*b)*x^2 - a)*cos(d*x + c))/((b^3*d^3*x^6 + 3*a*b^2*d^3
*x^4 + 3*a^2*b*d^3*x^2 + a^3*d^3)*cos(d*x + c)^2 + (b^3*d^3*x^6 + 3*a*b^2*d
^3*x^4 + 3*a^2*b*d^3*x^2 + a^3*d^3)*sin(d*x + c)^2), x) + ((d^2*x^3*sin(c)
+ d*x^2*cos(c) - 2*x*sin(c))*cos(d*x + c)^2 + (d^2*x^3*sin(c) + d*x^2*cos(c
) - 2*x*sin(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^2*cos(c)^2 + b^2*sin(c
)^2)*d^3*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^2 + (a^2*cos(c)^2 + a^
2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^4 +
2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3
)*sin(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 291, normalized size = 0.68

$$-\frac{\left(2i\,bx^2-(-i\,bx^2-i\,a)\sqrt{\frac{ad^2}{b}}+2i\,a\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(2i\,bx^2-(i\,bx^2+i\,a)\sqrt{\frac{ad^2}{b}}+2i\,a\right)\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\left(-2i\,bx^2-(i\,bx^2+i\,a)\sqrt{\frac{ad^2}{b}}-2i\,a\right)\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(-2i\,bx^2-(-i\,bx^2-i\,a)\sqrt{\frac{ad^2}{b}}-2i\,a\right)\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c-\sqrt{\frac{ad^2}{b}}\right)}-4\,a\sin\left(dx+c\right)}{8\,(b^3x^2+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((2*I*b*x^2 - (-I*b*x^2 - I*a)*sqrt(a*d^2/b) + 2*I*a)*Ei(I*d*x - sqrt(
a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (2*I*b*x^2 - (I*b*x^2 + I*a)*sqrt(a*d^2
/b) + 2*I*a)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (-2*I*b*x^
2 - (I*b*x^2 + I*a)*sqrt(a*d^2/b) - 2*I*a)*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I
*c + sqrt(a*d^2/b)) + (-2*I*b*x^2 - (-I*b*x^2 - I*a)*sqrt(a*d^2/b) - 2*I*a)
*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*a*sin(d*x + c))/(b
^3*x^2 + a*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sin\left(c+dx\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*sin(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sin\left(c+d\,x\right)}{{\left(b\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sin(c + d*x))/(a + b*x^2)^2,x)
```

```
[Out] int((x^3*sin(c + d*x))/(a + b*x^2)^2, x)
```

3.67 $\int \frac{x^2 \sin(c+dx)}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=416

$$\frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^2}+\frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4b^2}-\frac{\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(\right.}{4\sqrt{-a}\,b^{3/2}}$$

```
[Out] 1/4*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))/b^2+1/4*d*Ci
(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2))/b^2-1/2*x*sin(d*x+c
)/b/(b*x^2+a)-1/4*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2)
)/b^2-1/4*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/b^2+1/
4*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/
2)-1/4*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a
)^(1/2)-1/4*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/b^(3/2
)/(-a)^(1/2)+1/4*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/
b^(3/2)/(-a)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3424, 3414, 3384, 3380, 3383, 3427}

$$\frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4\sqrt{-a}\,b^{3/2}}+\frac{d\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^2}+\frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4b^2}+\frac{d\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^2}-\frac{d\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4b^2}-\frac{x\sin(c+dx)}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
4*b^2) + (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
+ d*x])/(4*b^2) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a
]*d)/Sqrt[b]])/(4*Sqrt[-a]*b^(3/2)) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d
*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*Sqrt[-a]*b^(3/2)) - (x*Sin[c + d*x])/
(2*b*(a + b*x^2)) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)
/Sqrt[b] - d*x])/(4*Sqrt[-a]*b^(3/2)) + (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*Si
nIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^2) - (Cos[c - (Sqrt[-a]*d)/Sqrt
[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2)) - (d*Sin
[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^2)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2\sin(c+dx)}{(a+bx^2)^2}\,dx = -\frac{x\sin(c+dx)}{2b(a+bx^2)} + \frac{\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{2b} + \frac{d\int \frac{x\cos(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x\sin(c+dx)}{2b(a+bx^2)} + \frac{\int \left(\frac{\sqrt{-a}\,\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{-a}\,\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{2b} + \frac{d\int \left(-\frac{\cos}{2\sqrt{b}\left(\sqrt{\right.}\right.}\right.}{}$$

$$= -\frac{x\sin(c+dx)}{2b(a+bx^2)} - \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4\sqrt{-a}\,b} - \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{4\sqrt{-a}\,b} - \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4b^{3/2}} + $$

$$= -\frac{x\sin(c+dx)}{2b(a+bx^2)} - \frac{\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{4\sqrt{-a}\,b} + \frac{\left(d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right.}{4b^3}$$

$$= \frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^2} + \frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4b^2} - $$

Mathematica [C] Result contains complex when optimal does not.
time = 0.47, size = 583, normalized size = 1.40

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] ((a + b*x^2)*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c - (I
*Sqrt[a]*d)/Sqrt[b]] + I*Sqrt[b]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]) + (a + b*x
^2)*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c + (I*Sqrt[
a]*d)/Sqrt[b]] - I*Sqrt[b]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]) - 2*Sqrt[a]*b*x*
Sin[c + d*x] + I*a*Sqrt[b]*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I
*Sqrt[a])/Sqrt[b] + x)] + I*b^(3/2)*x^2*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinI
ntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - a^(3/2)*d*Sin[c - (I*Sqrt[a]*d)/Sqrt
[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - Sqrt[a]*b*d*x^2*Sin[c - (I*
Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*a*Sqrt[b]*
Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + I
*b^(3/2)*x^2*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[
b] - d*x] + a^(3/2)*d*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]
*d)/Sqrt[b] - d*x] + Sqrt[a]*b*d*x^2*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinInte
gral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(4*Sqrt[a]*b^2*(a + b*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1779 vs. 2(318) = 636.
time = 0.13, size = 1780, normalized size = 4.28

method	result
risch	$-\frac{d\,\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}\,\mathrm{expIntegral}\left(1,\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{8b^2}-\frac{d\,\mathrm{e}^{\frac{ibc-d\sqrt{ab}}{b}}\,\mathrm{expIntegral}\left(1,\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)}{8b^2}-$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(d^4*c^2*(sin(d*x+c)*(1/2/a/d^2*(d*x+c)-1/2*c/a/d^2)/(d^2*a+b*c^2-2*b
*c*(d*x+c)+b*(d*x+c)^2)-1/4/a/d^2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(
d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)
+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/d^2/b/((d*(-a*b)^(1/2)-c*b)/b+c
)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d
*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4/a/b/d^2*(-Si(d*x+c-(
d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)
+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/b/d^2*(Si(d*x+c+(d*(-a*b)^(1/2)
-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((
d*(-a*b)^(1/2)-c*b)/b)))+sin(d*x+c)*(-d^2*c^2/a*(d*x+c)+d^2*c*(a*d^2+b*c^2)
/a/b)/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)+1/2*d^2*c^2/a/b/(-(d*(-a*b)^(
1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b
)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/2*d^2*c^2
/a/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-
a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b
)/b))+1/2*d^2*c*(d^2*a+b*c^2-c*(d*(-a*b)^(1/2)+c*b))/a/b^2/(-(d*(-a*b)^(1/2
)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+
Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))+1/2*d^2*c*(d^
2*a+b*c^2+c*(d*(-a*b)^(1/2)-c*b))/a/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+
c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1
/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))+sin(d*x+c)*(-1/2*d^2*(a*d^2-b*c^2)
/a/b*(d*x+c)-1/2*d^2*c*(a*d^2+b*c^2)/a/b)/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x
+c)^2)-1/4*d^2*(a*d^2+b*c^2)/a/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d
*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/4*d^2*(a*d^2+b*c^2)/a/b^2/((d*(-a*b)
^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)
/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4*d^2*(
a*c*d^2+a*d^2*(d*(-a*b)^(1/2)+c*b)/b+b*c^3-c^2*(d*(-a*b)^(1/2)+c*b))/a/b^2/
(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b
)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b
))-1/4*d^2*(a*c*d^2-a*d^2*(d*(-a*b)^(1/2)-c*b)/b+b*c^3+c^2*(d*(-a*b)^(1/2)-
c*b))/a/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*si
n((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1
```

```
/2)-c*b)/b)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + 2*(cos(c)^2 + sin(c)^2)*x*
sin(d*x + c) + ((d*x^2*cos(c) - 2*x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c)
- 2*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^2*cos(c)^2 + b^2*sin(
c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^2 + (a^2*cos(c)^2 + a
^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^4 +
 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^
2)*sin(d*x + c)^2)*integrate(-(2*a*d*x*cos(d*x + c) - (3*b*x^2 - a)*sin(d*x
 + c))/(b^3*d^2*x^6 + 3*a*b^2*d^2*x^4 + 3*a^2*b*d^2*x^2 + a^3*d^2), x) + 2*
(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d
^2*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c)^2
 + b^2*sin(c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^2 + (a^2*c
os(c)^2 + a^2*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(2*a*d*x*cos(d*x +
c) - (3*b*x^2 - a)*sin(d*x + c))/((b^3*d^2*x^6 + 3*a*b^2*d^2*x^4 + 3*a^2*b*
d^2*x^2 + a^3*d^2)*cos(d*x + c)^2 + (b^3*d^2*x^6 + 3*a*b^2*d^2*x^4 + 3*a^2*
b*d^2*x^2 + a^3*d^2)*sin(d*x + c)^2), x) + ((d*x^2*sin(c) + 2*x*cos(c))*cos
(d*x + c)^2 + (d*x^2*sin(c) + 2*x*cos(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(
((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^
2*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c)^2
+ b^2*sin(c)^2)*d^2*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^2 + (a^2*co
s(c)^2 + a^2*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.36, size = 333, normalized size = 0.80

$$-\frac{4\,abdx\sin\left(dx+c\right)-\left(abd^2x^2+a^2d^2+\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right){\rm Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\left(abd^2x^2+a^2d^2-\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right){\rm Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}-\left(abd^2x^2+a^2d^2+\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right){\rm Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\left(abd^2x^2+a^2d^2-\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right){\rm Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c-\sqrt{\frac{ad^2}{b}}\right)}}{8\,\left(ab^3dx^2+a^2b^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*b*d*x*sin(d*x + c) - (a*b*d^2*x^2 + a^2*d^2 + (b^2*x^2 + a*b)*sqr
t(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^
2 + a^2*d^2 - (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I
*c - sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*d^2 + (b^2*x^2 + a*b)*sqrt(a*d^2/b
))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2
*d^2 - (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c -
sqrt(a*d^2/b)))/(a*b^3*d*x^2 + a^2*b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sin (c + dx)}{(a + bx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sin (c + d\,x)}{(b\,x^2 + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sin(c + d*x))/(a + b*x^2)^2,x)
```

```
[Out] int((x^2*sin(c + d*x))/(a + b*x^2)^2, x)
```

3.68 $\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=239

$$\frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{\sin(c+dx)}{2b\,(a+bx^2)}+\frac{d\sin\left(c+\right.}{}$$

```
[Out] -1/2*sin(d*x+c)/b/(b*x^2+a)-1/4*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)
^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)+1/4*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(
c+d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)+1/4*d*Si(d*x+d*(-a)^(1/2)/b^(1/2
))*sin(c-d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)-1/4*d*Si(d*x-d*(-a)^(1/2)
/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/b^(3/2)/(-a)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3422, 3415, 3384, 3380, 3383}

$$\frac{d\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}\,b^{3/2}}+\frac{d\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}\,b^{3/2}}+\frac{d\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{\sin(c+dx)}{2b\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
4*Sqrt[-a]*b^(3/2)) - (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2)) - Sin[c + d*x]/(2*b*(a + b*x^2))
+ (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(4*Sqrt[-a]*b^(3/2)) + (d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[
-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx &= -\frac{\sin(c+dx)}{2b(a+bx^2)}+\frac{d\int \frac{\cos(c+dx)}{a+bx^2}\,dx}{2b} \\
&= -\frac{\sin(c+dx)}{2b(a+bx^2)}+\frac{d\int \left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{2b} \\
&= -\frac{\sin(c+dx)}{2b(a+bx^2)}-\frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4\sqrt{-a}\,b}-\frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{4\sqrt{-a}\,b} \\
&= -\frac{\sin(c+dx)}{2b(a+bx^2)}-\frac{\left(d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{4\sqrt{-a}\,b}-\frac{\left(d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right.\right.}{4\sqrt{\vphantom{a}}} \\
&= \frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}\,b^{3/2}}-\frac{\mathrm{s}}{2}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.29, size = 309, normalized size = 1.29

$$-\frac{i\left(d(a+bx^2)\cos\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-d(a+bx^2)\cos\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-2i\sqrt{a}\sqrt{b}\sin(c+dx)+ad\sin\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\operatorname{Si}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+bdx^2\sin\left(c-\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\operatorname{Si}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+ad\sin\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{i\sqrt{a}\,d}{\sqrt{b}}-dx\right)+bdx^2\sin\left(c+\frac{i\sqrt{a}\,d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{i\sqrt{a}\,d}{\sqrt{b}}-dx\right)\right)}{4\sqrt{a}\,b^{3/2}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] ((-1/4*I)*(d*(a + b*x^2)*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*(((-I
)*Sqrt[a])/Sqrt[b] + x)] - d*(a + b*x^2)*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*Cos
Integral[d*((I*Sqrt[a])/Sqrt[b] + x)] - (2*I)*Sqrt[a]*Sqrt[b]*Sin[c + d*x]
+ a*d*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x
)] + b*d*x^2*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt
[b] + x)] + a*d*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sq
rt[b] - d*x] + b*d*x^2*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a
]*d)/Sqrt[b] - d*x]))/(Sqrt[a]*b^(3/2)*(a + b*x^2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1092 vs. $2(181) = 362$.
time = 0.08, size = 1093, normalized size = 4.57 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(sin(d*x+c)*(1/2*d^2*c/a*(d*x+c)-1/2*d^2*(a*d^2+b*c^2)/a/b)/(d^2*a+b*
c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)-1/4*d^2*c/a/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(S
i(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a
*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/4*d^2*c/a/b/((d*(-a*b)^(1/
2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-
Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4*d^2*(d^2*
a+b*c^2-c*(d*(-a*b)^(1/2)+c*b))/a/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+
c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1
/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-1/4*d^2*(d^2*a+b*c^2+c*(d*(-a*b)^(
1/2)-c*b))/a/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/
b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*
b)^(1/2)-c*b)/b))-c*d^4*(sin(d*x+c)*(1/2/a/d^2*(d*x+c)-1/2*c/a/d^2)/(d^2*a+
b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)-1/4/a/d^2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(S
i(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a
*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/d^2/b/((d*(-a*b)^(1/2)
-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci
(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4/a/b/d^2*(-S
i(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a
*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/b/d^2*(Si(d*x+c+(d*(-a
*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)
/b)*cos((d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*c
os(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*
x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)
*d)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(a*b*cos(c)^2
 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)*i
ntegrate(1/2*(3*b*x^2 - a)*cos(d*x + c)/(b^3*d*x^6 + 3*a*b^2*d*x^4 + 3*a^2*
b*d*x^2 + a^3*d), x) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(a*b*cos
(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)
^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)
*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(3*
b*x^2 - a)*cos(d*x + c)/((b^3*d*x^6 + 3*a*b^2*d*x^4 + 3*a^2*b*d*x^2 + a^3*d
)*cos(d*x + c)^2 + (b^3*d*x^6 + 3*a*b^2*d*x^4 + 3*a^2*b*d*x^2 + a^3*d)*sin(
d*x + c)^2), x) + (x*cos(d*x + c)^2*sin(c) + x*sin(d*x + c)^2*sin(c))*sin(d
*x + 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(a*b*cos(c)^2 + a*b*si
n(c)^2)*d*x^2 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^2*cos
(c)^2 + b^2*sin(c)^2)*d*x^4 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^2 + (a^2*
cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.36, size = 244, normalized size = 1.02

$$\frac{(i\,bx^2+i\,a)\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+(-i\,bx^2-i\,a)\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+(-i\,bx^2-i\,a)\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}+(i\,bx^2+i\,a)\sqrt{\frac{ad^2}{b}}\,\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c-\sqrt{\frac{ad^2}{b}}\right)}-4\,a\sin\left(dx+c\right)}{8\,(ab^2x^2+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((I*b*x^2 + I*a)*sqrt(a*d^2/b)*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(
a*d^2/b)) + (-I*b*x^2 - I*a)*sqrt(a*d^2/b)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c
 - sqrt(a*d^2/b)) + (-I*b*x^2 - I*a)*sqrt(a*d^2/b)*Ei(-I*d*x - sqrt(a*d^2/b
))*e^(-I*c + sqrt(a*d^2/b)) + (I*b*x^2 + I*a)*sqrt(a*d^2/b)*Ei(-I*d*x + sqr
t(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*a*sin(d*x + c))/(a*b^2*x^2 + a^2*b
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin\left(c+dx\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*sin(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \sin(c + d x)}{(b x^2 + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(c + d*x))/(a + b*x^2)^2,x)
```

```
[Out] int((x*sin(c + d*x))/(a + b*x^2)^2, x)
```

3.69 $\int \frac{\sin(c+dx)}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=476

$$-\frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4ab}-\frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4ab}+\frac{\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(\right.}{4(-a)^{3/2}\sqrt{b}}$$

```
[Out] -1/4*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))/a/b-1/4*d*C
i(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2))/a/b+1/4*d*Si(d*x+d
*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a/b+1/4*d*Si(d*x-d*(-a)^(1
/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/a/b-1/4*cos(c+d*(-a)^(1/2)/b^(1/2)
)*Si(d*x-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)+1/4*cos(c-d*(-a)^(1/2)/b^
(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)+1/4*Ci(d*x+d*(-a)^(1
/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*Ci(-d*x+d*(
-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*sin(d
*x+c)/a/b^(1/2)/((-a)^(1/2)-x*b^(1/2))+1/4*sin(d*x+c)/a/b^(1/2)/((-a)^(1/2)
+x*b^(1/2))
```

Rubi [A]

time = 0.59, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3414, 3378, 3384, 3380, 3383}

$$-\frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4ab}-\frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4ab}-\frac{d\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4ab}+\frac{d\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4ab}+\frac{\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x^2)^2,x]
```

```
[Out] -1/4*(d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*
x])/(a*b) - (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[
b] + d*x])/(4*a*b) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt
[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b
] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) - Sin[c + d*
x]/(4*a*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Sin[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-
a] + Sqrt[b]*x)) + (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*S
inIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a*b) + (Cos[c - (Sqrt[-a]*d)/Sqr
t[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (d*
Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a
*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+bx^2)^2}\,dx = \int \left(-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\,\sqrt{b}\,-bx\right)^2} - \frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\,\sqrt{b}\,+bx\right)^2} - \frac{b\sin(c+dx)}{2a\,(-ab-b^2x^2)} \right) dx$$

$$= -\frac{b\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\,\sqrt{b}\,-bx\right)^2}\,dx}{4a} - \frac{b\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\,\sqrt{b}\,+bx\right)^2}\,dx}{4a} - \frac{b\int \frac{\sin(c+dx)}{-ab-b^2x^2}\,dx}{2a}$$

$$= -\frac{\sin(c+dx)}{4a\sqrt{b}\,\left(\sqrt{-a}\,-\sqrt{b}\,x\right)} + \frac{\sin(c+dx)}{4a\sqrt{b}\,\left(\sqrt{-a}\,+\sqrt{b}\,x\right)} - \frac{b\int \left(-\frac{\sqrt{-a}\,\sin(c+dx)}{2ab\left(\sqrt{-a}\,-\sqrt{b}\,x\right)} - \frac{\cdots}{2ab \cdots}\right.}{2a}$$

$$= -\frac{\sin(c+dx)}{4a\sqrt{b}\,\left(\sqrt{-a}\,-\sqrt{b}\,x\right)} + \frac{\sin(c+dx)}{4a\sqrt{b}\,\left(\sqrt{-a}\,+\sqrt{b}\,x\right)} + \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}\,-\sqrt{b}\,x}\,dx}{4(-a)^{3/2}} + \frac{\int \frac{\sin(c+\cdots}{\sqrt{-a}\,+\cdots}}{4(-a\cdots}$$

$$= -\frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4ab} - \frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4ab} - \cdots$$

$$= -\frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4ab} - \frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4ab} + \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.45, size = 585, normalized size = 1.23

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*x^2)^2,x]
```

```
[Out] (-((a + b*x^2)*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c -
(I*Sqrt[a]*d)/Sqrt[b]] - I*Sqrt[b]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]])) - (a +
b*x^2)*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c + (I*Sq
rt[a]*d)/Sqrt[b]] + I*Sqrt[b]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]) + 2*Sqrt[a]*b
*x*Sin[c + d*x] + I*a*Sqrt[b]*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*
((I*Sqrt[a])/Sqrt[b] + x)] + I*b^(3/2)*x^2*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*S
inIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + a^(3/2)*d*Sin[c - (I*Sqrt[a]*d)/S
```

```
qrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + Sqrt[a]*b*d*x^2*Sin[c -
(I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*a*Sqrt[
b]*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x]
+ I*b^(3/2)*x^2*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sq
rt[b] - d*x] - a^(3/2)*d*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt
[a]*d)/Sqrt[b] - d*x] - Sqrt[a]*b*d*x^2*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinI
ntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(4*a^(3/2)*b*(a + b*x^2))
```

Maple [A]

time = 0.06, size = 491, normalized size = 1.03

method	result
derivativedivides	$d^3\left(\frac{\sin(dx+c)\left(\frac{dx+c}{2a\,d^2}-\frac{c}{2a\,d^2}\right)}{d^2a+b\,c^2-2bc(dx+c)+b(dx+c)^2}-\frac{\mathrm{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{cosineIntegral}(}{4a\,d^2b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}\right.$
default	$d^3\left(\frac{\sin(dx+c)\left(\frac{dx+c}{2a\,d^2}-\frac{c}{2a\,d^2}\right)}{d^2a+b\,c^2-2bc(dx+c)+b(dx+c)^2}-\frac{\mathrm{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\mathrm{cosineIntegral}(}{4a\,d^2b\left(-\frac{d\sqrt{-ab}+cb}{b}+c\right)}\right.$
risch	$\frac{d\,e^{\frac{ibc+d\sqrt{ab}}{b}}\,\mathrm{expIntegral}\left(1,\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{8ab}+\frac{d\,e^{\frac{ibc-d\sqrt{ab}}{b}}\,\mathrm{expIntegral}\left(1,\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)}{8ab}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(sin(d*x+c)*(1/2/a/d^2*(d*x+c)-1/2*c/a/d^2)/(d^2*a+b*c^2-2*b*c*(d*x+c)+
b*(d*x+c)^2)-1/4/a/d^2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1
/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*si
n((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/d^2/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c
+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/
2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4/a/b/d^2*(-Si(d*x+c-(d*(-a*b)^(1
/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*co
s((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/b/d^2*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*si
n((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1
/2)-c*b)/b)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

[Out] integrate(sin(d*x + c)/(b*x^2 + a)^2, x)

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 333, normalized size = 0.70

$$\frac{4\,abdx\sin\left(dx+c\right)-\left(abd^2x^2+a^2d^2-\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\left(abd^2x^2+a^2d^2+\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}-\left(abd^2x^2+a^2d^2-\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\left(abd^2x^2+a^2d^2+\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c-\sqrt{\frac{ad^2}{b}}\right)}}{8\left(a^2b^2dx^2+a^3bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/8*(4*a*b*d*x*sin(d*x + c) - (a*b*d^2*x^2 + a^2*d^2 - (b^2*x^2 + a*b)*sqrt
(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^2
 + a^2*d^2 + (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*
c - sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*d^2 - (b^2*x^2 + a*b)*sqrt(a*d^2/b)
)*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*
d^2 + (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - s
qrt(a*d^2/b)))/(a^2*b^2*d*x^2 + a^3*b*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(c+dx\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/(b*x**2+a)**2,x)

[Out] Integral(sin(c + d*x)/(a + b*x**2)**2, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")

[Out] integrate(sin(d*x + c)/(b*x^2 + a)^2, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\sin\left(c+d\,x\right)}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c + d*x)/(a + b*x^2)^2,x)

[Out] int(sin(c + d*x)/(a + b*x^2)^2, x)

3.70 $\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=435

$$\frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\text{Ci}(dx)\sin(c)}{a^2}-\frac{\text{Ci}\left(\frac{\sqrt{-a}}{\sqrt{b}}\right.}{}$$

```
[Out] cos(c)*Si(d*x)/a^2-1/2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1
/2))/a^2-1/2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/a^2+C
i(d*x)*sin(c)/a^2+1/2*sin(d*x+c)/a/(b*x^2+a)-1/2*Ci(d*x+d*(-a)^(1/2)/b^(1/2
))*sin(c-d*(-a)^(1/2)/b^(1/2))/a^2-1/2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+
d*(-a)^(1/2)/b^(1/2))/a^2-1/4*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(
1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)+1/4*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+
d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)+1/4*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))
*sin(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)-1/4*d*Si(d*x-d*(-a)^(1/2)/b
^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(1/2)
```

Rubi [A]

time = 0.63, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3426, 3384, 3380, 3383, 3422, 3415}

$$-\frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2a^2}+\frac{\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a^2}+\frac{\sin(c)\text{CosIntegral}(dx)}{a^2}+\frac{\cos(c)\text{Si}(dx)}{a^2}+\frac{d\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{d\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{d\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sin(c+dx)}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^2)^2),x]
```

```
[Out] (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
4*(-a)^(3/2)*Sqrt[b]) - (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[
-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (CosIntegral[d*x]*Sin[c])/a
^2 - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]
)/(2*a^2) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/S
qrt[b]])/(2*a^2) + Sin[c + d*x]/(2*a*(a + b*x^2)) + (Cos[c]*SinIntegral[d*x
])/a^2 + (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x])/(2*a^2) + (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/S
qrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinI
ntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2) + (d*Sin[c - (Sqrt[-a]*d)/Sqrt
[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^2}\,dx = \int \left(\frac{\sin(c+dx)}{a^2 x} - \frac{bx\sin(c+dx)}{a\left(a+bx^2\right)^2} - \frac{bx\sin(c+dx)}{a^2\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^2} - \frac{b\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{a^2} - \frac{b\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a}$$

$$= \frac{\sin(c+dx)}{2a\left(a+bx^2\right)} - \frac{b\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{a^2} - \frac{d\int \frac{\cos(c+dx)}{a+bx^2}}{2a}$$

$$= \frac{\mathrm{Ci}(dx)\sin(c)}{a^2} + \frac{\sin(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cos(c)\mathrm{Si}(dx)}{a^2} + \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2a^2} - \frac{\sqrt{b}\int \frac{\sin(c}{\sqrt{-a}}}{2a^2}$$

$$= \frac{\mathrm{Ci}(dx)\sin(c)}{a^2} + \frac{\sin(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cos(c)\mathrm{Si}(dx)}{a^2} - \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{4(-a)^{3/2}} - \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}}}{4(-a)^{3/2}}$$

$$= \frac{\mathrm{Ci}(dx)\sin(c)}{a^2} - \frac{\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a^2} - \frac{\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)\sin\left(c+\right.}{2a^2}$$

$$= \frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.53, size = 650, normalized size = 1.49

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^2)^2),x]
```

```
[Out] (4*a*Sqrt[b]*CosIntegral[d*x]*Sin[c] + 4*b^(3/2)*x^2*CosIntegral[d*x]*Sin[c
] - I*(a + b*x^2)*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c
 - (I*Sqrt[a]*d)/Sqrt[b]] - (2*I)*Sqrt[b]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]) +
 I*(a + b*x^2)*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c
 + (I*Sqrt[a]*d)/Sqrt[b]] + (2*I)*Sqrt[b]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]) +
 2*a*Sqrt[b]*Sin[c + d*x] + 4*a*Sqrt[b]*Cos[c]*SinIntegral[d*x] + 4*b^(3/2)
*x^2*Cos[c]*SinIntegral[d*x] - 2*a*Sqrt[b]*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*S
```

```
inIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - 2*b^(3/2)*x^2*Cos[c - (I*Sqrt[a]*
d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*a^(3/2)*d*Sin[c -
(I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*Sqrt[a]
*b*d*x^2*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b]
+ x)] + 2*a*Sqrt[b]*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d
)/Sqrt[b] - d*x] + 2*b^(3/2)*x^2*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral
[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + I*a^(3/2)*d*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*
SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + I*Sqrt[a]*b*d*x^2*Sin[c + (I*Sqr
t[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(4*a^2*Sqrt[b]*(
a + b*x^2))
```

Maple [A]
time = 0.16, size = 478, normalized size = 1.10

method	result
derivativedivides	$\frac{\sin(dx+c)d^2}{2a\left(d^2a+b\,c^2-2bc(dx+c)+b(dx+c)^2\right)}-\frac{\text{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\text{cosineIntegral}\left(dx\right.}{2a^2}$
default	$\frac{\sin(dx+c)d^2}{2a\left(d^2a+b\,c^2-2bc(dx+c)+b(dx+c)^2\right)}-\frac{\text{sinIntegral}\left(dx+c-\frac{d\sqrt{-ab}+cb}{b}\right)\cos\left(\frac{d\sqrt{-ab}+cb}{b}\right)+\text{cosineIntegral}\left(dx\right.}{2a^2}$
risch	$\frac{i\,e^{\frac{ibc+d\sqrt{ab}}{b}}\,\text{expIntegral}\left(1,\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)d}{8a\sqrt{ab}}-\frac{i\,e^{\frac{ibc-d\sqrt{ab}}{b}}\,\text{expIntegral}\left(1,\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)d}{8a\sqrt{ab}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(d*x+c)*d^2/a/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)-1/2/a^2*(Si(d*
x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^
(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/2/a^2*(Si(d*x+c+(d*(-a*b)^(1/2
)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin(
(d*(-a*b)^(1/2)-c*b)/b))+1/a^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+1/4*d^2/a/b/
(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b
)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b
))+1/4*d^2/a/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)
*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)
^(1/2)-c*b)/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^2*x), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.40, size = 332, normalized size = 0.76

$$-\frac{4\,(i\,bx^2+i\,a)\mathrm{Ei}(i\,dx)\,e^{(i\,c)}+4\,(-i\,bx^2-i\,a)\mathrm{Ei}(-i\,dx)\,e^{(-i\,c)}+\left(-2i\,bx^2-(-i\,bx^2-i\,a)\sqrt{\frac{ad^2}{b}}-2i\,a\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(-2i\,bx^2-(i\,bx^2+i\,a)\sqrt{\frac{ad^2}{b}}-2i\,a\right)\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\left(2i\,bx^2-(i\,bx^2+i\,a)\sqrt{\frac{ad^2}{b}}+2i\,a\right)\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(2i\,bx^2-(-i\,bx^2-i\,a)\sqrt{\frac{ad^2}{b}}+2i\,a\right)\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c-\sqrt{\frac{ad^2}{b}}\right)}-4\,a\sin\left(dx+c\right)}{8\,(a^2bx^2+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(I*b*x^2 + I*a)*Ei(I*d*x)*e^(I*c) + 4*(-I*b*x^2 - I*a)*Ei(-I*d*x)*e
^(-I*c) + (-2*I*b*x^2 - (-I*b*x^2 - I*a)*sqrt(a*d^2/b) - 2*I*a)*Ei(I*d*x -
sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (-2*I*b*x^2 - (I*b*x^2 + I*a)*sqrt
(a*d^2/b) - 2*I*a)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (2*I
*b*x^2 - (I*b*x^2 + I*a)*sqrt(a*d^2/b) + 2*I*a)*Ei(-I*d*x - sqrt(a*d^2/b))*
e^(-I*c + sqrt(a*d^2/b)) + (2*I*b*x^2 - (-I*b*x^2 - I*a)*sqrt(a*d^2/b) + 2*
I*a)*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*a*sin(d*x + c)
)/(a^2*b*x^2 + a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{\left(c + dx \right)}}{x \left(a + bx^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x**2+a)**2,x)
```

```
[Out] Integral(sin(c + d*x)/(x*(a + b*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)}{x\,{\left(b\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x*(a + b*x^2)^2),x)
```

```
[Out] int(sin(c + d*x)/(x*(a + b*x^2)^2), x)
```

3.71 $\int \frac{\sin(c+dx)}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=501

$$\frac{d\cos(c)\mathrm{Ci}(dx)}{a^2}+\frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4a^2}+\frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4a^2}+\frac{3\sqrt{b}\,\mathrm{Ci}}{}$$

```
[Out] d*Ci(d*x)*cos(c)/a^2+1/4*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/
b^(1/2))/a^2+1/4*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2)
)/a^2-d*Si(d*x)*sin(c)/a^2-sin(d*x+c)/a^2/x-1/4*d*Si(d*x+d*(-a)^(1/2)/b^(1/
2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a^2-1/4*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(
c+d*(-a)^(1/2)/b^(1/2))/a^2-3/4*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(
1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)+3/4*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(
-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)+3/4*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(
c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)-3/4*Ci(-d*x+d*(-a)^(1/2)/b^(1/2)
)*sin(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(5/2)+1/4*sin(d*x+c)*b^(1/2)/a^2
/((-a)^(1/2)-x*b^(1/2))-1/4*sin(d*x+c)*b^(1/2)/a^2/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A]
time = 1.00, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3426, 3378, 3384, 3380, 3383, 3414}

$$\frac{d\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4a^2}+\frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4a^2}+\frac{d\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4a^2}-\frac{d\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4a^2}+\frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}\,x\right)}-\frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}\,x\right)}+\frac{d\cos(c)\mathrm{CosIntegral}(dx)}{a^2}-\frac{d\sin(c)\mathrm{Si}(dx)}{a^2}-\frac{\sin(c+dx)}{a^2x}+\frac{3\sqrt{b}\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}}-\frac{3\sqrt{b}\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}}+\frac{3\sqrt{b}\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}}+\frac{3\sqrt{b}\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{4(-a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a^2 + (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosInteg
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a^2) + (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]
*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a^2) + (3*Sqrt[b]*CosIntegral[
(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(5/2)) -
 (3*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sq
rt[b]])/(4*(-a)^(5/2)) - Sin[c + d*x]/(a^2*x) + (Sqrt[b]*Sin[c + d*x])/(4*a
^2*(Sqrt[-a] - Sqrt[b]*x)) - (Sqrt[b]*Sin[c + d*x])/(4*a^2*(Sqrt[-a] + Sqrt
[b]*x)) - (d*Sin[c]*SinIntegral[d*x])/a^2 + (3*Sqrt[b]*Cos[c + (Sqrt[-a]*d)
/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(5/2)) + (d*Sin[
c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a^2)
+ (3*Sqrt[b]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b]
 + d*x])/(4*(-a)^(5/2)) - (d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqr
t[-a]*d)/Sqrt[b] + d*x])/(4*a^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx = \int \left(\frac{\sin(c+dx)}{a^2x^2} - \frac{b\sin(c+dx)}{a\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^2\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a^2} - \frac{b\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{a^2} - \frac{b\int \frac{\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a}$$

$$= -\frac{\sin(c+dx)}{a^2x} - \frac{b\int\left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^2} - \frac{b\int\left(-\frac{b\sin(c}{4a\left(\sqrt{-a}\right.}\right.}{}$$

$$= -\frac{\sin(c+dx)}{a^2x} + \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{5/2}} + \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{5/2}} + \frac{b^2\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{4a^2}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a^2} - \frac{\sin(c+dx)}{a^2x} + \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d\sin}{}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a^2} + \frac{\sqrt{b}\,\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{5/2}} - \frac{\sqrt{b}\,\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a^2} + \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} + \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\right.}{4a^2}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a^2} + \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} + \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\right.}{4a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.62, size = 768, normalized size = 1.53

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (4*Sqrt[a]*d*x*(a + b*x^2)*Cos[c]*CosIntegral[d*x] + a^(3/2)*d*x*Cos[c - (I
*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + Sqrt[a]*b*d
```

```
*x^3*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)
] - (3*I)*a*Sqrt[b]*x*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin[c - (I*S
qrt[a]*d)/Sqrt[b]] - (3*I)*b^(3/2)*x^3*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] +
 x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] + x*(a + b*x^2)*CosIntegral[d*(((-I)*Sq
rt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]] + (3*I)*Sqrt
[b]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]) - 4*a^(3/2)*Sin[c + d*x] - 6*Sqrt[a]*b*
x^2*Sin[c + d*x] - 4*a^(3/2)*d*x*Sin[c]*SinIntegral[d*x] - 4*Sqrt[a]*b*d*x^
3*Sin[c]*SinIntegral[d*x] - (3*I)*a*Sqrt[b]*x*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]
]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - (3*I)*b^(3/2)*x^3*Cos[c - (I*S
qrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - a^(3/2)*d*x*S
in[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - Sq
rt[a]*b*d*x^3*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqr
t[b] + x)] - (3*I)*a*Sqrt[b]*x*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(
I*Sqrt[a]*d)/Sqrt[b] - d*x] - (3*I)*b^(3/2)*x^3*Cos[c + (I*Sqrt[a]*d)/Sqrt[
b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + a^(3/2)*d*x*Sin[c + (I*Sqrt[
a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + Sqrt[a]*b*d*x^3*S
in[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(4*
a^(5/2)*x*(a + b*x^2))
```

Maple [**A**]
time = 0.19, size = 761, normalized size = 1.52 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/a*b*d^2*(sin(d*x+c)*(1/2/a/d^2*(d*x+c)-1/2*c/a/d^2)/(d^2*a+b*c^2-2*b*
c*(d*x+c)+b*(d*x+c)^2)-1/4/a/d^2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d
*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/d^2/b/((d*(-a*b)^(1/2)-c*b)/b+c)
*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*
(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4/a/b/d^2*(-Si(d*x+c-(d
*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/b/d^2*(Si(d*x+c+(d*(-a*b)^(1/2)-
c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d
*(-a*b)^(1/2)-c*b)/b)))+1/a^2*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c
))-b/a^2*(-1/2/b/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)
/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a
*b)^(1/2)+c*b)/b))-1/2/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/
2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin
((d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")

[Out] integrate(sin(d*x + c)/((b*x^2 + a)^2*x^2), x)

Fricas [C] Result contains complex when optimal does not.
time = 0.39, size = 406, normalized size = 0.81

$$\frac{4\,(abd^2x^3+a^2d^2x)\mathrm{Ei}(i\,dx)\,e^{(ic)} + 4\,(abd^2x^3+a^2d^2x)\mathrm{Ei}(-i\,dx)\,e^{(-ic)} + \left(abd^2x^3+a^2d^2x-3\,(b^2x^3+abx)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic+\sqrt{\frac{ad^2}{b}}\right)} + \left(abd^2x^3+a^2d^2x+3\,(b^2x^3+abx)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic-\sqrt{\frac{ad^2}{b}}\right)} + \left(abd^2x^3+a^2d^2x-3\,(b^2x^3+abx)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-ic+\sqrt{\frac{ad^2}{b}}\right)} + \left(abd^2x^3+a^2d^2x+3\,(b^2x^3+abx)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-ic-\sqrt{\frac{ad^2}{b}}\right)} - 4\,(3\,abdx^2+2\,a^2d)\sin(dx+c)}{8\,(a^3bdx^3+a^4dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/8*(4*(a*b*d^2*x^3 + a^2*d^2*x)*Ei(I*d*x)*e^(I*c) + 4*(a*b*d^2*x^3 + a^2*d^2*x)*Ei(-I*d*x)*e^(-I*c) + (a*b*d^2*x^3 + a^2*d^2*x - 3*(b^2*x^3 + a*b*x)*sqrt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (a*b*d^2*x^3 + a^2*d^2*x + 3*(b^2*x^3 + a*b*x)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (a*b*d^2*x^3 + a^2*d^2*x - 3*(b^2*x^3 + a*b*x)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (a*b*d^2*x^3 + a^2*d^2*x + 3*(b^2*x^3 + a*b*x)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*(3*a*b*d*x^2 + 2*a^2*d)*sin(d*x + c))/(a^3*b*d*x^3 + a^4*d*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{x^2\,(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/x**2/(b*x**2+a)**2,x)

[Out] Integral(sin(c + d*x)/(x**2*(a + b*x**2)**2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")

[Out] integrate(sin(d*x + c)/((b*x^2 + a)^2*x^2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)}{x^2\,(b\,x^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] int(sin(c + d*x)/(x^2*(a + b*x^2)^2), x)
```

3.72 $\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=476

$$-\frac{dx\cos(c+dx)}{8b^2\left(a+bx^2\right)}+\frac{3d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}\,b^{5/2}}-\frac{3d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}\,b^{5/2}}-\frac{d^2\mathrm{C}}{}$$

```
[Out] -1/8*d*x*cos(d*x+c)/b^2/(b*x^2+a)-1/16*d^2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d
*x-d*(-a)^(1/2)/b^(1/2))/b^3-1/16*d^2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*
(-a)^(1/2)/b^(1/2))/b^3-1/4*x^2*sin(d*x+c)/b/(b*x^2+a)^2-1/4*sin(d*x+c)/b^2
/(b*x^2+a)-1/16*d^2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2)
)/b^3-1/16*d^2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/b^
3-3/16*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))/b^(5/2)/(
-a)^(1/2)+3/16*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2))/
b^(5/2)/(-a)^(1/2)+3/16*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b
^(1/2))/b^(5/2)/(-a)^(1/2)-3/16*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)
^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)
```

Rubi [A]
time = 0.80, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3424, 3422, 3415, 3384, 3380, 3383, 3425, 3426}

$$\frac{3d\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}\,b^{5/2}}-\frac{3d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}\,b^{5/2}}+\frac{3d\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}\,b^{5/2}}+\frac{3d\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16\sqrt{-a}\,b^{5/2}}-\frac{d^2\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{16b^3}-\frac{d^2\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16b^3}+\frac{d^2\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\mathrm{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16b^3}-\frac{d^2\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16b^3}-\frac{\sin(c+dx)}{4b^2\left(a+bx^2\right)}-\frac{dx\cos(c+dx)}{8b^2\left(a+bx^2\right)}-\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] -1/8*(d*x*Cos[c + d*x])/(b^2*(a + b*x^2)) + (3*d*Cos[c + (Sqrt[-a]*d)/Sqrt[
b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)) - (3*d*C
os[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*S
qrt[-a]*b^(5/2)) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sq
rt[-a]*d)/Sqrt[b]])/(16*b^3) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*b^3) - (x^2*Sin[c + d*x])/(4*b*(a + b*x
^2)^2) - Sin[c + d*x]/(4*b^2*(a + b*x^2)) + (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[
b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*b^3) + (3*d*Sin[c + (Sqrt[
-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/
2)) - (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x])/(16*b^3) + (3*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*
d)/Sqrt[b] + d*x])/(16*Sqrt[-a]*b^(5/2))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3425

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3\sin(c+dx)}{(a+bx^2)^3}\,dx = -\frac{x^2\sin(c+dx)}{4b\,(a+bx^2)^2} + \frac{\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx}{2b} + \frac{d\int \frac{x^2\cos(c+dx)}{(a+bx^2)^2}\,dx}{4b}$$

$$= -\frac{dx\cos(c+dx)}{8b^2\,(a+bx^2)} - \frac{x^2\sin(c+dx)}{4b\,(a+bx^2)^2} - \frac{\sin(c+dx)}{4b^2\,(a+bx^2)} + \frac{d\int \frac{\cos(c+dx)}{a+bx^2}\,dx}{8b^2} + \frac{d\int \frac{\cos(c+dx)}{a+bx^2}\,d}{4b^2}$$

$$= -\frac{dx\cos(c+dx)}{8b^2\,(a+bx^2)} - \frac{x^2\sin(c+dx)}{4b\,(a+bx^2)^2} - \frac{\sin(c+dx)}{4b^2\,(a+bx^2)} + \frac{d\int \left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{-}}{2a\left(\sqrt{}\right.}\right.}{8b^2}$$

$$= -\frac{dx\cos(c+dx)}{8b^2\,(a+bx^2)} - \frac{x^2\sin(c+dx)}{4b\,(a+bx^2)^2} - \frac{\sin(c+dx)}{4b^2\,(a+bx^2)} - \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{16\sqrt{-a}\,b^2} - \frac{d\int \frac{\mathrm{co}}{\sqrt{-}}}{16\sqrt{}}$$

$$= -\frac{dx\cos(c+dx)}{8b^2\,(a+bx^2)} - \frac{x^2\sin(c+dx)}{4b\,(a+bx^2)^2} - \frac{\sin(c+dx)}{4b^2\,(a+bx^2)} - \frac{\left(d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\right.}{\sqrt{-}}}{16\sqrt{-a}\,b^2}$$

$$= -\frac{dx\cos(c+dx)}{8b^2\,(a+bx^2)} + \frac{3d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}\,b^{5/2}} - \frac{3d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right.}{16\sqrt{-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.35, size = 647, normalized size = 1.36

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] ((-2*Cos[d*x]*(d*x*(a + b*x^2)*Cos[c] + 2*(a + 2*b*x^2)*Sin[c]))/(a + b*x^2
)^2 + (2*(-2*(a + 2*b*x^2)*Cos[c] + d*x*(a + b*x^2)*Sin[c])*Sin[d*x])/(a +
b*x^2)^2 + (d^2*Cos[c]*((-I)*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Si
nh[(Sqrt[a]*d)/Sqrt[b]] + I*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sinh[(
Sqrt[a]*d)/Sqrt[b]] + Cosh[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[d*((I*Sqrt[a]
```

```
)/Sqrt[b] + x)] + SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])))/b + (3*d*Cos[
c]*((-I)*Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] +
x)] + I*Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]
+ Sinh[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + Si
nIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])))/(Sqrt[a]*Sqrt[b]) - (3*d*Sin[c]*(
CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + Cos
Integral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + I*Cosh[(S
qrt[a]*d)/Sqrt[b]]*(SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + SinIntegral[
(I*Sqrt[a]*d)/Sqrt[b] - d*x])))/(Sqrt[a]*Sqrt[b]) - (d^2*Sin[c]*(Cosh[(Sqrt
[a]*d)/Sqrt[b]]*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + Cosh[(Sqrt[a]
*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*Sinh[(Sqrt[a]*d)/
Sqrt[b]]*(SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + SinIntegral[(I*Sqrt[a]
*d)/Sqrt[b] - d*x])))/b)/(16*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3352 vs. $2(374) = 748$.
time = 0.82, size = 3353, normalized size = 7.04

method	result
risch	$-\frac{i d^2 e^{\frac{ibc+d\sqrt{ab}}{b}} \operatorname{expIntegral}\left(1, \frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{32b^3} - \frac{i d^2 e^{\frac{ibc-d\sqrt{ab}}{b}} \operatorname{expIntegral}\left(1, \frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)}{32b^3}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-d^6*c^3*(-1/8*sin(d*x+c)*(5*a*c*d^2-5*a*d^2*(d*x+c)+3*b*c^3-9*b*c^2
*(d*x+c)+9*b*c*(d*x+c)^2-3*b*(d*x+c)^3)/a^2/d^4/(d^2*a+b*c^2-2*b*c*(d*x+c)+
b*(d*x+c)^2)^2+1/8*cos(d*x+c)/a/b/d^2/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^
2)-1/16*(a*d^2+3*b)/a^2/d^4/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-
a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b
)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/16*(a*d^2+3*b)/a^2/d^4/b^2/((d*(-a*b)^(
1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b
)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-3/16/a^2/d^
4/b*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c
-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-3/16/a^2/d^4/b*(Si(d*
x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^
(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)))-3/8*sin(d*x+c)*d^2*c^2*(2*a^2*d
^4+5*a*b*c^2*d^2-5*a*b*c*d^2*(d*x+c)+3*b^2*c^4-9*b^2*c^3*(d*x+c)+9*b^2*c^2*
(d*x+c)^2-3*b^2*c*(d*x+c)^3)/b/a^2/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)^
2+3/8*cos(d*x+c)*c^2*d^4/a/b*(d*x+c)/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2
)-3/16*d^2*c^2*(a*d^2*(d*(-a*b)^(1/2)+c*b)/b+3*c*b)/b^2/a^2/(-(d*(-a*b)^(1/
2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+
```

Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-3/16*d^2*c^2*
(-a*d^2*(d*(-a*b)^(1/2)-c*b)/b+3*c*b)/b^2/a^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(S
i(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a
*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-3/16*d^2*c^2*(d^2*a+3*b*c^2-
3*c*(d*(-a*b)^(1/2)+c*b))/b^2/a^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d
*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-3/16*d^2*c^2*(d^2*a+3*b*c^2+3*c*(d*(-a
*b)^(1/2)-c*b))/b^2/a^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2
)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos(
(d*(-a*b)^(1/2)-c*b)/b))+3/8*sin(d*x+c)*d^2*c*(3*a^2*c*d^4+a^2*d^4*(d*x+c)+
6*a*b*c^3*d^2-8*a*b*c^2*d^2*(d*x+c)+3*a*b*c*d^2*(d*x+c)^2-a*b*d^2*(d*x+c)^3
+3*b^2*c^5-9*b^2*c^4*(d*x+c)+9*b^2*c^3*(d*x+c)^2-3*b^2*c^2*(d*x+c)^3)/b/a^2
/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)^2+3/8*cos(d*x+c)*d^4*c*(d^2*a+b*c^
2-2*b*c*(d*x+c))/a/b^2/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)-3/16*d^2*c*(
a^2*d^4+a*b*c^2*d^2-2*a*c*d^2*(d*(-a*b)^(1/2)+c*b)-a*b*d^2-3*b^2*c^2)/a^2/b
^3/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a
*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)
/b))-3/16*d^2*c*(a^2*d^4+a*b*c^2*d^2+2*a*c*d^2*(d*(-a*b)^(1/2)-c*b)-a*b*d^2
-3*b^2*c^2)/a^2/b^3/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*
b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(
-a*b)^(1/2)-c*b)/b))+3/16*d^2*c*(3*a*c*d^2-a*d^2*(d*(-a*b)^(1/2)+c*b)/b+3*b
*c^3-3*c^2*(d*(-a*b)^(1/2)+c*b))/b^2/a^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d
x+c-(d(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)
^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))+3/16*d^2*c*(3*a*c*d^2+a*d^2*(d*
(-a*b)^(1/2)-c*b)/b+3*b*c^3+3*c^2*(d*(-a*b)^(1/2)-c*b))/b^2/a^2/((d*(-a*b)^
(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/
b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))-1/8*sin(d*
x+c)*d^2*(2*a^3*d^6+7*a^2*b*c^2*d^4-5*a^2*b*c*d^4*(d*x+c)+4*a^2*b*d^4*(d*x+
c)^2+8*a*b^2*c^4*d^2-14*a*b^2*c^3*d^2*(d*x+c)+9*a*b^2*c^2*d^2*(d*x+c)^2-3*a
*b^2*c*d^2*(d*x+c)^3+3*b^3*c^6-9*b^3*c^5*(d*x+c)+9*b^3*c^4*(d*x+c)^2-3*b^3*
c^3*(d*x+c)^3)/a^2/b^2/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)^2-1/8*cos(d*
x+c)*d^4*(2*a*c*d^2+a*d^2*(d*x+c)+2*b*c^3-3*b*c^2*(d*x+c))/a/b^2/(d^2*a+b*c
^2-2*b*c*(d*x+c)+b*(d*x+c)^2)+1/16*d^2*(2*a^2*c*d^4+a^2*d^4*(d*(-a*b)^(1/2)
+c*b)/b+2*a*b*c^3*d^2-3*a*c^2*d^2*(d*(-a*b)^(1/2)+c*b)-3*a*b*c*d^2-3*b^2*c^
3)/a^2/b^3/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*co
s((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1
/2)+c*b)/b))+1/16*d^2*(2*a^2*c*d^4-a^2*d^4*(d*(-a*b)^(1/2)-c*b)/b+2*a*b*c^3
*d^2+3*a*c^2*d^2*(d*(-a*b)^(1/2)-c*b)-3*a*b*c*d^2-3*b^2*c^3)/a^2/b^3/((d*(-
a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-
c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-3/16*
d^2*(a^2*d^4+2*a*b*c^2*d^2-a*c*d^2*(d*(-a*b)^(1/2)+c*b)+b^2*c^4-b*c^3*(d*(-
a*b)^(1/2)+c*b))/a^2/b^3/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(
1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*c
os((d*(-a*b)^(1/2)+c*b)/b))-3/16*d^2*(a^2*d^4+2*a*b*c^2*d^2+a*c*d^2*(d*(-a*
b)^(1/2)-c*b)+b^2*c^4+b*c^3*(d*(-a*b)^(1/2)-c*b))/a^2/b^3/((d*(-a*b)^(1/2)-

```
c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(
d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*(cos(c)^2 + sin(c)^2)*d*x^2*sin(d*x + c) + ((d^2*x^3*cos(c) - 3*d*x
^2*sin(c) - 12*x*cos(c))*cos(d*x + c)^2 + (d^2*x^3*cos(c) - 3*d*x^2*sin(c)
- 12*x*cos(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((cos(c)^2 + sin(c)^2)*d^2*
x^3 - 12*(cos(c)^2 + sin(c)^2)*x)*cos(d*x + c) - 2*(((b^3*cos(c)^2 + b^3*si
n(c)^2)*d^3*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^4 + 3*(a^2*b*co
s(c)^2 + a^2*b*sin(c)^2)*d^3*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*cos(d
*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^6 + 3*(a*b^2*cos(c)^2 + a*
b^2*sin(c)^2)*d^3*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^2 + (a^3*
cos(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(3*(3*a*d*x*sin(d*x
+ c) + ((a*d^2 + 10*b)*x^2 - 2*a)*cos(d*x + c))/(b^4*d^3*x^8 + 4*a*b^3*d^3*
x^6 + 6*a^2*b^2*d^3*x^4 + 4*a^3*b*d^3*x^2 + a^4*d^3), x) - 2*(((b^3*cos(c)^
2 + b^3*sin(c)^2)*d^3*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^4 + 3
*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*
d^3)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^6 + 3*(a*b^2*cos
(c)^2 + a*b^2*sin(c)^2)*d^3*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x
^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(3*(3*a*d*
x*sin(d*x + c) + ((a*d^2 + 10*b)*x^2 - 2*a)*cos(d*x + c))/((b^4*d^3*x^8 + 4
*a*b^3*d^3*x^6 + 6*a^2*b^2*d^3*x^4 + 4*a^3*b*d^3*x^2 + a^4*d^3)*cos(d*x + c
)^2 + (b^4*d^3*x^8 + 4*a*b^3*d^3*x^6 + 6*a^2*b^2*d^3*x^4 + 4*a^3*b*d^3*x^2
+ a^4*d^3)*sin(d*x + c)^2), x) + ((d^2*x^3*sin(c) + 3*d*x^2*cos(c) - 12*x*s
in(c))*cos(d*x + c)^2 + (d^2*x^3*sin(c) + 3*d*x^2*cos(c) - 12*x*sin(c))*sin
(d*x + c)^2)*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^6 + 3*(a
*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^
2)*d^3*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^3*cos(
c)^2 + b^3*sin(c)^2)*d^3*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^4
+ 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^
2)*d^3)*sin(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 492, normalized size = 1.03

$$-\frac{\left(-i\,ab^2d^2x^4-2i\,a^2bd^2x^2-i\,a^3d^2+3\left(-i\,b^3x^4-2i\,ab^2x^2-i\,a^2b\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(-i\,ab^2d^2x^4-2i\,a^2bd^2x^2-i\,a^3d^2+3\left(i\,b^3x^4+2i\,ab^2x^2+i\,a^2b\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\left(i\,ab^2d^2x^4+2i\,a^2bd^2x^2+i\,a^3d^2+3\left(i\,b^3x^4+2i\,ab^2x^2+i\,a^2b\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(i\,ab^2d^2x^4+2i\,a^2bd^2x^2+i\,a^3d^2+3\left(-i\,b^3x^4-2i\,ab^2x^2-i\,a^2b\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c-\sqrt{\frac{ad^2}{b}}\right)}+4\left(ab^2dx^3+a^2bdx\right)\cos\left(dx+c\right)+8\left(2\,ab^2x^2+a^2b\right)\sin\left(dx+c\right)}{32\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*((-I*a*b^2*d^2*x^4 - 2*I*a^2*b*d^2*x^2 - I*a^3*d^2 + 3*(-I*b^3*x^4 -
2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c +
sqrt(a*d^2/b)) + (-I*a*b^2*d^2*x^4 - 2*I*a^2*b*d^2*x^2 - I*a^3*d^2 + 3*(I*b
^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*
e^(I*c - sqrt(a*d^2/b)) + (I*a*b^2*d^2*x^4 + 2*I*a^2*b*d^2*x^2 + I*a^3*d^2
+ 3*(I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a
*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (I*a*b^2*d^2*x^4 + 2*I*a^2*b*d^2*x^2 +
I*a^3*d^2 + 3*(-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2/b))*Ei(-I*d
*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 4*(a*b^2*d*x^3 + a^2*b*d*x)*
cos(d*x + c) + 8*(2*a*b^2*x^2 + a^2*b)*sin(d*x + c))/(a*b^5*x^4 + 2*a^2*b^4
*x^2 + a^3*b^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sin (c + dx)}{(a + bx^2)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x**2)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*sin(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\, \sin (c + d\,x)}{{(b\,x^2 + a)}^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sin(c + d*x))/(a + b*x^2)^3,x)
```

```
[Out] int((x^3*sin(c + d*x))/(a + b*x^2)^3, x)
```

3.73 $\int \frac{x^2 \sin(c+dx)}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=746

$$-\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)}-\frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16ab^2}-\frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{16ab^2}+\frac{\text{Ci}\left(\sqrt{-}\right.}{\sqrt{}}$$

```
[Out] -1/8*d*cos(d*x+c)/b^2/(b*x^2+a)-1/16*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d
*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a
)^(1/2)/b^(1/2))/a/b^2-1/16*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)
/b^(1/2))/(-a)^(3/2)/b^(3/2)+1/16*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)
^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/4*x*sin(d*x+c)/b/(b*x^2+a)^2+1/16*Ci(d
*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)+1/1
6*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*Ci(
-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)+1
/16*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d
^2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1
/2)+1/16*d^2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/b^(5/
2)/(-a)^(1/2)+1/16*d^2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1
/2))/b^(5/2)/(-a)^(1/2)-1/16*d^2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)
^(1/2)/b^(1/2))/b^(5/2)/(-a)^(1/2)-1/16*sin(d*x+c)/a/b^(3/2)/((-a)^(1/2)-x*
b^(1/2))+1/16*sin(d*x+c)/a/b^(3/2)/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A]
time = 0.89, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3424, 3414, 3378, 3384, 3380, 3383, 3423}

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] -1/8*(d*Cos[c + d*x])/(b^2*(a + b*x^2)) - (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*
CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) - (d*Cos[c - (Sqrt[-a]*
d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2) + (CosInteg
ral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/
2)*b^(3/2)) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a
]*d)/Sqrt[b]])/(16*Sqrt[-a]*b^(5/2)) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)*b^(3/2)) - (d^2*CosInteg
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*Sqrt[-a]
*b^(5/2)) - Sin[c + d*x]/(16*a*b^(3/2)*(Sqrt[-a] - Sqrt[b]*x)) + Sin[c + d*
```

```
x]/(16*a*b^(3/2)*(Sqrt[-a] + Sqrt[b]*x)) - (x*Sin[c + d*x])/(4*b*(a + b*x^2
)^2) + (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*
x])/(16*(-a)^(3/2)*b^(3/2)) + (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)) - (d*Sin[c + (Sqrt[-a]
*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) + (Cos[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3
/2)*b^(3/2)) + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/
Sqrt[b] + d*x])/(16*Sqrt[-a]*b^(5/2)) + (d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*Si
nIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3423

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_
), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1))),
x] + Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sin[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
```

```
ntegerQ[n] || GtQ[e, 0])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rubi steps

$$\int \frac{x^2 \sin(c+dx)}{\left(a+bx^2\right)^3}\,dx = -\frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{\int \frac{\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{4b} + \frac{d\int \frac{x\cos(c+dx)}{\left(a+bx^2\right)^2}\,dx}{4b}$$

$$= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{\int \left(-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \cdots\right.}{4b}$$

$$= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a} - \frac{\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a} - \cdots$$

$$= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{x\sin(c+\cdots}{4b\left(a+bx\cdots\right.}$$

$$= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{x\sin(c+\cdots}{4b\left(a+bx\cdots\right.}$$

$$= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\cdots\right.}{16ab^2}$$

$$= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\cdots\right.}{16ab^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.86, size = 927, normalized size = 1.24

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] ((-2*a^(5/2)*d*Cos[c]*Cos[d*x])/(a + b*x^2)^2 - (2*a^(3/2)*b*d*x^2*Cos[c]*C
os[d*x])/(a + b*x^2)^2 - (2*a^(3/2)*b*x*Cos[d*x]*Sin[c])/(a + b*x^2)^2 + (2
*Sqrt[a]*b^2*x^3*Cos[d*x]*Sin[c])/(a + b*x^2)^2 + (CosIntegral[d*((I*Sqrt[a
])/Sqrt[b] + x)]*(-(Sqrt[a]*Sqrt[b]*d*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]) + I*(
b - a*d^2)*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[b] + (I*CosIntegral[d*(((-
I)*Sqrt[a])/Sqrt[b] + x)]*(I*Sqrt[a]*Sqrt[b]*d*Cos[c + (I*Sqrt[a]*d)/Sqrt[b
]] + (-b + a*d^2)*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[b] - (2*a^(3/2)*b*x
*Cos[c]*Sin[d*x])/(a + b*x^2)^2 + (2*Sqrt[a]*b^2*x^3*Cos[c]*Sin[d*x])/(a +
b*x^2)^2 + (2*a^(5/2)*d*Sin[c]*Sin[d*x])/(a + b*x^2)^2 + (2*a^(3/2)*b*d*x^2
*Sin[c]*Sin[d*x])/(a + b*x^2)^2 + I*Sqrt[b]*Cos[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]
]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - (I*a*d^2*Cos[c]*Cosh[(Sqrt[a]*
d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)])/Sqrt[b] + Sqrt[a]*d*C
osh[(Sqrt[a]*d)/Sqrt[b]]*Sin[c]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] -
I*Sqrt[a]*d*Cos[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqr
t[b] + x)] - Sqrt[b]*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqr
t[a])/Sqrt[b] + x)] + (a*d^2*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d
*((I*Sqrt[a])/Sqrt[b] + x)])/Sqrt[b] + I*Sqrt[b]*Cos[c]*Cosh[(Sqrt[a]*d)/Sq
rt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - (I*a*d^2*Cos[c]*Cosh[(Sqr
t[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/Sqrt[b] - Sqrt[a
]*d*Cosh[(Sqrt[a]*d)/Sqrt[b]]*Sin[c]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*
x] - I*Sqrt[a]*d*Cos[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)
/Sqrt[b] - d*x] + Sqrt[b]*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*S
qrt[a]*d)/Sqrt[b] - d*x] - (a*d^2*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinInteg
ral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/Sqrt[b])/(16*a^(3/2)*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2283 vs. $2(574) = 1148$.
time = 0.19, size = 2284, normalized size = 3.06

method	result
risch	$\frac{d\,\mathrm{e}^{\frac{ibc+d\sqrt{ab}}{b}}\,\mathrm{expIntegral}\left(1,\frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{32b^2a}+\frac{d\,\mathrm{e}^{\frac{ibc-d\sqrt{ab}}{b}}\,\mathrm{expIntegral}\left(1,\frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)}{32b^2a}+$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(d^6*c^2*(-1/8*sin(d*x+c)*(5*a*c*d^2-5*a*d^2*(d*x+c)+3*b*c^3-9*b*c^2*
(d*x+c)+9*b*c*(d*x+c)^2-3*b*(d*x+c)^3)/a^2/d^4/(d^2*a+b*c^2-2*b*c*(d*x+c)+b
*(d*x+c)^2)^2+1/8*cos(d*x+c)/a/b/d^2/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2
)-1/16*(a*d^2+3*b)/a^2/d^4/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a
*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)
/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/16*(a*d^2+3*b)/a^2/d^4/b^2/((d*(-a*b)^(1
/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)
-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-3/16/a^2/d^4
/b*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-
(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-3/16/a^2/d^4/b*(Si(d*x
+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(
1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)))+1/4*sin(d*x+c)*d^2*c*(2*a^2*d^4+
5*a*b*c^2*d^2-5*a*b*c*d^2*(d*x+c)+3*b^2*c^4-9*b^2*c^3*(d*x+c)+9*b^2*c^2*(d*
x+c)^2-3*b^2*c*(d*x+c)^3)/b/a^2/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)^2-1
/4*cos(d*x+c)*c*d^4/a/b*(d*x+c)/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)+1/8
*d^2*c*(a*d^2*(d*(-a*b)^(1/2)+c*b)/b+3*c*b)/b^2/a^2/(-(d*(-a*b)^(1/2)+c*b)/
b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c
-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/8*d^2*c*(-a*d^2*(d*
(-a*b)^(1/2)-c*b)/b+3*c*b)/b^2/a^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*
(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c
*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))+1/8*d^2*c*(d^2*a+3*b*c^2-3*c*(d*(-a*b)^
(1/2)+c*b))/b^2/a^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d
*(-a*b)^(1/2)+c*b)/b))+1/8*d^2*c*(d^2*a+3*b*c^2+3*c*(d*(-a*b)^(1/2)-c*b))/b
^2/a^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*
(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c
*b)/b))-1/8*sin(d*x+c)*d^2*(3*a^2*c*d^4+a^2*d^4*(d*x+c)+6*a*b*c^3*d^2-8*a*b
*c^2*d^2*(d*x+c)+3*a*b*c*d^2*(d*x+c)^2-a*b*d^2*(d*x+c)^3+3*b^2*c^5-9*b^2*c^
4*(d*x+c)+9*b^2*c^3*(d*x+c)^2-3*b^2*c^2*(d*x+c)^3)/b/a^2/(d^2*a+b*c^2-2*b*c
*(d*x+c)+b*(d*x+c)^2)^2-1/8*cos(d*x+c)*d^4*(d^2*a+b*c^2-2*b*c*(d*x+c))/a/b^
2/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)+1/16*d^2*(a^2*d^4+a*b*c^2*d^2-2*a
*c*d^2*(d*(-a*b)^(1/2)+c*b)-a*b*d^2-3*b^2*c^2)/a^2/b^3/(-(d*(-a*b)^(1/2)+c*
b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*
x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/16*d^2*(a^2*d^4+
a*b*c^2*d^2+2*a*c*d^2*(d*(-a*b)^(1/2)-c*b)-a*b*d^2-3*b^2*c^2)/a^2/b^3/((d*(
-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)
-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/16
*d^2*(3*a*c*d^2-a*d^2*(d*(-a*b)^(1/2)+c*b)/b+3*b*c^3-3*c^2*(d*(-a*b)^(1/2)+
c*b))/b^2/a^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b
)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b
)^(1/2)+c*b)/b))-1/16*d^2*(3*a*c*d^2+a*d^2*(d*(-a*b)^(1/2)-c*b)/b+3*b*c^3+3
*c^2*(d*(-a*b)^(1/2)-c*b))/b^2/a^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*
(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c
```

```
*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + 4*(cos(c)^2 + sin(c)^2)*x*
sin(d*x + c) + ((d*x^2*cos(c) - 4*x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c)
- 4*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^3*cos(c)^2 + b^3*sin(
c)^2)*d^2*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^4 + 3*(a^2*b*cos(
c)^2 + a^2*b*sin(c)^2)*d^2*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*cos(d*x
 + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^6 + 3*(a*b^2*cos(c)^2 + a*b^
2*sin(c)^2)*d^2*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^2 + (a^3*co
s(c)^2 + a^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(3*a*d*x*cos(d*x + c
) - 2*(5*b*x^2 - a)*sin(d*x + c))/(b^4*d^2*x^8 + 4*a*b^3*d^2*x^6 + 6*a^2*b^
2*d^2*x^4 + 4*a^3*b*d^2*x^2 + a^4*d^2), x) + 2*(((b^3*cos(c)^2 + b^3*sin(c)
^2)*d^2*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^4 + 3*(a^2*b*cos(c)
^2 + a^2*b*sin(c)^2)*d^2*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*cos(d*x +
 c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*
sin(c)^2)*d^2*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^2 + (a^3*cos(
c)^2 + a^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(3*a*d*x*cos(d*x + c)
- 2*(5*b*x^2 - a)*sin(d*x + c))/((b^4*d^2*x^8 + 4*a*b^3*d^2*x^6 + 6*a^2*b^2
*d^2*x^4 + 4*a^3*b*d^2*x^2 + a^4*d^2)*cos(d*x + c)^2 + (b^4*d^2*x^8 + 4*a*b
^3*d^2*x^6 + 6*a^2*b^2*d^2*x^4 + 4*a^3*b*d^2*x^2 + a^4*d^2)*sin(d*x + c)^2)
, x) + ((d*x^2*sin(c) + 4*x*cos(c))*cos(d*x + c)^2 + (d*x^2*sin(c) + 4*x*co
s(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x
^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^4 + 3*(a^2*b*cos(c)^2 + a^2*
b*sin(c)^2)*d^2*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*cos(d*x + c)^2 + (
(b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)
*d^2*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^2 + (a^3*cos(c)^2 + a^
3*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.40, size = 604, normalized size = 0.81

$$-\frac{\left(ab^2d^2x^4+2\,a^2bd^2x^2+a^3d^2+\left(a^3d^2+(ab^2d^2-b^3)x^4-a^2b+2\,(a^2bd^2-ab^2)x^2\right)\sqrt{\frac{ad^2}{b}}\right){\rm Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic+\sqrt{\frac{ad^2}{b}}\right)}+\left(ab^2d^2x^4+2\,a^2bd^2x^2+a^3d^2-\left(a^3d^2+(ab^2d^2-b^3)x^4-a^2b+2\,(a^2bd^2-ab^2)x^2\right)\sqrt{\frac{ad^2}{b}}\right){\rm Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic-\sqrt{\frac{ad^2}{b}}\right)}+\left(ab^2d^2x^4+2\,a^2bd^2x^2+a^3d^2+\left(a^3d^2+(ab^2d^2-b^3)x^4-a^2b+2\,(a^2bd^2-ab^2)x^2\right)\sqrt{\frac{ad^2}{b}}\right){\rm Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-ic+\sqrt{\frac{ad^2}{b}}\right)}+\left(ab^2d^2x^4+2\,a^2bd^2x^2+a^3d^2-\left(a^3d^2+(ab^2d^2-b^3)x^4-a^2b+2\,(a^2bd^2-ab^2)x^2\right)\sqrt{\frac{ad^2}{b}}\right){\rm Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-ic-\sqrt{\frac{ad^2}{b}}\right)}+4\,(a^2bd^2x^2+a^3d^2)\cos\left(dx+c\right)-4\,(ab^2dx^3-a^2bdx)\sin\left(dx+c\right)}{32\,(a^2b^4dx^4+2\,a^3b^3dx^2+a^4b^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 -
 b^3)*x^4 - a^2*b + 2*(a^2*b*d^2 - a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(I*d*x - sq
```

```
rt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a
^3*d^2 - (a^3*d^2 + (a*b^2*d^2 - b^3)*x^4 - a^2*b + 2*(a^2*b*d^2 - a*b^2)*x
^2)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (a*b
^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 - b^3)*x^4 -
 a^2*b + 2*(a^2*b*d^2 - a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b
))*e^(-I*c + sqrt(a*d^2/b)) + (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2 -
(a^3*d^2 + (a*b^2*d^2 - b^3)*x^4 - a^2*b + 2*(a^2*b*d^2 - a*b^2)*x^2)*sqrt(
a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 4*(a^2*b*d^
2*x^2 + a^3*d^2)*cos(d*x + c) - 4*(a*b^2*d*x^3 - a^2*b*d*x)*sin(d*x + c))/(
a^2*b^4*d*x^4 + 2*a^3*b^3*d*x^2 + a^4*b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x**2)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sin(c+d\,x)}{{(b\,x^2+a)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sin(c + d*x))/(a + b*x^2)^3,x)
```

```
[Out] int((x^2*sin(c + d*x))/(a + b*x^2)^3, x)
```

3.74 $\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=512

$$-\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)}-\frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}+\frac{d\cos\left(c-\right.}{}$$

```
[Out] 1/16*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/
b^(3/2)-1/16*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2))/(-
a)^(3/2)/b^(3/2)+1/16*d^2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b
^(1/2))/a/b^2+1/16*d^2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1
/2))/a/b^2-1/4*sin(d*x+c)/b/(b*x^2+a)^2+1/16*d^2*Ci(d*x+d*(-a)^(1/2)/b^(1/2
))*sin(c-d*(-a)^(1/2)/b^(1/2))/a/b^2-1/16*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))*si
n(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)+1/16*d^2*Ci(-d*x+d*(-a)^(1/2)/
b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/a/b^2+1/16*d*Si(d*x-d*(-a)^(1/2)/b^(1/
2))*sin(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/16*d*cos(d*x+c)/a/b^(3
/2)/((-a)^(1/2)-x*b^(1/2))+1/16*d*cos(d*x+c)/a/b^(3/2)/((-a)^(1/2)+x*b^(1/2
))
```

Rubi [A]
time = 0.60, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3422, 3415, 3378, 3384, 3380, 3383}

$$-\frac{d\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}+\frac{d\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}}-\frac{d\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}-\frac{d\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}-\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)}+\frac{d^2\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{16ab^2}+\frac{d^2\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16ab^2}-\frac{d^2\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16ab^2}+\frac{d^2\cos\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Si}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16ab^2}-\frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] -1/16*(d*Cos[c + d*x])/(a*b^(3/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Cos[c + d*x]
)/(16*a*b^(3/2)*(Sqrt[-a] + Sqrt[b]*x)) - (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*
CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) + (d*Cos[c
 - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^
(3/2)*b^(3/2)) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt
[-a]*d)/Sqrt[b]])/(16*a*b^2) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*a*b^2) - Sin[c + d*x]/(4*b*(a + b*x^2)^
2) - (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x])/(16*a*b^2) - (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d
)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[
b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2) - (d*Sin[c - (Sqrt[
-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(
3/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx = -\frac{\sin(c+dx)}{4b\,(a+bx^2)^2} + \frac{d\int \frac{\cos(c+dx)}{(a+bx^2)^2}\,dx}{4b}$$

$$= -\frac{\sin(c+dx)}{4b\,(a+bx^2)^2} + \frac{d\int \left(-\frac{b\cos(c+dx)}{4a\left(\sqrt{-a}\,\sqrt{b}-bx\right)^2} - \frac{b\cos(c+dx)}{4a\left(\sqrt{-a}\,\sqrt{b}+bx\right)^2} - \frac{b\cos(c+dx)}{2a(-ab-b^2x^2)}\right) dx}{4b}$$

$$= -\frac{\sin(c+dx)}{4b\,(a+bx^2)^2} - \frac{d\int \frac{\cos(c+dx)}{\left(\sqrt{-a}\,\sqrt{b}-bx\right)^2}\,dx}{16a} - \frac{d\int \frac{\cos(c+dx)}{\left(\sqrt{-a}\,\sqrt{b}+bx\right)^2}\,dx}{16a} - \frac{d\int \frac{\cos(c+dx)}{-ab-b^2x^2}\,d}{8a}$$

$$= -\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{\sin(c+dx)}{4b\,(a+bx^2)^2} - \frac{d\int\left(-\frac{}{2ab}\right.}{}$$

$$= -\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{\sin(c+dx)}{4b\,(a+bx^2)^2} + \frac{d\int \frac{\cos(c}{\sqrt{-a}}}{16(-a}$$

$$= -\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} + \frac{d^2\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)\sin\left(\right.}{16ab^2}$$

$$= -\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ci}\left(\right.}{16(-a)^{3/2}b^{3/2}}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 1.24, size = 634, normalized size = 1.24

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] ((2*Cos[d*x]*(d*x*(a + b*x^2)*Cos[c] - 2*a*Sin[c]))/(a + b*x^2)^2 - (2*(2*a
*Cos[c] + d*x*(a + b*x^2)*Sin[c])*Sin[d*x])/(a + b*x^2)^2 + (d^2*Cos[c]*(I*
CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] - I*C
osIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + Cosh[(S
```

```
qrt[a]*d)/Sqrt[b]]*(SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - SinIntegral[
(I*Sqrt[a]*d)/Sqrt[b] - d*x])))/b + (d*Cos[c]*((-I)*Cosh[(Sqrt[a]*d)/Sqrt[b
]]*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + I*Cosh[(Sqrt[a]*d)/Sqrt[b]
]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + Sinh[(Sqrt[a]*d)/Sqrt[b]]*(-Si
nIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + SinIntegral[(I*Sqrt[a]*d)/Sqrt[b]
- d*x])))/(Sqrt[a]*Sqrt[b]) - (d*Sin[c]*(CosIntegral[d*(((-I)*Sqrt[a])/Sqrt
[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + CosIntegral[d*((I*Sqrt[a])/Sqrt[b] +
x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + I*Cosh[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[d*
((I*Sqrt[a])/Sqrt[b] + x)] + SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])))/(S
qrt[a]*Sqrt[b]) + (d^2*Sin[c]*(Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*(((-
I)*Sqrt[a])/Sqrt[b] + x)] + Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqr
t[a])/Sqrt[b] + x)] + I*Sinh[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[d*((I*Sqrt[a
])/Sqrt[b] + x)] + SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])))/b)/(16*a*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1359 vs. $2(398) = 796$.

time = 0.13, size = 1360, normalized size = 2.66

method	result
risch	$\frac{i d^2 e^{\frac{ibc+d\sqrt{ab}}{b}} \operatorname{expIntegral}\left(1, \frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{32b^2 a} + \frac{i d^2 e^{\frac{ibc-d\sqrt{ab}}{b}} \operatorname{expIntegral}\left(1, \frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)}{32b^2 a} -$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(-1/8*sin(d*x+c)*d^2*(2*a^2*d^4+5*a*b*c^2*d^2-5*a*b*c*d^2*(d*x+c)+3*b
^2*c^4-9*b^2*c^3*(d*x+c)+9*b^2*c^2*(d*x+c)^2-3*b^2*c*(d*x+c)^3)/a^2/b/(d^2*
a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)^2+1/8*cos(d*x+c)*d^4/a/b*(d*x+c)/(d^2*a+
b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)-1/16*d^2*(a*d^2*(d*(-a*b)^(1/2)+c*b)/b+3*c
*b)/a^2/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*c
os((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(
1/2)+c*b)/b))-1/16*d^2*(-a*d^2*(d*(-a*b)^(1/2)-c*b)/b+3*c*b)/a^2/b^2/((d*(-
a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-
c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/16*
d^2*(d^2*a+3*b*c^2-3*c*(d*(-a*b)^(1/2)+c*b))/a^2/b^2/(-(d*(-a*b)^(1/2)+c*b)
/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x
+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-1/16*d^2*(d^2*a+3*b
*c^2+3*c*(d*(-a*b)^(1/2)-c*b))/a^2/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c
+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/
2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))-c*d^6*(-1/8*sin(d*x+c)*(5*a*c*d^2-5
*a*d^2*(d*x+c)+3*b*c^3-9*b*c^2*(d*x+c)+9*b*c*(d*x+c)^2-3*b*(d*x+c)^3)/a^2/d
^4/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)^2+1/8*cos(d*x+c)/a/b/d^2/(d^2*a+
```

```
b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)-1/16*(a*d^2+3*b)/a^2/d^4/b^2/(-(d*(-a*b)^(
1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b
)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/16*(a*d^2
+3*b)/a^2/d^4/b^2/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)
/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a
*b)^(1/2)-c*b)/b))-3/16/a^2/d^4/b*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d
*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+
c*b)/b))-3/16/a^2/d^4/b*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/
2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*c
os(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^3*cos(c)^2 + b^3*sin(c)^2)*d*
x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b
*sin(c)^2)*d*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^3*
cos(c)^2 + b^3*sin(c)^2)*d*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^4
+ 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)
*d)*sin(d*x + c)^2)*integrate(1/2*(5*b*x^2 - a)*cos(d*x + c)/(b^4*d*x^8 + 4
*a*b^3*d*x^6 + 6*a^2*b^2*d*x^4 + 4*a^3*b*d*x^2 + a^4*d), x) + 2*(((b^3*cos(
c)^2 + b^3*sin(c)^2)*d*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^4 + 3*
(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*
cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^6 + 3*(a*b^2*cos(c)^2 +
 a*b^2*sin(c)^2)*d*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^2 + (a^3*c
os(c)^2 + a^3*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(5*b*x^2 - a)*cos(
d*x + c)/((b^4*d*x^8 + 4*a*b^3*d*x^6 + 6*a^2*b^2*d*x^4 + 4*a^3*b*d*x^2 + a^
4*d)*cos(d*x + c)^2 + (b^4*d*x^8 + 4*a*b^3*d*x^6 + 6*a^2*b^2*d*x^4 + 4*a^3*
b*d*x^2 + a^4*d)*sin(d*x + c)^2), x) + (x*cos(d*x + c)^2*sin(c) + x*sin(d*x
 + c)^2*sin(c))*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^6 + 3*(
a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^4 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2
)*d*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^3*cos(c)^2
+ b^3*sin(c)^2)*d*x^6 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^4 + 3*(a^2*
b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^2 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*sin(d
*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 483, normalized size = 0.94

$$-\frac{8\,a^2b\sin\left(dx+c\right)+\left(i\,ab^2d^2x^4+2i\,a^2bd^2x^2+i\,a^3d^2-\left(i\,b^3x^4+2i\,ab^2x^2+i\,a^2b\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(i\,ab^2d^2x^4+2i\,a^2bd^2x^2+i\,a^3d^2-\left(-i\,b^3x^4-2i\,ab^2x^2-i\,a^2b\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\left(-i\,ab^2d^2x^4-2i\,a^2bd^2x^2-i\,a^3d^2-\left(-i\,b^3x^4-2i\,ab^2x^2-i\,a^2b\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(-i\,ab^2d^2x^4-2i\,a^2bd^2x^2-i\,a^3d^2-\left(i\,b^3x^4+2i\,ab^2x^2+i\,a^2b\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c-\sqrt{\frac{ad^2}{b}}\right)}-4\left(ab^2dx^3+a^2bdx\right)\cos\left(dx+c\right)}{32\left(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*a^2*b*sin(d*x + c) + (I*a*b^2*d^2*x^4 + 2*I*a^2*b*d^2*x^2 + I*a^3*
d^2 - (I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x - sqrt(
a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (I*a*b^2*d^2*x^4 + 2*I*a^2*b*d^2*x^2 +
I*a^3*d^2 - (-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x
+ sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (-I*a*b^2*d^2*x^4 - 2*I*a^2*b*d^
2*x^2 - I*a^3*d^2 - (-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2/b))*E
i(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (-I*a*b^2*d^2*x^4 - 2*
I*a^2*b*d^2*x^2 - I*a^3*d^2 - (I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*
d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*(a*b^2*d*x^
3 + a^2*b*d*x)*cos(d*x + c))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x**2)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sin(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sin\left(c+d\,x\right)}{{\left(b\,x^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(c + d*x))/(a + b*x^2)^3,x)
```

```
[Out] int((x*sin(c + d*x))/(a + b*x^2)^3, x)
```

3.75 $\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=856

$$\frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}\,x\right)}-\frac{3d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16a^2b}-\frac{3d}{}$$

```
[Out] -3/16*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))/a^2/b-3/16
*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2))/a^2/b-1/16*d^2
*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/(-a)^(3/2)/b^(3/2
)+1/16*d^2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(3
/2)/b^(3/2)+1/16*d^2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2
))/(-a)^(3/2)/b^(3/2)+3/16*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2
)/b^(1/2))/a^2/b-1/16*d^2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/
b^(1/2))/(-a)^(3/2)/b^(3/2)+3/16*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a
)^(1/2)/b^(1/2))/a^2/b+3/16*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)
/b^(1/2))/(-a)^(5/2)/b^(1/2)-3/16*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)
^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-3/16*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-
d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+3/16*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))
*sin(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-1/16*sin(d*x+c)/(-a)^(3/2)/
b^(1/2)/((-a)^(1/2)-x*b^(1/2))^2+1/16*d*cos(d*x+c)/(-a)^(3/2)/b/((-a)^(1/2)
-x*b^(1/2))-3/16*sin(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)-x*b^(1/2))+1/16*sin(d*x
+c)/(-a)^(3/2)/b^(1/2)/((-a)^(1/2)+x*b^(1/2))^2+1/16*d*cos(d*x+c)/(-a)^(3/2
)/b/((-a)^(1/2)+x*b^(1/2))+3/16*sin(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)+x*b^(1/2
))
```

Rubi [A]
time = 0.89, antiderivative size = 856, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3414, 3378, 3384, 3380, 3383}

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x^2)^3,x]
```

```
[Out] (d*Cos[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[-a] - Sqrt[b]*x)) + (d*Cos[c + d*x]
)/(16*(-a)^(3/2)*b*(Sqrt[-a] + Sqrt[b]*x)) - (3*d*Cos[c + (Sqrt[-a]*d)/Sqrt
[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (3*d*Cos[c - (Sq
rt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b) - (3
*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16
*(-a)^(5/2)*Sqrt[b]) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c -
```

```
 (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)*b^(3/2)) + (3*CosIntegral[(Sqrt[-a]*
d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) -
(d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])
/(16*(-a)^(3/2)*b^(3/2)) - Sin[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(Sqrt[-a] -
Sqrt[b]*x)^2) - (3*Sin[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) +
Sin[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)^2) + (3*Sin[c +
d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (3*Cos[c + (Sqrt[-a]*d)/Sqr
t[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (d
^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
16*(-a)^(3/2)*b^(3/2)) - (3*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sq
rt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (3*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*Sin
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (d^2*Cos[c
- (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(
3/2)*b^(3/2)) + (3*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)
/Sqrt[b] + d*x])/(16*a^2*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+bx^2)^3}\,dx = \int \left(-\frac{b^{3/2}\sin(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\,\sqrt{b}-bx\right)^3} - \frac{3b\sin(c+dx)}{16a^2\left(\sqrt{-a}\,\sqrt{b}-bx\right)^2} - \frac{b^{3/2}\sin(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\,\sqrt{b}\right.}\right.$$

$$= -\frac{(3b)\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\,\sqrt{b}-bx\right)^2}\,dx}{16a^2} - \frac{(3b)\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\,\sqrt{b}+bx\right)^2}\,dx}{16a^2} - \frac{(3b)\int \frac{\sin(c+dx)}{-ab-b^2x^2}\,dx}{8a^2} - \frac{b^{3/2}}{}$$

$$= -\frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)^2} - \frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sin(c+}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-}\right.}$$

$$= \frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}\right.}$$

$$= \frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{3d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{C}}{16a^2 b}$$

$$= \frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{3d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{C}}{16a^2 b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.71, size = 932, normalized size = 1.09

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*x^2)^3,x]
```

```
[Out] ((2*a^2*Sqrt[b]*d*Cos[c]*Cos[d*x])/(a + b*x^2)^2 + (2*a*b^(3/2)*d*x^2*Cos[c
]*Cos[d*x])/(a + b*x^2)^2 + (10*a*b^(3/2)*x*Cos[d*x]*Sin[c])/(a + b*x^2)^2
+ (6*b^(5/2)*x^3*Cos[d*x]*Sin[c])/(a + b*x^2)^2 + (I*CosIntegral[d*((I*Sqrt
[a])/Sqrt[b] + x)]*((3*I)*Sqrt[a]*Sqrt[b]*d*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]
+ (3*b + a*d^2)*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[a] - (I*CosIntegral[d
```

```
*(((-I)*Sqrt[a])/Sqrt[b] + x)]*((-3*I)*Sqrt[a]*Sqrt[b]*d*Cos[c + (I*Sqrt[a]
*d)/Sqrt[b]] + (3*b + a*d^2)*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[a] + (10
*a*b^(3/2)*x*Cos[c]*Sin[d*x])/(a + b*x^2)^2 + (6*b^(5/2)*x^3*Cos[c]*Sin[d*x
])/(a + b*x^2)^2 - (2*a^2*Sqrt[b]*d*Sin[c]*Sin[d*x])/(a + b*x^2)^2 - (2*a*b
^(3/2)*d*x^2*Sin[c]*Sin[d*x])/(a + b*x^2)^2 + ((3*I)*b*Cos[c]*Cosh[(Sqrt[a]
*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)])/Sqrt[a] + I*Sqrt[a]*
d^2*Cos[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x
)] + 3*Sqrt[b]*d*Cosh[(Sqrt[a]*d)/Sqrt[b]]*Sin[c]*SinIntegral[d*((I*Sqrt[a]
)/Sqrt[b] + x)] - (3*I)*Sqrt[b]*d*Cos[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinInteg
ral[d*((I*Sqrt[a])/Sqrt[b] + x)] - (3*b*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*Si
nIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)])/Sqrt[a] - Sqrt[a]*d^2*Sin[c]*Sinh[(
Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + ((3*I)*b*Cos
[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/Sqr
t[a] + I*Sqrt[a]*d^2*Cos[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a
]*d)/Sqrt[b] - d*x] - 3*Sqrt[b]*d*Cosh[(Sqrt[a]*d)/Sqrt[b]]*Sin[c]*SinInteg
ral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - (3*I)*Sqrt[b]*d*Cos[c]*Sinh[(Sqrt[a]*d)/
Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + (3*b*Sin[c]*Sinh[(Sqrt[
a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/Sqrt[a] + Sqrt[a]*
d^2*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*
x])/(16*a^2*b^(3/2))
```

Maple [**A**]
time = 0.09, size = 598, normalized size = 0.70 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^5*(-1/8*sin(d*x+c)*(5*a*c*d^2-5*a*d^2*(d*x+c)+3*b*c^3-9*b*c^2*(d*x+c)+9*b
*c*(d*x+c)^2-3*b*(d*x+c)^3)/a^2/d^4/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)
^2+1/8*cos(d*x+c)/a/b/d^2/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)-1/16*(a*d
^2+3*b)/a^2/d^4/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c
*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*
(-a*b)^(1/2)+c*b)/b))-1/16*(a*d^2+3*b)/a^2/d^4/b^2/((d*(-a*b)^(1/2)-c*b)/b+
c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(
d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-3/16/a^2/d^4/b*(-Si(d*x
+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(
1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-3/16/a^2/d^4/b*(Si(d*x+c+(d*(-a*b
)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b
)*cos((d*(-a*b)^(1/2)-c*b)/b)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*x^2 + a)^3, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.40, size = 611, normalized size = 0.71

$$-\frac{\left(3\,ab^2d^2x^4+6\,a^2bd^2x^2+3\,a^3d^2-\left(a^3d^2+\left(ab^2d^2+3\,b^3\right)x^4+3\,a^2b+2\,\left(a^2bd^2+3\,ab^2\right)x^2\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(3\,ab^2d^2x^4+6\,a^2bd^2x^2+3\,a^3d^2+\left(a^3d^2+\left(ab^2d^2+3\,b^3\right)x^4+3\,a^2b+2\,\left(a^2bd^2+3\,ab^2\right)x^2\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\left(3\,ab^2d^2x^4+6\,a^2bd^2x^2+3\,a^3d^2-\left(a^3d^2+\left(ab^2d^2+3\,b^3\right)x^4+3\,a^2b+2\,\left(a^2bd^2+3\,ab^2\right)x^2\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(3\,ab^2d^2x^4+6\,a^2bd^2x^2+3\,a^3d^2+\left(a^3d^2+\left(ab^2d^2+3\,b^3\right)x^4+3\,a^2b+2\,\left(a^2bd^2+3\,ab^2\right)x^2\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c-\sqrt{\frac{ad^2}{b}}\right)}-4\,\left(a^2bd^2x^2+a^3d^2\right)\cos\left(dx+c\right)-4\,\left(3\,ab^2dx^3+5\,a^2bdx\right)\sin\left(dx+c\right)}{32\,\left(a^3b^3dx^4+2\,a^4b^2dx^2+a^5bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*((3*a*b^2*d^2*x^4 + 6*a^2*b*d^2*x^2 + 3*a^3*d^2 - (a^3*d^2 + (a*b^2*d
^2 + 3*b^3)*x^4 + 3*a^2*b + 2*(a^2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(
I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (3*a*b^2*d^2*x^4 + 6*a^2*b
*d^2*x^2 + 3*a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 + 3*b^3)*x^4 + 3*a^2*b + 2*(a^
2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - s
qrt(a*d^2/b)) + (3*a*b^2*d^2*x^4 + 6*a^2*b*d^2*x^2 + 3*a^3*d^2 - (a^3*d^2 +
 (a*b^2*d^2 + 3*b^3)*x^4 + 3*a^2*b + 2*(a^2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^
2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (3*a*b^2*d^2*x^
4 + 6*a^2*b*d^2*x^2 + 3*a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 + 3*b^3)*x^4 + 3*a^
2*b + 2*(a^2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b)
)*e^(-I*c - sqrt(a*d^2/b)) - 4*(a^2*b*d^2*x^2 + a^3*d^2)*cos(d*x + c) - 4*(
3*a*b^2*d*x^3 + 5*a^2*b*d*x)*sin(d*x + c))/(a^3*b^3*d*x^4 + 2*a^4*b^2*d*x^2
 + a^5*b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c+dx\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*x**2)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)}{{\left(b\,x^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*x^2)^3,x)
[Out] int(sin(c + d*x)/(a + b*x^2)^3, x)
```

3.76 $\int \frac{\sin(c+dx)}{x(a+bx^2)^3}\, dx$

Optimal. Leaf size=730

$$\frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)}-\frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}\,x\right)}-\frac{5d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}+\frac{5d\cos\left(\right.}{}$$

```
[Out] cos(c)*Si(d*x)/a^3-1/2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1
/2))/a^3-1/16*d^2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/
a^2/b-1/2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/a^3-1/16
*d^2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/a^2/b+Ci(d*x)
*sin(c)/a^3+1/4*sin(d*x+c)/a/(b*x^2+a)^2+1/2*sin(d*x+c)/a^2/(b*x^2+a)-1/2*C
i(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a^3-1/16*d^2*Ci(d*x
+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a^2/b-1/2*Ci(-d*x+d*(-a)
^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/a^3-1/16*d^2*Ci(-d*x+d*(-a)^(1/
2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/a^2/b+5/16*d*Ci(d*x+d*(-a)^(1/2)/b^
(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-5/16*d*Ci(-d*x+d*(-a)
^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-5/16*d*Si(d*
x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+5/16
*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1
/2)+1/16*d*cos(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)-x*b^(1/2))-1/16*d*cos(d*x+c)/
a^2/b^(1/2)/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A]
time = 1.26, antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3426, 3384, 3380, 3383, 3422, 3415, 3378}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^2)^3),x]
```

```
[Out] (d*Cos[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) - (d*Cos[c + d*x])
/(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (5*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b
]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (5*d*
Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*
(-a)^(5/2)*Sqrt[b]) + (CosIntegral[d*x]*Sin[c])/a^3 - (CosIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a^3) - (d^2*CosIntegr
al[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^2*b) -
(CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*
a^3) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sq
```

```
rt[b]])/(16*a^2*b) + Sin[c + d*x]/(4*a*(a + b*x^2)^2) + Sin[c + d*x]/(2*a^2
*(a + b*x^2)) + (Cos[c]*SinIntegral[d*x])/a^3 + (Cos[c + (Sqrt[-a]*d)/Sqrt[
b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) + (d^2*Cos[c + (Sqrt[-
a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (5*d*S
in[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(
-a)^(5/2)*Sqrt[b]) - (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d
)/Sqrt[b] + d*x])/(2*a^3) - (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[
(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b) - (5*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]
]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b])
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
```

```
ntegerQ[n] || GtQ[e, 0])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^3}\,dx = \int \left(\frac{\sin(c+dx)}{a^3x} - \frac{bx\sin(c+dx)}{a\left(a+bx^2\right)^3} - \frac{bx\sin(c+dx)}{a^2\left(a+bx^2\right)^2} - \frac{bx\sin(c+dx)}{a^3\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x}dx}{a^3} - \frac{b\int \frac{x\sin(c+dx)}{a+bx^2}dx}{a^3} - \frac{b\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^2}dx}{a^2} - \frac{b\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^3}dx}{a}$$

$$= \frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)} - \frac{b\int\left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)d}{a^3}$$

$$= \frac{\mathrm{Ci}(dx)\sin(c)}{a^3} + \frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cos(c)\mathrm{Si}(dx)}{a^3} + \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{2a^3}$$

$$= \frac{\mathrm{Ci}(dx)\sin(c)}{a^3} + \frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cos(c)\mathrm{Si}(dx)}{a^3} + \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{4(-a)^{5/2}} +$$

$$= \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{\mathrm{Ci}(dx)\sin(c)}{a^3} - \frac{\mathrm{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}\right.}{}$$

$$= \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Ci}\left(\right.}{4(-a)^{5/2}\sqrt{b}}$$

$$= \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Ci}\left(\right.}{4(-a)^{5/2}\sqrt{b}}$$

$$= \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{5d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Ci}\left(\right.}{16(-a)^{5/2}\sqrt{b}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.68, size = 924, normalized size = 1.27

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^2)^3),x]
```

```
[Out] ((-2*a^2*d*x*Cos[c + d*x])/(a + b*x^2)^2 - (2*a*b*d*x^3*Cos[c + d*x])/(a +
b*x^2)^2 + (16*a^2*CosIntegral[d*x]*Sin[c])/(a + b*x^2)^2 + (32*a*b*x^2*Cos
Integral[d*x]*Sin[c])/(a + b*x^2)^2 + (16*b^2*x^4*CosIntegral[d*x]*Sin[c])/
(a + b*x^2)^2 - (CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*((5*I)*Sqrt[a]*Sq
rt[b]*d*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]] + (8*b + a*d^2)*Sin[c - (I*Sqrt[a]*d
)/Sqrt[b]]))/b - (CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*((-5*I)*Sqrt[
a]*Sqrt[b]*d*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]] + (8*b + a*d^2)*Sin[c + (I*Sqrt
[a]*d)/Sqrt[b]]))/b + (12*a^2*Sin[c + d*x])/(a + b*x^2)^2 + (8*a*b*x^2*Sin[
c + d*x])/(a + b*x^2)^2 + (16*a^2*Cos[c]*SinIntegral[d*x])/(a + b*x^2)^2 +
(32*a*b*x^2*Cos[c]*SinIntegral[d*x])/(a + b*x^2)^2 + (16*b^2*x^4*Cos[c]*Sin
Integral[d*x])/(a + b*x^2)^2 - 8*Cos[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegr
al[d*((I*Sqrt[a])/Sqrt[b] + x)] - (a*d^2*Cos[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]]*S
inIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)])/b + ((5*I)*Sqrt[a]*d*Cosh[(Sqrt[a]
*d)/Sqrt[b]]*Sin[c]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)])/Sqrt[b] + (5*
Sqrt[a]*d*Cos[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[
b] + x)])/Sqrt[b] - (8*I)*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((
I*Sqrt[a])/Sqrt[b] + x)] - (I*a*d^2*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinInt
egral[d*((I*Sqrt[a])/Sqrt[b] + x)])/b + 8*Cos[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]]*
SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + (a*d^2*Cos[c]*Cosh[(Sqrt[a]*d)/S
qrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/b + ((5*I)*Sqrt[a]*d*Cosh
[(Sqrt[a]*d)/Sqrt[b]]*Sin[c]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/Sqrt
[b] - (5*Sqrt[a]*d*Cos[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*
d)/Sqrt[b] - d*x])/Sqrt[b] - (8*I)*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinInte
gral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - (I*a*d^2*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b
]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/b)/(16*a^3)
```

Maple [A]
time = 0.31, size = 580, normalized size = 0.79 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sin(d*x+c)*d^2*(3*d^2*a+2*b*c^2-4*b*c*(d*x+c)+2*b*(d*x+c)^2)/a^2/(d^2*a
+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)^2-1/8*cos(d*x+c)*d^3*x/(d^2*a+b*c^2-2*b*c
*(d*x+c)+b*(d*x+c)^2)/a^2-1/16*(a*d^2+8*b)/b/a^3*(Si(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d
*(-a*b)^(1/2)+c*b)/b))-1/16*(a*d^2+8*b)/b/a^3*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b
)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-
a*b)^(1/2)-c*b)/b))+1/a^3*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+5/16*d^2/a^2/b/(-
(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^
(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))
+5/16*d^2/a^2/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b
```

```
)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b
)^(1/2)-c*b)/b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.50, size = 647, normalized size = 0.89

$$-\frac{16\,(i\,b^3x^4+2i\,ab^2x^2+i\,a^2b){\rm Ei}(i\,dx)\,e^{(ic)}+16\,(-i\,b^3x^4-2i\,ab^2x^2-i\,a^2b){\rm Ei}(-i\,dx)\,e^{(-ic)}+\left(-i\,a^3d^2-i\,(ab^2d^2+8\,b^3)x^4-8i\,a^2b-2i\,(a^2bd^2+8\,ab^2)x^2+5\,(i\,b^3x^4+2i\,ab^2x^2+i\,a^2b)\sqrt{\frac{ad^2}{b}}\right){\rm Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic+\sqrt{\frac{ad^2}{b}}\right)}+\left(-i\,a^3d^2-i\,(ab^2d^2+8\,b^3)x^4-8i\,a^2b-2i\,(a^2bd^2+8\,ab^2)x^2+5\,(-i\,b^3x^4-2i\,ab^2x^2-i\,a^2b)\sqrt{\frac{ad^2}{b}}\right){\rm Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic-\sqrt{\frac{ad^2}{b}}\right)}+\left(i\,a^3d^2+i\,(ab^2d^2+8\,b^3)x^4+8i\,a^2b+2i\,(a^2bd^2+8\,ab^2)x^2+5\,(-i\,b^3x^4-2i\,ab^2x^2-i\,a^2b)\sqrt{\frac{ad^2}{b}}\right){\rm Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-ic+\sqrt{\frac{ad^2}{b}}\right)}+\left(i\,a^3d^2+i\,(ab^2d^2+8\,b^3)x^4+8i\,a^2b+2i\,(a^2bd^2+8\,ab^2)x^2+5\,(i\,b^3x^4+2i\,ab^2x^2+i\,a^2b)\sqrt{\frac{ad^2}{b}}\right){\rm Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-ic-\sqrt{\frac{ad^2}{b}}\right)}+4\,(ab^2dx^3+a^2bdx)\cos(dx+c)-8\,(2\,ab^2x^2+3\,a^2b)\sin(dx+c)}{32\,(a^3b^3x^4+2\,a^4b^2x^2+a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(16*(I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*Ei(I*d*x)*e^(I*c) + 16*(-I*
b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b)*Ei(-I*d*x)*e^(-I*c) + (-I*a^3*d^2 - I*(a
*b^2*d^2 + 8*b^3)*x^4 - 8*I*a^2*b - 2*I*(a^2*b*d^2 + 8*a*b^2)*x^2 + 5*(I*b^
3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e
^(I*c + sqrt(a*d^2/b)) + (-I*a^3*d^2 - I*(a*b^2*d^2 + 8*b^3)*x^4 - 8*I*a^2*
b - 2*I*(a^2*b*d^2 + 8*a*b^2)*x^2 + 5*(-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b
)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (I*a^3
*d^2 + I*(a*b^2*d^2 + 8*b^3)*x^4 + 8*I*a^2*b + 2*I*(a^2*b*d^2 + 8*a*b^2)*x^
2 + 5*(-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^2/b))*Ei(-I*d*x - sqr
t(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (I*a^3*d^2 + I*(a*b^2*d^2 + 8*b^3)*x
^4 + 8*I*a^2*b + 2*I*(a^2*b*d^2 + 8*a*b^2)*x^2 + 5*(I*b^3*x^4 + 2*I*a*b^2*x
^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^
2/b)) + 4*(a*b^2*d*x^3 + a^2*b*d*x)*cos(d*x + c) - 8*(2*a*b^2*x^2 + 3*a^2*b
)*sin(d*x + c))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{\left(c + dx \right)}}{x \left(a + bx^{2}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x**2+a)**3,x)
```

```
[Out] Integral(sin(c + d*x)/(x*(a + b*x**2)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)}{x\,{(b\,x^2+a)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x*(a + b*x^2)^3),x)
```

```
[Out] int(sin(c + d*x)/(x*(a + b*x^2)^3), x)
```

3.77 $\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=875

$$\frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)}+\frac{d\cos(c)\mathrm{Ci}(dx)}{a^3}+\frac{7d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}}{\sqrt{b}}\right.}{16a^3}$$

```
[Out] d*Ci(d*x)*cos(c)/a^3+7/16*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)
/b^(1/2))/a^3+7/16*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/
2))/a^3-d*Si(d*x)*sin(c)/a^3-sin(d*x+c)/a^3/x-7/16*d*Si(d*x+d*(-a)^(1/2)/b^
(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a^3-7/16*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*
sin(c+d*(-a)^(1/2)/b^(1/2))/a^3-1/16*d^2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x
-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+1/16*d^2*cos(c-d*(-a)^(1/2)/b^(1/
2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+1/16*d^2*Ci(d*x+d*(-a)^
(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)-1/16*d^2*Ci(-
d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/(-a)^(5/2)/b^(1/2)+15
/16*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(
7/2)-15/16*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))*b^(1/2)
/(-a)^(7/2)-15/16*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))*
b^(1/2)/(-a)^(7/2)+15/16*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b
^(1/2))*b^(1/2)/(-a)^(7/2)-1/16*sin(d*x+c)*b^(1/2)/(-a)^(5/2)/((-a)^(1/2)-x
*b^(1/2))^2+1/16*d*cos(d*x+c)/(-a)^(5/2)/((-a)^(1/2)-x*b^(1/2))+7/16*sin(d*
x+c)*b^(1/2)/a^3/((-a)^(1/2)-x*b^(1/2))+1/16*sin(d*x+c)*b^(1/2)/(-a)^(5/2)/
((-a)^(1/2)+x*b^(1/2))^2+1/16*d*cos(d*x+c)/(-a)^(5/2)/((-a)^(1/2)+x*b^(1/2)
)-7/16*sin(d*x+c)*b^(1/2)/a^3/((-a)^(1/2)+x*b^(1/2))
```

Rubi [A]

time = 2.03, antiderivative size = 875, normalized size of antiderivative = 1.00, number of steps used = 60, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3426, 3378, 3384, 3380, 3383, 3414}

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

```
[Out] (d*Cos[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Cos[c + d*x])/
(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)) + (d*Cos[c]*CosIntegral[d*x])/a^3 +
(7*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(16*a^3) + (7*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x])/(16*a^3) - (15*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]
*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) + (d^2*CosIntegral[(Sqrt[-a
```

```
]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b])
+ (15*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/
Sqrt[b]])/(16*(-a)^(7/2)) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Si
n[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) - Sin[c + d*x]/(a^3*x)
 - (Sqrt[b]*Sin[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)^2) + (7*Sqr
t[b]*Sin[c + d*x])/(16*a^3*(Sqrt[-a] - Sqrt[b]*x)) + (Sqrt[b]*Sin[c + d*x])
/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)^2) - (7*Sqrt[b]*Sin[c + d*x])/(16*a^
3*(Sqrt[-a] + Sqrt[b]*x)) - (d*Sin[c]*SinIntegral[d*x])/a^3 - (15*Sqrt[b]*C
os[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(
-a)^(7/2)) + (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (7*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*
SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) - (15*Sqrt[b]*Cos[c - (Sq
rt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2))
 + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*
x])/(16*(-a)^(5/2)*Sqrt[b]) - (7*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
```

```
x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx = \int\left(\frac{\sin(c+dx)}{a^3x^2} - \frac{b\sin(c+dx)}{a\left(a+bx^2\right)^3} - \frac{b\sin(c+dx)}{a^2\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int\frac{\sin(c+dx)}{x^2}\,dx}{a^3} - \frac{b\int\frac{\sin(c+dx)}{a+bx^2}\,dx}{a^3} - \frac{b\int\frac{\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a^2} - \frac{b\int\frac{\sin(c+dx)}{\left(a+bx^2\right)^3}\,dx}{a}$$

$$= -\frac{\sin(c+dx)}{a^3x} - \frac{b\int\left(\frac{\sqrt{-a}\,\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{-a}\,\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}\,x\right)}\right)dx}{a^3} - \frac{b\int\left(-\frac{b\sin(c+\cdots}{4a\left(\sqrt{-a}\sqrt{\cdots}\right.}\cdots\right.}{\cdots}$$

$$= -\frac{\sin(c+dx)}{a^3x} - \frac{b\int\frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}\,x}\,dx}{2(-a)^{7/2}} - \frac{b\int\frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}\,x}\,dx}{2(-a)^{7/2}} + \frac{\left(3b^2\right)\int\frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\cdots}{16a^3}\cdots$$

$$= \frac{d\cos(c)\operatorname{Ci}(dx)}{a^3} - \frac{\sin(c+dx)}{a^3x} - \frac{\sqrt{b}\,\sin(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)^2} + \frac{7\sqrt{b}\,\sin(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}\,x\right)\cdots}\cdots$$

$$= \frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} + \frac{d\cos(c)\operatorname{Ci}(dx)}{a^3} - \frac{\sqrt{b}\,\operatorname{Ci}\cdots}{\cdots}$$

$$= \frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} + \frac{d\cos(c)\operatorname{Ci}(dx)}{a^3} + \frac{7d\cos\cdots}{\cdots}$$

$$= \frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}\,x\right)} + \frac{d\cos(c)\operatorname{Ci}(dx)}{a^3} + \frac{7d\cos\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.81, size = 1177, normalized size = 1.35

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

```
[Out] ((-2*a^(5/2)*d*Cos[c]*Cos[d*x])/(a + b*x^2)^2 - (2*a^(3/2)*b*d*x^2*Cos[c]*C
os[d*x])/(a + b*x^2)^2 + 16*Sqrt[a]*d*Cos[c]*CosIntegral[d*x] + 7*Sqrt[a]*d
*Cos[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]
- (16*a^(5/2)*Cos[d*x]*Sin[c])/(x*(a + b*x^2)^2) - (50*a^(3/2)*b*x*Cos[d*x]
*Sin[c])/(a + b*x^2)^2 - (30*Sqrt[a]*b^2*x^3*Cos[d*x]*Sin[c])/(a + b*x^2)^2
 - (15*I)*Sqrt[b]*Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt
[b] + x)]*Sin[c] - (I*a*d^2*Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqr
t[a])/Sqrt[b] + x)]*Sin[c])/Sqrt[b] + (CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b
] + x)]*(7*Sqrt[a]*Sqrt[b]*d*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]] + I*(15*b + a*d
^2)*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[b] - (16*a^(5/2)*Cos[c]*Sin[d*x])
/(x*(a + b*x^2)^2) - (50*a^(3/2)*b*x*Cos[c]*Sin[d*x])/(a + b*x^2)^2 - (30*S
qrt[a]*b^2*x^3*Cos[c]*Sin[d*x])/(a + b*x^2)^2 + (2*a^(5/2)*d*Sin[c]*Sin[d*x
])/(a + b*x^2)^2 + (2*a^(3/2)*b*d*x^2*Sin[c]*Sin[d*x])/(a + b*x^2)^2 - 15*S
qrt[b]*Cos[c]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqr
t[b]] - (a*d^2*Cos[c]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a
]*d)/Sqrt[b]])/Sqrt[b] + (7*I)*Sqrt[a]*d*CosIntegral[d*((I*Sqrt[a])/Sqrt[b]
 + x)]*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]] - 16*Sqrt[a]*d*Sin[c]*SinIntegral[d
*x] - (15*I)*Sqrt[b]*Cos[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqr
t[a])/Sqrt[b] + x)] - (I*a*d^2*Cos[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral
[d*((I*Sqrt[a])/Sqrt[b] + x)])/Sqrt[b] - 7*Sqrt[a]*d*Cosh[(Sqrt[a]*d)/Sqrt[
b]]*Sin[c]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + (7*I)*Sqrt[a]*d*Cos[c
]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + 15*S
qrt[b]*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b]
+ x)] + (a*d^2*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/
Sqrt[b] + x)])/Sqrt[b] - (15*I)*Sqrt[b]*Cos[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]]*Si
nIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - (I*a*d^2*Cos[c]*Cosh[(Sqrt[a]*d)/S
qrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/Sqrt[b] + 7*Sqrt[a]*d*Cos
h[(Sqrt[a]*d)/Sqrt[b]]*Sin[c]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + (7
*I)*Sqrt[a]*d*Cos[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sq
rt[b] - d*x] - 15*Sqrt[b]*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*S
qrt[a]*d)/Sqrt[b] - d*x] - (a*d^2*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinInteg
ral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/Sqrt[b])/(16*a^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(671) = 1342$.
time = 0.37, size = 1363, normalized size = 1.56

method	result
risch	$\frac{d^2 e^{\frac{ibc+d\sqrt{ab}}{b}} \text{expIntegral}\left(1, \frac{ibc+d\sqrt{ab}-b(idx+ic)}{b}\right)}{32a^2\sqrt{ab}} - \frac{d^2 e^{\frac{ibc-d\sqrt{ab}}{b}} \text{expIntegral}\left(1, \frac{ibc-d\sqrt{ab}-b(idx+ic)}{b}\right)}{32a^2\sqrt{ab}} -$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/a*b*d^4*(-1/8*sin(d*x+c)*(5*a*c*d^2-5*a*d^2*(d*x+c)+3*b*c^3-9*b*c^2*(
d*x+c)+9*b*c*(d*x+c)^2-3*b*(d*x+c)^3)/a^2/d^4/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*
(d*x+c)^2)^2+1/8*cos(d*x+c)/a/b/d^2/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)
-1/16*(a*d^2+3*b)/a^2/d^4/b^2/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*
b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/
b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/16*(a*d^2+3*b)/a^2/d^4/b^2/((d*(-a*b)^(1/
2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-
Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-3/16/a^2/d^4/
b*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(
d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-3/16/a^2/d^4/b*(Si(d*x+
c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1
/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)))-b*d^2/a^2*(sin(d*x+c)*(1/2/a/d^2*
(d*x+c)-1/2*c/a/d^2)/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)-1/4/a/d^2/b/(-
(d*(-a*b)^(1/2)+c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(
1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-
1/4/a/d^2/b/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*co
s((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1
/2)-c*b)/b))-1/4/a/b/d^2*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(
1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-
1/4/a/b/d^2*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+C
i(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)))+1/a^3*(-sin(d
*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))-1/a^3*b*(-1/2/b/(-(d*(-a*b)^(1/2)+
c*b)/b+c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(
d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/2/b/((d*(-a*b)
^(1/2)-c*b)/b+c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)
/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.48, size = 720, normalized size = 0.82

$$\frac{16\,(ab^2d^2x^5 + 2\,a^2bd^2x^3 + a^3d^2x)\operatorname{Ei}(i\,dx)\,e^{(i\,c)} + 16\,(ab^2d^2x^5 + 2\,a^2bd^2x^3 + a^3d^2x)\operatorname{Ei}(-i\,dx)\,e^{(-i\,c)} + \left(7\,ab^2d^2x^5 + 14\,a^2bd^2x^3 + 7\,a^3d^2x - ((ab^2d^2 + 15\,b^3)x^5 + 2\,(a^2bd^2 + 15\,ab^2)x^3 + (a^3d^2 + 15\,a^2b)x)\sqrt{\frac{ad^2}{b}}\right)\operatorname{Ei}\left(i\,dx - \sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c + \sqrt{\frac{ad^2}{b}}\right)} + \left(7\,ab^2d^2x^5 + 14\,a^2bd^2x^3 + 7\,a^3d^2x + ((ab^2d^2 + 15\,b^3)x^5 + 2\,(a^2bd^2 + 15\,ab^2)x^3 + (a^3d^2 + 15\,a^2b)x)\sqrt{\frac{ad^2}{b}}\right)\operatorname{Ei}\left(i\,dx + \sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c - \sqrt{\frac{ad^2}{b}}\right)} + \left(7\,ab^2d^2x^5 + 14\,a^2bd^2x^3 + 7\,a^3d^2x - ((ab^2d^2 + 15\,b^3)x^5 + 2\,(a^2bd^2 + 15\,ab^2)x^3 + (a^3d^2 + 15\,a^2b)x)\sqrt{\frac{ad^2}{b}}\right)\operatorname{Ei}\left(-i\,dx - \sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c + \sqrt{\frac{ad^2}{b}}\right)} + \left(7\,ab^2d^2x^5 + 14\,a^2bd^2x^3 + 7\,a^3d^2x + ((ab^2d^2 + 15\,b^3)x^5 + 2\,(a^2bd^2 + 15\,ab^2)x^3 + (a^3d^2 + 15\,a^2b)x)\sqrt{\frac{ad^2}{b}}\right)\operatorname{Ei}\left(-i\,dx + \sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c - \sqrt{\frac{ad^2}{b}}\right)} - 4\,(a^2bd^2x^3 + a^3d^2x)\cos(dx + c) - 4\,(15\,ab^2dx^4 + 25\,a^2bdx^2 + 8\,a^3d)\sin(dx + c)}{32\,(a^4b^2dx^5 + 2\,a^5bdx^3 + a^6dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(16*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*Ei(I*d*x)*e^(I*c) +
16*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*Ei(-I*d*x)*e^(-I*c) + (7*a
*b^2*d^2*x^5 + 14*a^2*b*d^2*x^3 + 7*a^3*d^2*x - ((a*b^2*d^2 + 15*b^3)*x^5 +
 2*(a^2*b*d^2 + 15*a*b^2)*x^3 + (a^3*d^2 + 15*a^2*b)*x)*sqrt(a*d^2/b))*Ei(I
*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (7*a*b^2*d^2*x^5 + 14*a^2*b
*d^2*x^3 + 7*a^3*d^2*x + ((a*b^2*d^2 + 15*b^3)*x^5 + 2*(a^2*b*d^2 + 15*a*b^
2)*x^3 + (a^3*d^2 + 15*a^2*b)*x)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e
^(I*c - sqrt(a*d^2/b)) + (7*a*b^2*d^2*x^5 + 14*a^2*b*d^2*x^3 + 7*a^3*d^2*x
- ((a*b^2*d^2 + 15*b^3)*x^5 + 2*(a^2*b*d^2 + 15*a*b^2)*x^3 + (a^3*d^2 + 15*
a^2*b)*x)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)
) + (7*a*b^2*d^2*x^5 + 14*a^2*b*d^2*x^3 + 7*a^3*d^2*x + ((a*b^2*d^2 + 15*b^
3)*x^5 + 2*(a^2*b*d^2 + 15*a*b^2)*x^3 + (a^3*d^2 + 15*a^2*b)*x)*sqrt(a*d^2/
b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*(a^2*b*d^2*x^3
+ a^3*d^2*x)*cos(d*x + c) - 4*(15*a*b^2*d*x^4 + 25*a^2*b*d*x^2 + 8*a^3*d)*s
in(d*x + c))/(a^4*b^2*d*x^5 + 2*a^5*b*d*x^3 + a^6*d*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)}{x^2\,{\left(b\,x^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x^2*(a + b*x^2)^3),x)
[Out] int(sin(c + d*x)/(x^2*(a + b*x^2)^3), x)
```

3.78 $\int \frac{\sin(c+dx)}{x^3(a+bx^2)^3}\,dx$

Optimal. Leaf size=791

$$-\frac{d\cos(c+dx)}{2a^3x}-\frac{\sqrt{b}\,d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}\,x\right)}+\frac{\sqrt{b}\,d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}\,x\right)}-\frac{9\sqrt{b}\,d\cos\left(c+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-d\right.}{16(-a)^{7/2}}$$

[Out] -1/2*d*cos(d*x+c)/a^3/x-3*b*cos(c)*Si(d*x)/a^4-1/2*d^2*cos(c)*Si(d*x)/a^3+3/2*b*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/a^4+1/16*d^2*cos(c+d*(-a)^(1/2)/b^(1/2))*Si(d*x-d*(-a)^(1/2)/b^(1/2))/a^3+3/2*b*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/a^4+1/16*d^2*cos(c-d*(-a)^(1/2)/b^(1/2))*Si(d*x+d*(-a)^(1/2)/b^(1/2))/a^3-3*b*Ci(d*x)*sin(c)/a^4-1/2*d^2*Ci(d*x)*sin(c)/a^3-1/2*sin(d*x+c)/a^3/x^2-1/4*b*sin(d*x+c)/a^2/(b*x^2+a)^2-b*sin(d*x+c)/a^3/(b*x^2+a)+3/2*b*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a^4+1/16*d^2*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))/a^3+3/2*b*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/a^4+1/16*d^2*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))/a^3+9/16*d*Ci(d*x+d*(-a)^(1/2)/b^(1/2))*cos(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)-9/16*d*Ci(-d*x+d*(-a)^(1/2)/b^(1/2))*cos(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)-9/16*d*Si(d*x+d*(-a)^(1/2)/b^(1/2))*sin(c-d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)+9/16*d*Si(d*x-d*(-a)^(1/2)/b^(1/2))*sin(c+d*(-a)^(1/2)/b^(1/2))*b^(1/2)/(-a)^(7/2)-1/16*d*cos(d*x+c)*b^(1/2)/a^3/((-a)^(1/2)-x*b^(1/2))+1/16*d*cos(d*x+c)*b^(1/2)/a^3/((-a)^(1/2)+x*b^(1/2))

Rubi [A]
time = 1.38, antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3426, 3378, 3384, 3380, 3383, 3422, 3415}

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]/(x^3*(a + b*x^2)^3),x]

[Out] -1/2*(d*Cos[c + d*x])/(a^3*x) - (Sqrt[b]*d*Cos[c + d*x])/(16*a^3*(Sqrt[-a] - Sqrt[b]*x)) + (Sqrt[b]*d*Cos[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) - (9*Sqrt[b]*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(7/2)) + (9*Sqrt[b]*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (3*b*CosIntegral[d*x]*Sin[c])/a^4 - (d^2*CosIntegral[d*x]*Sin[c])/(2*a^3) + (3*b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a^4) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^3)

```
 + (3*b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b
]])/(2*a^4) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a
]*d)/Sqrt[b]])/(16*a^3) - Sin[c + d*x]/(2*a^3*x^2) - (b*Sin[c + d*x])/(4*a^
2*(a + b*x^2)^2) - (b*Sin[c + d*x])/(a^3*(a + b*x^2)) - (3*b*Cos[c]*SinInte
gral[d*x])/a^4 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a^3) - (3*b*Cos[c + (Sqrt
[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) - (d^2*Co
s[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^
3) - (9*Sqrt[b]*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] - d*x])/(16*(-a)^(7/2)) + (3*b*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinInteg
ral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^4) + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b
]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3) - (9*Sqrt[b]*d*Sin[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7
/2))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx = \int \left(\frac{\sin(c+dx)}{a^3x^3} - \frac{3b\sin(c+dx)}{a^4x} + \frac{b^2x\sin(c+dx)}{a^2\left(a+bx^2\right)^3} + \frac{2b^2x\sin(c+dx)}{a^3\left(a+bx^2\right)^2} + \frac{3b^2x\sin(c+ \cdots}{a^4\left(a+bx \cdots\right.}\right.$$

$$= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a^3} - \frac{(3b)\int \frac{\sin(c+dx)}{x}\,dx}{a^4} + \frac{\left(3b^2\right)\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{a^4} + \frac{\left(2b^2\right)\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a^3} + \frac{b^2\int \cdots}{\cdots}$$

$$= -\frac{\sin(c+dx)}{2a^3x^2} - \frac{b\sin(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)} + \frac{\left(3b^2\right)\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\cdots}{2\sqrt{\cdots}}\right.}{a^4}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{3b\mathrm{Ci}(dx)\sin(c)}{a^4} - \frac{\sin(c+dx)}{2a^3x^2} - \frac{b\sin(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3\cdots}{\cdots}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{3b\mathrm{Ci}(dx)\sin(c)}{a^4} - \frac{\sin(c+dx)}{2a^3x^2} - \frac{b\sin(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3\cdots}{\cdots}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{\sqrt{b}\,d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{b}\,d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{3b\mathrm{Ci}(dx)\sin(c)}{a^4} - \cdots$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{\sqrt{b}\,d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{b}\,d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{\sqrt{b}\,d\cos\left(c+\frac{\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{\sqrt{b}\,d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{b}\,d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{\sqrt{b}\,d\cos\left(c+\frac{\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{\sqrt{b}\,d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}\,x\right)} + \frac{\sqrt{b}\,d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}\,x\right)} - \frac{9\sqrt{b}\,d\cos\left(c+\cdots\right.}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.72, size = 995, normalized size = 1.26

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x^2)^3),x]
```

```
[Out] ((-2*a*Cos[d*x]*(d*x*(4*a^2 + 7*a*b*x^2 + 3*b^2*x^4)*Cos[c] + 2*(2*a^2 + 9*
a*b*x^2 + 6*b^2*x^4)*Sin[c]))/(x^2*(a + b*x^2)^2) + (2*a*(-2*(2*a^2 + 9*a*b
*x^2 + 6*b^2*x^4)*Cos[c] + d*x*(4*a^2 + 7*a*b*x^2 + 3*b^2*x^4)*Sin[c])*Sin[
d*x])/(x^2*(a + b*x^2)^2) - 8*(6*b + a*d^2)*(CosIntegral[d*x]*Sin[c] + Cos[
c]*SinIntegral[d*x]) + 24*b*Cos[c]*(I*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b]
 + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] - I*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x
)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + Cosh[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[d*((I
*Sqrt[a])/Sqrt[b] + x)] - SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])) + a*d^
2*Cos[c]*(I*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sq
rt[b]] - I*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b
]] + Cosh[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] -
SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])) + 9*Sqrt[a]*Sqrt[b]*d*Cos[c]*((-
I)*Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] +
I*Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + Sinh
[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + SinInteg
ral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])) - 9*Sqrt[a]*Sqrt[b]*d*Sin[c]*(CosIntegra
l[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + CosIntegral[d
*((I*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + I*Cosh[(Sqrt[a]*d)/
Sqrt[b]]*(SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + SinIntegral[(I*Sqrt[a]
*d)/Sqrt[b] - d*x])) + 24*b*Sin[c]*(Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d
*(((-I)*Sqrt[a])/Sqrt[b] + x)] + Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((
I*Sqrt[a])/Sqrt[b] + x)] + I*Sinh[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[d*((I*S
qrt[a])/Sqrt[b] + x)] + SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])) + a*d^2*
Sin[c]*(Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x
)] + Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I
*Sinh[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + SinI
ntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])))/(16*a^4)
```

Maple [**A**]
time = 1.84, size = 697, normalized size = 0.88 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/4*sin(d*x+c)*(2*a^2*d^4+9*a*b*c^2*d^2-18*a*b*c*d^2*(d*x+c)+9*a*b*d^
2*(d*x+c)^2+6*b^2*c^4-24*b^2*c^3*(d*x+c)+36*b^2*c^2*(d*x+c)^2-24*b^2*c*(d*x
+c)^3+6*b^2*(d*x+c)^4)/a^3/d^2/x^2/(d^2*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)^
2-1/8*cos(d*x+c)*(4*d^2*a+3*b*c^2-6*b*c*(d*x+c)+3*b*(d*x+c)^2)/a^3/d/x/(d^2
*a+b*c^2-2*b*c*(d*x+c)+b*(d*x+c)^2)+1/16*(a*d^2+24*b)/d^2/a^4*(Si(d*x+c-(d*
(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c
*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/16*(a*d^2+24*b)/d^2/a^4*(Si(d*x+c+(d*
(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c
*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/2/a^4*(a*d^2+6*b)/d^2*(Si(d*x)*cos(c)
```

```
+Ci(d*x)*sin(c))-9/16/a^3/(-(d*(-a*b)^(1/2)+c*b)/b+c)*(-Si(d*x+c-(d*(-a*b)^
(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*
cos((d*(-a*b)^(1/2)+c*b)/b))-9/16/a^3/((d*(-a*b)^(1/2)-c*b)/b+c)*(Si(d*x+c+
(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2
)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

Fricas [**C**] Result contains complex when optimal does not.
time = 0.47, size = 768, normalized size = 0.97

$$-\frac{8\left(-i\left(ab^2d^2+6b^3\right)x^6-2i\left(a^2bd^2+6ab^2\right)x^4-i\left(a^3d^2+6a^2b\right)x^2\right)\mathrm{Ei}(idx)e^{(ic)}+8\left(i\left(ab^2d^2+6b^3\right)x^6+2i\left(a^2bd^2+6ab^2\right)x^4+i\left(a^3d^2+6a^2b\right)x^2\right)\mathrm{Ei}(-idx)e^{(-ic)}+\left(i\left(ab^2d^2+24b^3\right)x^6+2i\left(a^2bd^2+24ab^2\right)x^4+i\left(a^3d^2+24a^2b\right)x^2+9\left(-ib^3x^6-2iab^2x^4-ia^2bx^2\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(idx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic+\sqrt{\frac{ad^2}{b}}\right)}+\left(i\left(ab^2d^2+24b^3\right)x^6+2i\left(a^2bd^2+24ab^2\right)x^4+i\left(a^3d^2+24a^2b\right)x^2+9\left(ib^3x^6+2iab^2x^4+ia^2bx^2\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(idx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic-\sqrt{\frac{ad^2}{b}}\right)}+\left(-i\left(ab^2d^2+24b^3\right)x^6-2i\left(a^2bd^2+24ab^2\right)x^4-i\left(a^3d^2+24a^2b\right)x^2+9\left(ib^3x^6+2iab^2x^4+ia^2bx^2\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(-idx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-ic+\sqrt{\frac{ad^2}{b}}\right)}+\left(-i\left(ab^2d^2+24b^3\right)x^6-2i\left(a^2bd^2+24ab^2\right)x^4-i\left(a^3d^2+24a^2b\right)x^2+9\left(-ib^3x^6-2iab^2x^4-ia^2bx^2\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(-idx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-ic-\sqrt{\frac{ad^2}{b}}\right)}+4\left(3ab^2dx^5+7a^2bdx^3+4a^3dx\right)\cos(dx+c)+8\left(6ab^2x^4+9a^2bx^2+2a^3\right)\sin(dx+c)}{32\left(a^4b^2x^6+2a^5bx^4+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*(-I*(a*b^2*d^2 + 6*b^3)*x^6 - 2*I*(a^2*b*d^2 + 6*a*b^2)*x^4 - I*(a
^3*d^2 + 6*a^2*b)*x^2)*Ei(I*d*x)*e^(I*c) + 8*(I*(a*b^2*d^2 + 6*b^3)*x^6 + 2
*I*(a^2*b*d^2 + 6*a*b^2)*x^4 + I*(a^3*d^2 + 6*a^2*b)*x^2)*Ei(-I*d*x)*e^(-I*
c) + (I*(a*b^2*d^2 + 24*b^3)*x^6 + 2*I*(a^2*b*d^2 + 24*a*b^2)*x^4 + I*(a^3*
d^2 + 24*a^2*b)*x^2 + 9*(-I*b^3*x^6 - 2*I*a*b^2*x^4 - I*a^2*b*x^2)*sqrt(a*d
^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (I*(a*b^2*d^2 +
24*b^3)*x^6 + 2*I*(a^2*b*d^2 + 24*a*b^2)*x^4 + I*(a^3*d^2 + 24*a^2*b)*x^2 +
 9*(I*b^3*x^6 + 2*I*a*b^2*x^4 + I*a^2*b*x^2)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt
(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (-I*(a*b^2*d^2 + 24*b^3)*x^6 - 2*I*(a^
2*b*d^2 + 24*a*b^2)*x^4 - I*(a^3*d^2 + 24*a^2*b)*x^2 + 9*(I*b^3*x^6 + 2*I*a
*b^2*x^4 + I*a^2*b*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c +
 sqrt(a*d^2/b)) + (-I*(a*b^2*d^2 + 24*b^3)*x^6 - 2*I*(a^2*b*d^2 + 24*a*b^2)
*x^4 - I*(a^3*d^2 + 24*a^2*b)*x^2 + 9*(-I*b^3*x^6 - 2*I*a*b^2*x^4 - I*a^2*b
*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) +
4*(3*a*b^2*d*x^5 + 7*a^2*b*d*x^3 + 4*a^3*d*x)*cos(d*x + c) + 8*(6*a*b^2*x^4
 + 9*a^2*b*x^2 + 2*a^3)*sin(d*x + c))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**3/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)}{x^3\,{\left(b\,x^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x^3*(a + b*x^2)^3),x)
```

```
[Out] int(sin(c + d*x)/(x^3*(a + b*x^2)^3), x)
```

3.79 $\int x^3(a+bx^3)\sin(c+dx)\,dx$

Optimal. Leaf size=156

$$\frac{720b\cos(c+dx)}{d^7}+\frac{6ax\cos(c+dx)}{d^3}-\frac{360bx^2\cos(c+dx)}{d^5}-\frac{ax^3\cos(c+dx)}{d}+\frac{30bx^4\cos(c+dx)}{d^3}-\frac{bx^6\cos(c+dx)}{d}$$

```
[Out] 720*b*cos(d*x+c)/d^7+6*a*x*cos(d*x+c)/d^3-360*b*x^2*cos(d*x+c)/d^5-a*x^3*co
s(d*x+c)/d+30*b*x^4*cos(d*x+c)/d^3-b*x^6*cos(d*x+c)/d-6*a*sin(d*x+c)/d^4+72
0*b*x*sin(d*x+c)/d^6+3*a*x^2*sin(d*x+c)/d^2-120*b*x^3*sin(d*x+c)/d^4+6*b*x^
5*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.16, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3420, 3377, 2717, 2718}

$$-\frac{6a\sin(c+dx)}{d^4}+\frac{6ax\cos(c+dx)}{d^3}+\frac{3ax^2\sin(c+dx)}{d^2}-\frac{ax^3\cos(c+dx)}{d}+\frac{720b\cos(c+dx)}{d^7}+\frac{720bx\sin(c+dx)}{d^6}-\frac{360bx^2\cos(c+dx)}{d^5}-\frac{120bx^3\sin(c+dx)}{d^4}+\frac{30bx^4\cos(c+dx)}{d^3}+\frac{6bx^5\sin(c+dx)}{d^2}-\frac{bx^6\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (720*b*Cos[c + d*x])/d^7 + (6*a*x*Cos[c + d*x])/d^3 - (360*b*x^2*Cos[c + d*
x])/d^5 - (a*x^3*Cos[c + d*x])/d + (30*b*x^4*Cos[c + d*x])/d^3 - (b*x^6*Cos
[c + d*x])/d - (6*a*Sin[c + d*x])/d^4 + (720*b*x*Sin[c + d*x])/d^6 + (3*a*x
^2*Sin[c + d*x])/d^2 - (120*b*x^3*Sin[c + d*x])/d^4 + (6*b*x^5*Sin[c + d*x]
)/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(a+bx^3\right)\sin(c+dx)\,dx &= \int \left(ax^3\sin(c+dx)+bx^6\sin(c+dx)\right)\,dx \\
&= a\int x^3\sin(c+dx)\,dx+b\int x^6\sin(c+dx)\,dx \\
&= -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^6\cos(c+dx)}{d}+\frac{(3a)\int x^2\cos(c+dx)\,dx}{d}+\frac{(6b)\int x^5
\\
&= -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^6\cos(c+dx)}{d}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{6bx^5\sin(c+dx)}{d^2}
\\
&= \frac{6ax\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}+\frac{30bx^4\cos(c+dx)}{d^3}-\frac{bx^6\cos(c+dx)}{d}
\\
&= \frac{6ax\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}+\frac{30bx^4\cos(c+dx)}{d^3}-\frac{bx^6\cos(c+dx)}{d}
\\
&= \frac{6ax\cos(c+dx)}{d^3}-\frac{360bx^2\cos(c+dx)}{d^5}-\frac{ax^3\cos(c+dx)}{d}+\frac{30bx^4\cos(c+
\\
&= \frac{6ax\cos(c+dx)}{d^3}-\frac{360bx^2\cos(c+dx)}{d^5}-\frac{ax^3\cos(c+dx)}{d}+\frac{30bx^4\cos(c+
\\
&= \frac{720b\cos(c+dx)}{d^7}+\frac{6ax\cos(c+dx)}{d^3}-\frac{360bx^2\cos(c+dx)}{d^5}-\frac{ax^3\cos(c+dx
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 101, normalized size = 0.65

$$
\frac{-((ad^4x(-6+d^2x^2)+b(-720+360d^2x^2-30d^4x^4+d^6x^6))\cos(c+dx))+3d(ad^2(-2+d^2x^2)+2bx(120-20d^2x^2+d^4x^4))\sin(c+dx)}{d^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (-((a*d^4*x*(-6 + d^2*x^2) + b*(-720 + 360*d^2*x^2 - 30*d^4*x^4 + d^6*x^6))
*Cos[c + d*x]) + 3*d*(a*d^2*(-2 + d^2*x^2) + 2*b*x*(120 - 20*d^2*x^2 + d^4*
x^4))*Sin[c + d*x])/d^7
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(156) = 312$.
time = 0.06, size = 556, normalized size = 3.56 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(a*c^3*cos(d*x+c)+3*a*c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-3*a*c*(-(d*
x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+a*(-(d*x+c)^3*cos(d*x+
c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))-1/d^3*b*c^6*co
s(d*x+c)-6/d^3*b*c^5*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+15/d^3*b*c^4*(-(d*x+c)
^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-20/d^3*b*c^3*(-(d*x+c)^3*c
os(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+15/d^3*
b*c^2*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)
-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))-6/d^3*b*c*(-(d*x+c)^5*cos(d*x+c)+5*(d
*x+c)^4*sin(d*x+c)+20*(d*x+c)^3*cos(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(
d*x+c)-120*(d*x+c)*cos(d*x+c))+1/d^3*b*(-(d*x+c)^6*cos(d*x+c)+6*(d*x+c)^5*s
in(d*x+c)+30*(d*x+c)^4*cos(d*x+c)-120*(d*x+c)^3*sin(d*x+c)-360*(d*x+c)^2*co
s(d*x+c)+720*cos(d*x+c)+720*(d*x+c)*sin(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(156) = 312$.
time = 0.33, size = 449, normalized size = 2.88

$$\frac{ac^3\cos(dx+c) - \frac{bc^6\cos(dx+c)}{d^3} - 3\left((dx+c)\cos(dx+c) - \sin(dx+c)\right)ac^2 + \frac{6\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)bc^5}{d^3} + 3\left(\left((dx+c)^2-2\right)\cos(dx+c) - 2(dx+c)\sin(dx+c)\right)ac - \frac{15\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)bc^4}{d^3} - \left(\left((dx+c)^3-6dx-6c\right)\cos(dx+c) - 3\left((dx+c)^2-2\right)\sin(dx+c)\right)a + \frac{20\left(\left((dx+c)^3-6dx-6c\right)\cos(dx+c)-3\left((dx+c)^2-2\right)\sin(dx+c)\right)bc^3}{d^3} - \frac{15\left(\left((dx+c)^4-12(dx+c)^2+24\right)\cos(dx+c)-4\left((dx+c)^3-6dx-6c\right)\sin(dx+c)\right)bc^2}{d^3} + \frac{6\left(\left((dx+c)^5-20(dx+c)^3+120dx+120c\right)\cos(dx+c)-5\left((dx+c)^4-12(dx+c)^2+24\right)\sin(dx+c)\right)bc}{d^3} - \frac{\left(\left((dx+c)^6-30(dx+c)^4+360(dx+c)^2-720\right)\cos(dx+c)-6\left((dx+c)^5-20(dx+c)^3+120dx+120c\right)\sin(dx+c)\right)b}{d^3}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c^3*cos(d*x + c) - b*c^6*cos(d*x + c)/d^3 - 3*((d*x + c)*cos(d*x + c) -
sin(d*x + c))*a*c^2 + 6*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^5/d^3 +
 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*c - 15*(((
d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^4/d^3 - (((d*x
 + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a +
 20*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x
 + c))*b*c^3/d^3 - 15*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4
*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b*c^2/d^3 + 6*(((d*x + c)^5 - 20
*(d*x + c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c
)^2 + 24)*sin(d*x + c))*b*c/d^3 - (((d*x + c)^6 - 30*(d*x + c)^4 + 360*(d*x
 + c)^2 - 720)*cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 1
20*c)*sin(d*x + c))*b/d^3)/d^4
```

Fricas [A]
time = 0.36, size = 104, normalized size = 0.67

$$-\frac{\left(bd^6x^6 + ad^6x^3 - 30\,bd^4x^4 - 6\,ad^4x + 360\,bd^2x^2 - 720\,b\right)\cos(dx+c) - 3\left(2\,bd^5x^5 + ad^5x^2 - 40\,bd^3x^3 - 2\,ad^3 + 240\,bdx\right)\sin(dx+c)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^6*x^6 + a*d^6*x^3 - 30*b*d^4*x^4 - 6*a*d^4*x + 360*b*d^2*x^2 - 720*b
)*cos(d*x + c) - 3*(2*b*d^5*x^5 + a*d^5*x^2 - 40*b*d^3*x^3 - 2*a*d^3 + 240*
b*d*x)*sin(d*x + c))/d^7
```

Sympy [A]
time = 0.68, size = 185, normalized size = 1.19

$$\begin{cases} -\frac{ax^3\cos(c+dx)}{d} + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{6ax\cos(c+dx)}{d^3} - \frac{6a\sin(c+dx)}{d^4} - \frac{bx^6\cos(c+dx)}{d} + \frac{6bx^5\sin(c+dx)}{d^2} + \frac{30bx^4\cos(c+dx)}{d^3} - \frac{120bx^3\sin(c+dx)}{d^4} - \frac{360bx^2\cos(c+dx)}{d^5} + \frac{720bx\sin(c+dx)}{d^6} + \frac{720b\cos(c+dx)}{d^7} & \text{for}\ d \neq 0 \\ \left(\frac{ax^4}{4} + \frac{bx^7}{7}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**3*cos(c + d*x)/d + 3*a*x**2*sin(c + d*x)/d**2 + 6*a*x*cos(
c + d*x)/d**3 - 6*a*sin(c + d*x)/d**4 - b*x**6*cos(c + d*x)/d + 6*b*x**5*si
n(c + d*x)/d**2 + 30*b*x**4*cos(c + d*x)/d**3 - 120*b*x**3*sin(c + d*x)/d**
4 - 360*b*x**2*cos(c + d*x)/d**5 + 720*b*x*sin(c + d*x)/d**6 + 720*b*cos(c
+ d*x)/d**7, Ne(d, 0)), ((a*x**4/4 + b*x**7/7)*sin(c), True))
```

Giac [A]
time = 5.04, size = 106, normalized size = 0.68

$$-\frac{(bd^6x^6 + ad^6x^3 - 30\,bd^4x^4 - 6\,ad^4x + 360\,bd^2x^2 - 720\,b)\cos(dx+c)}{d^7} + \frac{3\,(2\,bd^5x^5 + ad^5x^2 - 40\,bd^3x^3 - 2\,ad^3 + 240\,bdx)\sin(dx+c)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^6*x^6 + a*d^6*x^3 - 30*b*d^4*x^4 - 6*a*d^4*x + 360*b*d^2*x^2 - 720*b)
*cos(d*x + c)/d^7 + 3*(2*b*d^5*x^5 + a*d^5*x^2 - 40*b*d^3*x^3 - 2*a*d^3 + 2
40*b*d*x)*sin(d*x + c)/d^7
```

Mupad [B]
time = 0.59, size = 151, normalized size = 0.97

$$\frac{d^4\,(6\,a\,x\,\cos(c+d\,x) + 30\,b\,x^4\,\cos(c+d\,x)) + 720\,b\,\cos(c+d\,x) - d^6\,(a\,x^3\,\cos(c+d\,x) + b\,x^6\,\cos(c+d\,x)) + d^5\,(3\,a\,x^2\,\sin(c+d\,x) + 6\,b\,x^5\,\sin(c+d\,x)) - d^3\,(6\,a\,\sin(c+d\,x) + 120\,b\,x^3\,\sin(c+d\,x)) + 720\,b\,d\,x\,\sin(c+d\,x) - 360\,b\,d^2\,x^2\,\cos(c+d\,x)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(c + d*x)*(a + b*x^3),x)
```

```
[Out] (d^4*(6*a*x*cos(c + d*x) + 30*b*x^4*cos(c + d*x)) + 720*b*cos(c + d*x) - d^
6*(a*x^3*cos(c + d*x) + b*x^6*cos(c + d*x)) + d^5*(3*a*x^2*sin(c + d*x) + 6
*b*x^5*sin(c + d*x)) - d^3*(6*a*sin(c + d*x) + 120*b*x^3*sin(c + d*x)) + 72
0*b*d*x*sin(c + d*x) - 360*b*d^2*x^2*cos(c + d*x))/d^7
```

3.80 $\int x^2(a+bx^3)\sin(c+dx)\,dx$

Optimal. Leaf size=126

$$\frac{2a\cos(c+dx)}{d^3}-\frac{120bx\cos(c+dx)}{d^5}-\frac{ax^2\cos(c+dx)}{d}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{bx^5\cos(c+dx)}{d}+\frac{120b\sin(c+dx)}{d^6}$$

```
[Out] 2*a*cos(d*x+c)/d^3-120*b*x*cos(d*x+c)/d^5-a*x^2*cos(d*x+c)/d+20*b*x^3*cos(d
*x+c)/d^3-b*x^5*cos(d*x+c)/d+120*b*sin(d*x+c)/d^6+2*a*x*sin(d*x+c)/d^2-60*b
*x^2*sin(d*x+c)/d^4+5*b*x^4*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3420, 3377, 2718, 2717}

$$\frac{2a\cos(c+dx)}{d^3}+\frac{2ax\sin(c+dx)}{d^2}-\frac{ax^2\cos(c+dx)}{d}+\frac{120b\sin(c+dx)}{d^6}-\frac{120bx\cos(c+dx)}{d^5}-\frac{60bx^2\sin(c+dx)}{d^4}+\frac{20bx^3\cos(c+dx)}{d^3}+\frac{5bx^4\sin(c+dx)}{d^2}-\frac{bx^5\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (2*a*Cos[c + d*x])/d^3 - (120*b*x*Cos[c + d*x])/d^5 - (a*x^2*Cos[c + d*x])/
d + (20*b*x^3*Cos[c + d*x])/d^3 - (b*x^5*Cos[c + d*x])/d + (120*b*Sin[c + d
*x])/d^6 + (2*a*x*Sin[c + d*x])/d^2 - (60*b*x^2*Sin[c + d*x])/d^4 + (5*b*x^
4*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+bx^3\right)\sin(c+dx)\,dx &= \int \left(ax^2\sin(c+dx)+bx^5\sin(c+dx)\right)\,dx\\
&= a\int x^2\sin(c+dx)\,dx+b\int x^5\sin(c+dx)\,dx\\
&= -\frac{ax^2\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{(2a)\int x\cos(c+dx)\,dx}{d}+\frac{(5b)\int x^4
\\
&= -\frac{ax^2\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{2ax\sin(c+dx)}{d^2}+\frac{5bx^4\sin(c+dx)}{d^2}
\\
&= \frac{2a\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{bx^5\cos(c+dx)}{d}+
\\
&= \frac{2a\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{bx^5\cos(c+dx)}{d}+
\\
&= \frac{2a\cos(c+dx)}{d^3}-\frac{120bx\cos(c+dx)}{d^5}-\frac{ax^2\cos(c+dx)}{d}+\frac{20bx^3\cos(c+dx)}{d^3}
\\
&= \frac{2a\cos(c+dx)}{d^3}-\frac{120bx\cos(c+dx)}{d^5}-\frac{ax^2\cos(c+dx)}{d}+\frac{20bx^3\cos(c+dx)}{d^3}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 84, normalized size = 0.67

$$\frac{-d(ad^2(-2+d^2x^2)+bx(120-20d^2x^2+d^4x^4))\cos(c+dx)+(2ad^4x+5b(24-12d^2x^2+d^4x^4))\sin(c+dx)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (-(d*(a*d^2*(-2 + d^2*x^2) + b*x*(120 - 20*d^2*x^2 + d^4*x^4))*Cos[c + d*x]
) + (2*a*d^4*x + 5*b*(24 - 12*d^2*x^2 + d^4*x^4))*Sin[c + d*x])/d^6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(126) = 252$.
time = 0.05, size = 392, normalized size = 3.11 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(-a*c^2*cos(d*x+c)-2*a*c*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+a*(-(d*x+c)^
2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+1/d^3*b*c^5*cos(d*x+c)+5/d^
3*b*c^4*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-10/d^3*b*c^3*(-(d*x+c)^2*cos(d*x+c)
```

```
+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+10/d^3*b*c^2*(-(d*x+c)^3*cos(d*x+c)+3*(
d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))-5/d^3*b*c*(-(d*x+c)^
4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)-2
4*(d*x+c)*sin(d*x+c))+1/d^3*b*(-(d*x+c)^5*cos(d*x+c)+5*(d*x+c)^4*sin(d*x+c)
+20*(d*x+c)^3*cos(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(d*x+c)-120*(d*x+c)
*cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(126) = 252$.
time = 0.33, size = 326, normalized size = 2.59

$$-\frac{ac^2\cos(dx+c) - \frac{bc^5\cos(dx+c)}{d^3} - 2\left((dx+c)\cos(dx+c) - \sin(dx+c)\right)ac + \frac{5\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)bc^4}{d^3} + \left(\left((dx+c)^2-2\right)\cos(dx+c) - 2(dx+c)\sin(dx+c)\right)a - \frac{10\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)bc^3}{d^3} + \frac{10\left(\left((dx+c)^3-6\,dx-6\,c\right)\cos(dx+c)-3\left((dx+c)^2-2\right)\sin(dx+c)\right)bc^2}{d^3} - \frac{5\left(\left((dx+c)^4-12\,(dx+c)^2+24\right)\cos(dx+c)-4\left((dx+c)^3-6\,dx-6\,c\right)\sin(dx+c)\right)bc}{d^3} + \frac{\left(\left((dx+c)^5-20\,(dx+c)^3+120\,dx+120\,c\right)\cos(dx+c)-5\left((dx+c)^4-12\,(dx+c)^2+24\right)\sin(dx+c)\right)b}{d^3}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*c^2*cos(d*x + c) - b*c^5*cos(d*x + c)/d^3 - 2*((d*x + c)*cos(d*x + c) -
 sin(d*x + c))*a*c + 5*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^4/d^3 +
(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a - 10*(((d*x +
 c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^3/d^3 + 10*(((d*x +
 c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b*c^2
/d^3 - 5*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3
 - 6*d*x - 6*c)*sin(d*x + c))*b*c/d^3 + (((d*x + c)^5 - 20*(d*x + c)^3 + 12
0*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 + 24)*sin(d*x
 + c))*b/d^3)/d^3
```

Fricas [A]
time = 0.38, size = 87, normalized size = 0.69

$$-\frac{\left(bd^5x^5 + ad^5x^2 - 20\,bd^3x^3 - 2\,ad^3 + 120\,bdx\right)\cos(dx+c) - \left(5\,bd^4x^4 + 2\,ad^4x - 60\,bd^2x^2 + 120\,b\right)\sin(dx+c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^5*x^5 + a*d^5*x^2 - 20*b*d^3*x^3 - 2*a*d^3 + 120*b*d*x)*cos(d*x + c)
 - (5*b*d^4*x^4 + 2*a*d^4*x - 60*b*d^2*x^2 + 120*b)*sin(d*x + c))/d^6
```

Sympy [A]
time = 0.44, size = 151, normalized size = 1.20

$$\begin{cases} -\frac{ax^2\cos(c+dx)}{d} + \frac{2ax\sin(c+dx)}{d^2} + \frac{2a\cos(c+dx)}{d^3} - \frac{bx^5\cos(c+dx)}{d} + \frac{5bx^4\sin(c+dx)}{d^2} + \frac{20bx^3\cos(c+dx)}{d^3} - \frac{60bx^2\sin(c+dx)}{d^4} - \frac{120bx\cos(c+dx)}{d^5} + \frac{120b\sin(c+dx)}{d^6} & \text{for } d \neq 0 \\ \left(\frac{ax^3}{3} + \frac{bx^6}{6}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**2*cos(c + d*x)/d + 2*a*x*sin(c + d*x)/d**2 + 2*a*cos(c + d
*x)/d**3 - b*x**5*cos(c + d*x)/d + 5*b*x**4*sin(c + d*x)/d**2 + 20*b*x**3*c
os(c + d*x)/d**3 - 60*b*x**2*sin(c + d*x)/d**4 - 120*b*x*cos(c + d*x)/d**5
+ 120*b*sin(c + d*x)/d**6, Ne(d, 0)), ((a*x**3/3 + b*x**6/6)*sin(c), True))
```

Giac [A]
time = 4.73, size = 88, normalized size = 0.70

$$-\frac{(bd^5x^5 + ad^5x^2 - 20\,bd^3x^3 - 2\,ad^3 + 120\,bdx)\cos(dx + c)}{d^6} + \frac{(5\,bd^4x^4 + 2\,ad^4x - 60\,bd^2x^2 + 120\,b)\sin(dx + c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^5*x^5 + a*d^5*x^2 - 20*b*d^3*x^3 - 2*a*d^3 + 120*b*d*x)*cos(d*x + c)/
d^6 + (5*b*d^4*x^4 + 2*a*d^4*x - 60*b*d^2*x^2 + 120*b)*sin(d*x + c)/d^6
```

Mupad [B]
time = 4.95, size = 121, normalized size = 0.96

$$\frac{120\,b\,\sin(c+d\,x) + d^4\,(5\,b\,x^4\,\sin(c+d\,x) + 2\,a\,x\,\sin(c+d\,x)) - d^5\,(a\,x^2\,\cos(c+d\,x) + b\,x^5\,\cos(c+d\,x)) + d^3\,(2\,a\,\cos(c+d\,x) + 20\,b\,x^3\,\cos(c+d\,x)) - 60\,b\,d^2\,x^2\,\sin(c+d\,x) - 120\,b\,d\,x\,\cos(c+d\,x)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(c + d*x)*(a + b*x^3),x)
```

```
[Out] (120*b*sin(c + d*x) + d^4*(5*b*x^4*sin(c + d*x) + 2*a*x*sin(c + d*x)) - d^5
*(a*x^2*cos(c + d*x) + b*x^5*cos(c + d*x)) + d^3*(2*a*cos(c + d*x) + 20*b*x
^3*cos(c + d*x)) - 60*b*d^2*x^2*sin(c + d*x) - 120*b*d*x*cos(c + d*x))/d^6
```

3.81 $\int x(a+bx^3)\sin(c+dx)\,dx$

Optimal. Leaf size=95

$$-\frac{24b\cos(c+dx)}{d^5}-\frac{ax\cos(c+dx)}{d}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{bx^4\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}-\frac{24bx\sin(c+dx)}{d^4}+4$$

```
[Out] -24*b*cos(d*x+c)/d^5-a*x*cos(d*x+c)/d+12*b*x^2*cos(d*x+c)/d^3-b*x^4*cos(d*x
+c)/d+a*sin(d*x+c)/d^2-24*b*x*sin(d*x+c)/d^4+4*b*x^3*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3420, 3377, 2717, 2718}

$$\frac{a\sin(c+dx)}{d^2}-\frac{ax\cos(c+dx)}{d}-\frac{24b\cos(c+dx)}{d^5}-\frac{24bx\sin(c+dx)}{d^4}+\frac{12bx^2\cos(c+dx)}{d^3}+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{bx^4\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (-24*b*Cos[c + d*x])/d^5 - (a*x*Cos[c + d*x])/d + (12*b*x^2*Cos[c + d*x])/d
^3 - (b*x^4*Cos[c + d*x])/d + (a*Sin[c + d*x])/d^2 - (24*b*x*Sin[c + d*x])/
d^4 + (4*b*x^3*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x(a+bx^3)\sin(c+dx)\,dx = \int (ax\sin(c+dx)+bx^4\sin(c+dx))\,dx$$
$$= a\int x\sin(c+dx)\,dx + b\int x^4\sin(c+dx)\,dx$$
$$= -\frac{ax\cos(c+dx)}{d} - \frac{bx^4\cos(c+dx)}{d} + \frac{a\int\cos(c+dx)\,dx}{d} + \frac{(4b)\int x^3\cos(c+$$
$$= -\frac{ax\cos(c+dx)}{d} - \frac{bx^4\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} + \frac{4bx^3\sin(c+dx)}{d^2} - \frac{(1$$
$$= -\frac{ax\cos(c+dx)}{d} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{bx^4\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} +$$
$$= -\frac{ax\cos(c+dx)}{d} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{bx^4\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} -$$
$$= -\frac{24b\cos(c+dx)}{d^5} - \frac{ax\cos(c+dx)}{d} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{bx^4\cos(c+dx)}{d}$$

Mathematica [A]
time = 0.09, size = 66, normalized size = 0.69

$$\frac{-((ad^4x+b(24-12d^2x^2+d^4x^4))\cos(c+dx))+d(ad^2+4bx(-6+d^2x^2))\sin(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)*Sin[c + d*x],x]
[Out] (-((a*d^4*x + b*(24 - 12*d^2*x^2 + d^4*x^4))*Cos[c + d*x]) + d*(a*d^2 + 4*b
*x*(-6 + d^2*x^2))*Sin[c + d*x])/d^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(95) = 190.
time = 0.04, size = 258, normalized size = 2.72

method	result
risch	$-\frac{(b\,x^4 d^4+a\,d^4 x-12 d^2 x^2 b+24 b)\cos(dx+c)}{d^5}+\frac{(4 b\,d^2 x^3+d^2 a-24 b x)\sin(dx+c)}{d^4}$
norman	$\frac{\frac{ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b\,x^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{48b}{d^5}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}-\frac{ax}{d}+\frac{12b\,x^2}{d^3}-\frac{b\,x^4}{d}-\frac{48bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}-\frac{12b\,x^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{d^3}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
meijerg	$\frac{16b\sqrt{\pi}\,\sin(c)\left(-\frac{x\left(d^2\right)^{\frac{5}{2}}\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{10\sqrt{\pi}\,d^4}+\frac{\left(d^2\right)^{\frac{5}{2}}\left(\frac{5}{8}d^4x^4-\frac{15}{2}d^2x^2+15\right)\sin(dx)}{10\sqrt{\pi}\,d^5}\right)}{d^4\sqrt{d^2}}+\frac{16b\sqrt{\pi}\,\cos(c)\left(\frac{3}{2\sqrt{\pi}}\right.}{}$
derivativedivides	$\frac{ac\cos(dx+c)+a(\sin(dx+c)-(dx+c)\cos(dx+c))-\frac{b\,c^4\cos(dx+c)}{d^3}-\frac{4b\,c^3(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^3}+\frac{6b\,c^2\left(-(dx+c)^2\cos(c\right.}{}}{}$

default	$\frac{ac\cos(dx+c)+a(\sin(dx+c)-(dx+c)\cos(dx+c))-\frac{b\,c^4\cos(dx+c)}{d^3}-\frac{4b\,c^3(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^3}+\frac{6b\,c^2\left(-(dx+c)^2\cos(dx\cdots\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(a*c*cos(d*x+c)+a*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-1/d^3*b*c^4*cos(d*x
+c)-4/d^3*b*c^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+6/d^3*b*c^2*(-(d*x+c)^2*cos
(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-4/d^3*b*c*(-(d*x+c)^3*cos(d*x+c)
+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+1/d^3*b*(-(d*x+c
)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)
-24*(d*x+c)*sin(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(95) = 190$.
time = 0.31, size = 224, normalized size = 2.36

$$\frac{ac\cos(dx+c)-\frac{bc^4\cos(dx+c)}{d^3}-((dx+c)\cos(dx+c)-\sin(dx+c))a+\frac{4((dx+c)\cos(dx+c)-\sin(dx+c))bc^3}{d^3}-\frac{6\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)bc^2}{d^3}+\frac{4\left(\left((dx+c)^3-6dx-6c\right)\cos(dx+c)-3\left((dx+c)^2-2\right)\sin(dx+c)\right)bc}{d^3}-\frac{\left(\left((dx+c)^4-12(dx+c)^2+24\right)\cos(dx+c)-4\left((dx+c)^3-6dx-6c\right)\sin(dx+c)\right)b}{d^3}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c*cos(d*x + c) - b*c^4*cos(d*x + c)/d^3 - ((d*x + c)*cos(d*x + c) - sin(
d*x + c))*a + 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^3/d^3 - 6*(((d*
x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^2/d^3 + 4*(((d*x
 + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b*c
/d^3 - (((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 -
 6*d*x - 6*c)*sin(d*x + c))*b/d^3)/d^2
```

Fricas [A]
time = 0.40, size = 68, normalized size = 0.72

$$-\frac{(bd^4x^4 + ad^4x - 12\,bd^2x^2 + 24\,b)\cos(dx+c) - (4\,bd^3x^3 + ad^3 - 24\,bdx)\sin(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^4*x^4 + a*d^4*x - 12*b*d^2*x^2 + 24*b)*cos(d*x + c) - (4*b*d^3*x^3 +
 a*d^3 - 24*b*d*x)*sin(d*x + c))/d^5
```

Sympy [A]
time = 0.29, size = 116, normalized size = 1.22

$$\begin{cases} -\frac{ax\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} - \frac{bx^4\cos(c+dx)}{d} + \frac{4bx^3\sin(c+dx)}{d^2} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{24bx\sin(c+dx)}{d^4} - \frac{24b\cos(c+dx)}{d^5} & \text{for}\ d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^5}{5}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x*cos(c + d*x)/d + a*sin(c + d*x)/d**2 - b*x**4*cos(c + d*x)/
d + 4*b*x**3*sin(c + d*x)/d**2 + 12*b*x**2*cos(c + d*x)/d**3 - 24*b*x*sin(c
 + d*x)/d**4 - 24*b*cos(c + d*x)/d**5, Ne(d, 0)), ((a*x**2/2 + b*x**5/5)*si
n(c), True))
```

Giac [**A**]
time = 5.10, size = 69, normalized size = 0.73

$$-\frac{(bd^4x^4 + ad^4x - 12\,bd^2x^2 + 24\,b)\cos{(dx + c)}}{d^5} + \frac{(4\,bd^3x^3 + ad^3 - 24\,bdx)\sin{(dx + c)}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^4*x^4 + a*d^4*x - 12*b*d^2*x^2 + 24*b)*cos(d*x + c)/d^5 + (4*b*d^3*x^
3 + a*d^3 - 24*b*d*x)*sin(d*x + c)/d^5
```

Mupad [**B**]
time = 4.79, size = 92, normalized size = 0.97

$$-\frac{d^4\,\left(a\,x\,\cos\left(c+d\,x\right)+b\,x^4\,\cos\left(c+d\,x\right)\right)+24\,b\,\cos\left(c+d\,x\right)-d^3\,\left(a\,\sin\left(c+d\,x\right)+4\,b\,x^3\,\sin\left(c+d\,x\right)\right)+24\,b\,d\,x\,\sin\left(c+d\,x\right)-12\,b\,d^2\,x^2\,\cos\left(c+d\,x\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(c + d*x)*(a + b*x^3),x)
```

```
[Out] -(d^4*(a*x*cos(c + d*x) + b*x^4*cos(c + d*x)) + 24*b*cos(c + d*x) - d^3*(a*
sin(c + d*x) + 4*b*x^3*sin(c + d*x)) + 24*b*d*x*sin(c + d*x) - 12*b*d^2*x^2
*cos(c + d*x))/d^5
```

3.82 $\int (a+bx^3)\sin(c+dx)\,dx$

Optimal. Leaf size=68

$$-\frac{a\cos(c+dx)}{d}+\frac{6bx\cos(c+dx)}{d^3}-\frac{bx^3\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{3bx^2\sin(c+dx)}{d^2}$$

```
[Out] -a*cos(d*x+c)/d+6*b*x*cos(d*x+c)/d^3-b*x^3*cos(d*x+c)/d-6*b*sin(d*x+c)/d^4+
3*b*x^2*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3410, 2718, 3377, 2717}

$$-\frac{a\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+\frac{6bx\cos(c+dx)}{d^3}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{bx^3\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (6*b*x*Cos[c + d*x])/d^3 - (b*x^3*Cos[c + d*x])/d -
 (6*b*Sin[c + d*x])/d^4 + (3*b*x^2*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3410

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+bx^3)\sin(c+dx)\,dx &= \int \left(a\sin(c+dx)+bx^3\sin(c+dx)\right)\,dx \\
&= a\int \sin(c+dx)\,dx+b\int x^3\sin(c+dx)\,dx \\
&= -\frac{a\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}+\frac{(3b)\int x^2\cos(c+dx)\,dx}{d} \\
&= -\frac{a\cos(c+dx)}{d}-\frac{bx^3\cos(c+dx)}{d}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{(6b)\int x\sin(c+dx)\,c}{d^2} \\
&= -\frac{a\cos(c+dx)}{d}+\frac{6bx\cos(c+dx)}{d^3}-\frac{bx^3\cos(c+dx)}{d}+\frac{3bx^2\sin(c+dx)}{d^2}-(6 \\
&= -\frac{a\cos(c+dx)}{d}+\frac{6bx\cos(c+dx)}{d^3}-\frac{bx^3\cos(c+dx)}{d}-\frac{6b\sin(c+dx)}{d^4}+3bx^2
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 50, normalized size = 0.74

$$\frac{-d(ad^2+bx(-6+d^2x^2))\cos(c+dx)+3b(-2+d^2x^2)\sin(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (-(d*(a*d^2 + b*x*(-6 + d^2*x^2))*Cos[c + d*x]) + 3*b*(-2 + d^2*x^2)*Sin[c
 + d*x])/d^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(68) = 136.
time = 0.04, size = 159, normalized size = 2.34

method	result
risch	$-\frac{(b\,d^2x^3+d^2a-6bx)\cos(dx+c)}{d^3}+\frac{3b(d^2x^2-2)\sin(dx+c)}{d^4}$
norman	$\frac{\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b\,x^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{12b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^4}+\frac{6bx}{d^3}-\frac{b\,x^3}{d}-\frac{6bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d^3}+\frac{6b\,x^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d^2}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$
meijerg	$\frac{8b\sqrt{\pi}\,\sin(c)\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3d^2x^2}{2}+3\right)\cos(dx)}{4\sqrt{\pi}}-\frac{dx\left(-\frac{d^2x^2}{2}+3\right)\sin(dx)}{4\sqrt{\pi}}\right)}{d^4}+\frac{8b\sqrt{\pi}\,\cos(c)\left(\frac{xd\left(-\frac{5d^2x^2}{2}+15\right)\cos(dx)}{20\sqrt{\pi}}-\right.}{d^4}$
derivativedivides	$\frac{-a\cos(dx+c)+\frac{b\,c^3\cos(dx+c)}{d^3}+\frac{3b\,c^2(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^3}-\frac{3bc\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{d^3}}{d}$
default	$\frac{-a\cos(dx+c)+\frac{b\,c^3\cos(dx+c)}{d^3}+\frac{3b\,c^2(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^3}-\frac{3bc\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{d^3}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a*cos(d*x+c)+1/d^3*b*c^3*cos(d*x+c)+3/d^3*b*c^2*(sin(d*x+c)-(d*x+c)*c
os(d*x+c))-3/d^3*b*c*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+
c))+1/d^3*b*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d
*x+c)*cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(68) = 136$.
time = 0.30, size = 141, normalized size = 2.07

$$-\frac{a\cos\left(dx+c\right)-\frac{bc^{3}\cos\left(dx+c\right)}{d^{3}}+\frac{3\left(\left(dx+c\right)\cos\left(dx+c\right)-\sin\left(dx+c\right)\right)bc^{2}}{d^{3}}-\frac{3\left(\left(\left(dx+c\right)^{2}-2\right)\cos\left(dx+c\right)-2\left(dx+c\right)\sin\left(dx+c\right)\right)bc}{d^{3}}+\frac{\left(\left(\left(dx+c\right)^{3}-6\,dx-6\,c\right)\cos\left(dx+c\right)-3\left(\left(dx+c\right)^{2}-2\right)\sin\left(dx+c\right)\right)b}{d^{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*cos(d*x + c) - b*c^3*cos(d*x + c)/d^3 + 3*((d*x + c)*cos(d*x + c) - sin
(d*x + c))*b*c^2/d^3 - 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(
d*x + c))*b*c/d^3 + (((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c
)^2 - 2)*sin(d*x + c))*b/d^3)/d
```

Fricas [A]
time = 0.36, size = 52, normalized size = 0.76

$$-\frac{\left(bd^{3}x^{3}+ad^{3}-6\,bdx\right)\cos\left(dx+c\right)-3\left(bd^{2}x^{2}-2\,b\right)\sin\left(dx+c\right)}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^3*x^3 + a*d^3 - 6*b*d*x)*cos(d*x + c) - 3*(b*d^2*x^2 - 2*b)*sin(d*x
+ c))/d^4
```

Sympy [A]
time = 0.18, size = 82, normalized size = 1.21

$$\begin{cases} -\frac{a\cos\left(c+dx\right)}{d}-\frac{bx^{3}\cos\left(c+dx\right)}{d}+\frac{3bx^{2}\sin\left(c+dx\right)}{d^{2}}+\frac{6bx\cos\left(c+dx\right)}{d^{3}}-\frac{6b\sin\left(c+dx\right)}{d^{4}} & \text{for}\ d\neq 0\\ \left(ax+\frac{bx^{4}}{4}\right)\sin\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)/d - b*x**3*cos(c + d*x)/d + 3*b*x**2*sin(c + d*x
)/d**2 + 6*b*x*cos(c + d*x)/d**3 - 6*b*sin(c + d*x)/d**4, Ne(d, 0)), ((a*x
+ b*x**4/4)*sin(c), True))
```

Giac [A]
time = 5.12, size = 54, normalized size = 0.79

$$-\frac{(bd^3x^3 + ad^3 - 6\,bdx)\cos{(dx + c)}}{d^4} + \frac{3\,(bd^2x^2 - 2\,b)\sin{(dx + c)}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^3*x^3 + a*d^3 - 6*b*d*x)*cos(d*x + c)/d^4 + 3*(b*d^2*x^2 - 2*b)*sin(d
*x + c)/d^4
```

Mupad [B]
time = 0.11, size = 65, normalized size = 0.96

$$-\frac{6\,b\,\sin{(c+d\,x)} + d^3\,(a\,\cos{(c+d\,x)} + b\,x^3\,\cos{(c+d\,x)}) - 3\,b\,d^2\,x^2\,\sin{(c+d\,x)} - 6\,b\,d\,x\,\cos{(c+d\,x)}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*x^3),x)
```

```
[Out] -(6*b*sin(c + d*x) + d^3*(a*cos(c + d*x) + b*x^3*cos(c + d*x)) - 3*b*d^2*x^
2*sin(c + d*x) - 6*b*d*x*cos(c + d*x))/d^4
```

3.83 $\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx$

Optimal. Leaf size=57

$$\frac{2b\cos(c+dx)}{d^3}-\frac{bx^2\cos(c+dx)}{d}+a\text{Ci}(dx)\sin(c)+\frac{2bx\sin(c+dx)}{d^2}+a\cos(c)\text{Si}(dx)$$

```
[Out] 2*b*cos(d*x+c)/d^3-b*x^2*cos(d*x+c)/d+a*cos(c)*Si(d*x)+a*Ci(d*x)*sin(c)+2*b
*x*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3420, 3384, 3380, 3383, 3377, 2718}

$$a\sin(c)\text{CosIntegral}(dx)+a\cos(c)\text{Si}(dx)+\frac{2b\cos(c+dx)}{d^3}+\frac{2bx\sin(c+dx)}{d^2}-\frac{bx^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Sin[c + d*x])/x,x]
```

```
[Out] (2*b*Cos[c + d*x])/d^3 - (b*x^2*Cos[c + d*x])/d + a*CosIntegral[d*x]*Sin[c]
 + (2*b*x*Sin[c + d*x])/d^2 + a*Cos[c]*SinIntegral[d*x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx &= \int \left(\frac{a\sin(c+dx)}{x}+bx^2\sin(c+dx)\right)dx\\ &= a\int \frac{\sin(c+dx)}{x}\,dx+b\int x^2\sin(c+dx)\,dx\\ &= -\frac{bx^2\cos(c+dx)}{d}+\frac{(2b)\int x\cos(c+dx)\,dx}{d}+(a\cos(c))\int \frac{\sin(dx)}{x}\,dx+(a\,\mathrm{si}\\ &= -\frac{bx^2\cos(c+dx)}{d}+a\mathrm{Ci}(dx)\sin(c)+\frac{2bx\sin(c+dx)}{d^2}+a\cos(c)\mathrm{Si}(dx)-\frac{(2b)}{}\\ &= \frac{2b\cos(c+dx)}{d^3}-\frac{bx^2\cos(c+dx)}{d}+a\mathrm{Ci}(dx)\sin(c)+\frac{2bx\sin(c+dx)}{d^2}+a\cos(\end{aligned}$$

Mathematica [A]
time = 0.14, size = 50, normalized size = 0.88

$$a\mathrm{Ci}(dx)\sin(c)+\frac{b((2-d^2x^2)\cos(c+dx)+2dx\sin(c+dx))}{d^3}+a\cos(c)\mathrm{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Sin[c + d*x])/x,x]
```

```
[Out] a*CosIntegral[d*x]*Sin[c] + (b*((2 - d^2*x^2)*Cos[c + d*x] + 2*d*x*Sin[c +
d*x]))/d^3 + a*Cos[c]*SinIntegral[d*x]
```

Maple [A]
time = 0.06, size = 112, normalized size = 1.96

method	result
derivativedivides	$a(\mathrm{sinIntegral}\,(dx)\cos\,(c)+\mathrm{cosineIntegral}\,(dx)\sin\,(c))-\frac{3b\,c^2\cos(dx+c)}{d^3}-\frac{3bc(c+1)(\sin(dx+c)}{d}$

default	$a(\mathrm{sinIntegral}\,(dx)\cos{(c)}+\mathrm{cosineIntegral}\,(dx)\sin{(c)})-\frac{3b\,c^{2}\cos(dx+c)}{d^{3}}-\frac{3bc(c+1)(\sin(dx+c)-}{d^{3}}$
meijerg	$\frac{4b\sqrt{\pi}\ \sin(c)\left(\frac{x\left(d^{2}\right)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\ d^{2}}-\frac{\left(d^{2}\right)^{\frac{3}{2}}\left(-\frac{3d^{2}x^{2}}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\ d^{3}}\right)}{d^{2}\sqrt{d^{2}}}+\frac{4b\sqrt{\pi}\ \cos(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{d^{2}x^{2}}{2}+1\right)\cos(dx)}{2\sqrt{\pi}}+\frac{dx\,\mathrm{s}}{2\sqrt{}}\right.}{d^{3}}$
risch	$-\frac{\mathrm{e}^{-ic}\pi\,\mathrm{csgn}(dx)a}{2}+\mathrm{e}^{-ic}\,\mathrm{sinIntegral}\,(dx)\,a-\frac{i\,\mathrm{expIntegral}(1,-idx)\mathrm{e}^{-ic}a}{2}+\frac{ia\,\mathrm{e}^{ic}\,\mathrm{expIntegral}(1,-idx)}{2}+\frac{b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-3/d^3*b*c^2*cos(d*x+c)-3*b*c*(c+1)/d^3*(s
in(d*x+c)-(d*x+c)*cos(d*x+c))+(c^2+c+1)/d^3*b*(-(d*x+c)^2*cos(d*x+c)+2*cos(
d*x+c)+2*(d*x+c)*sin(d*x+c))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.75, size = 76, normalized size = 1.33

$$\frac{(a(-i\,\mathrm{Ei}(i\,dx)+i\,\mathrm{Ei}(-i\,dx))\cos{(c)}+a(\mathrm{Ei}(i\,dx)+\mathrm{Ei}(-i\,dx))\sin{(c)})d^{3}+4\,bdx\sin{(dx+c)}-2\,(bd^{2}x^{2}-2\,b)\cos{(dx+c)}}{2\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] 1/2*((a*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a*(Ei(I*d*x) + Ei(-I*d*x))*s
in(c))*d^3 + 4*b*d*x*sin(d*x + c) - 2*(b*d^2*x^2 - 2*b)*cos(d*x + c))/d^3
```

Fricas [A]
time = 0.43, size = 72, normalized size = 1.26

$$\frac{2\,ad^{3}\cos{(c)}\,\mathrm{Si}\,(dx)+4\,bdx\sin{(dx+c)}-2\,(bd^{2}x^{2}-2\,b)\cos{(dx+c)}+(ad^{3}\,\mathrm{Ci}\,(dx)+ad^{3}\,\mathrm{Ci}\,(-dx))\sin{(c)}}{2\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d^3*cos(c)*sin_integral(d*x) + 4*b*d*x*sin(d*x + c) - 2*(b*d^2*x^2
 - 2*b)*cos(d*x + c) + (a*d^3*cos_integral(d*x) + a*d^3*cos_integral(-d*x))
*sin(c))/d^3
```

Sympy [A]
time = 2.90, size = 83, normalized size = 1.46

$$a\sin{(c)}\,\mathrm{Ci}\,(dx)+a\cos{(c)}\,\mathrm{Si}\,(dx)+bx^{2}\left(\begin{cases}x\sin{(c)} & \text{for}\ d=0\\ -\frac{\cos{(c+dx)}}{d} & \text{otherwise}\end{cases}\right)-2b\left(\begin{cases}\frac{x^{3}\sin{(c)}}{3} & \text{for}\ d=0\\ -\frac{\begin{cases}\frac{x\sin{(c+dx)}}{d}+\frac{\cos{(c+dx)}}{d^{2}} & \text{for}\ d\neq 0\\ \frac{x^{2}\cos{(c)}}{2} & \text{otherwise}\end{cases}}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*sin(d*x+c)/x,x)
```

```
[Out] a*sin(c)*Ci(d*x) + a*cos(c)*Si(d*x) + b*x**2*Piecewise((x*sin(c), Eq(d, 0))
, (-cos(c + d*x)/d, True)) - 2*b*Piecewise((x**3*sin(c)/3, Eq(d, 0)), (-Pie
cewise((x*sin(c + d*x)/d + cos(c + d*x)/d**2, Ne(d, 0)), (x**2*cos(c)/2, Tr
ue))/d, True))
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.64, size = 510, normalized size = 8.95

$$-\frac{2 b d^2 x^2 \tan\left(\frac{1}{2} d x\right)^2 \tan\left(\frac{1}{2} c\right)^2 + a d^3 \Im(\mathrm{Ci}(d x)) \tan\left(\frac{1}{2} d x\right)^2 \tan\left(\frac{1}{2} c\right)^2 - a d^3 \Im(\mathrm{Ci}(-d x)) \tan\left(\frac{1}{2} d x\right)^2 \tan\left(\frac{1}{2} c\right)^2 + 2 a d^3 \mathrm{Si}(d x) \tan\left(\frac{1}{2} d x\right)^2 \tan\left(\frac{1}{2} c\right)^2 - 2 a d^3 \Re(\mathrm{Ci}(d x)) \tan\left(\frac{1}{2} d x\right)^2 \tan\left(\frac{1}{2} c\right) - 2 a d^3 \Re(\mathrm{Ci}(-d x)) \tan\left(\frac{1}{2} d x\right)^2 \tan\left(\frac{1}{2} c\right) - 2 b d^2 x^2 \tan\left(\frac{1}{2} d x\right)^2 - a d^3 \Im(\mathrm{Ci}(d x)) \tan\left(\frac{1}{2} d x\right)^2 + a d^3 \Im(\mathrm{Ci}(-d x)) \tan\left(\frac{1}{2} d x\right)^2 - 2 a d^3 \mathrm{Si}(d x) \tan\left(\frac{1}{2} d x\right)^2 - 8 b d^2 x^2 \tan\left(\frac{1}{2} d x\right) \tan\left(\frac{1}{2} c\right) - 2 b d^2 x^2 \tan\left(\frac{1}{2} c\right)^2 + a d^3 \Im(\mathrm{Ci}(d x)) \tan\left(\frac{1}{2} c\right)^2 - a d^3 \Im(\mathrm{Ci}(-d x)) \tan\left(\frac{1}{2} c\right)^2 + 2 a d^3 \mathrm{Si}(d x) \tan\left(\frac{1}{2} c\right)^2 - 2 a d^3 \Re(\mathrm{Ci}(d x)) \tan\left(\frac{1}{2} c\right) - 2 a d^3 \Re(\mathrm{Ci}(-d x)) \tan\left(\frac{1}{2} c\right) + 8 b d x \tan\left(\frac{1}{2} d x\right)^2 \tan\left(\frac{1}{2} c\right) + 8 b d x \tan\left(\frac{1}{2} d x\right) \tan\left(\frac{1}{2} c\right)^2 + 2 b d^2 x^2 - a d^3 \Im(\mathrm{Ci}(d x)) + a d^3 \Im(\mathrm{Ci}(-d x)) - 2 a d^3 \mathrm{Si}(d x) - 4 b \tan\left(\frac{1}{2} d x\right)^2 \tan\left(\frac{1}{2} c\right)^2 - 8 b d x \tan\left(\frac{1}{2} d x\right) - 8 b d x \tan\left(\frac{1}{2} c\right) + 4 b \tan\left(\frac{1}{2} d x\right)^2 + 16 b \tan\left(\frac{1}{2} d x\right) \tan\left(\frac{1}{2} c\right) + 4 b \tan\left(\frac{1}{2} c\right)^2 - 4 b}{2 \left(d^3 \tan\left(\frac{1}{2} d x\right)^2 \tan\left(\frac{1}{2} c\right)^2 + d^3 \tan\left(\frac{1}{2} d x\right)^2 + d^3 \tan\left(\frac{1}{2} c\right)^2 + d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*d^3*imag_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^3*imag_part(cos_integral(-d*x))*t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*a*d^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) -
2*a*d^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b*d^2*x
^2*tan(1/2*d*x)^2 - a*d^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + a*d
^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d^3*sin_integral(d*x)
*tan(1/2*d*x)^2 - 8*b*d^2*x^2*tan(1/2*d*x)*tan(1/2*c) - 2*b*d^2*x^2*tan(1/2
*c)^2 + a*d^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a*d^3*imag_part(c
os_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^3*sin_integral(d*x)*tan(1/2*c)^2 -
2*a*d^3*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^3*real_part(cos_int
egral(-d*x))*tan(1/2*c) + 8*b*d*x*tan(1/2*d*x)^2*tan(1/2*c) + 8*b*d*x*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*b*d^2*x^2 - a*d^3*imag_part(cos_integral(d*x)) + a
*d^3*imag_part(cos_integral(-d*x)) - 2*a*d^3*sin_integral(d*x) - 4*b*tan(1/
2*d*x)^2*tan(1/2*c)^2 - 8*b*d*x*tan(1/2*d*x) - 8*b*d*x*tan(1/2*c) + 4*b*tan
(1/2*d*x)^2 + 16*b*tan(1/2*d*x)*tan(1/2*c) + 4*b*tan(1/2*c)^2 - 4*b)/(d^3*t
an(1/2*d*x)^2*tan(1/2*c)^2 + d^3*tan(1/2*d*x)^2 + d^3*tan(1/2*c)^2 + d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$a\,\mathrm{cosint}(d\,x)\,\sin\left(c\right)+a\,\mathrm{sinint}(d\,x)\,\cos\left(c\right)+\frac{b\,\left(2\,\cos\left(c+d\,x\right)-d^2\,x^2\,\cos\left(c+d\,x\right)+2\,d\,x\,\sin\left(c+d\,x\right)\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^3))/x,x)
```

```
[Out] a*cosint(d*x)*sin(c) + a*sinint(d*x)*cos(c) + (b*(2*cos(c + d*x) - d^2*x^2*
cos(c + d*x) + 2*d*x*sin(c + d*x)))/d^3
```

3.84 $\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx$

Optimal. Leaf size=56

$$-\frac{bx\cos(c+dx)}{d}+ad\cos(c)\text{Ci}(dx)+\frac{b\sin(c+dx)}{d^2}-\frac{a\sin(c+dx)}{x}-ad\sin(c)\text{Si}(dx)$$

```
[Out] a*d*Ci(d*x)*cos(c)-b*x*cos(d*x+c)/d-a*d*Si(d*x)*sin(c)+b*sin(d*x+c)/d^2-a*s
in(d*x+c)/x
```

Rubi [A]
time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3420, 3378, 3384, 3380, 3383, 3377, 2717}

$$ad\cos(c)\text{CosIntegral}(dx)-ad\sin(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{x}+\frac{b\sin(c+dx)}{d^2}-\frac{bx\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b*x*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] + (b*Sin[c + d*x])/d^
2 - (a*Sin[c + d*x])/x - a*d*Sin[c]*SinIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^2}+bx\sin(c+dx)\right)\,dx\\
&= a\int \frac{\sin(c+dx)}{x^2}\,dx + b\int x\sin(c+dx)\,dx\\
&= -\frac{bx\cos(c+dx)}{d}-\frac{a\sin(c+dx)}{x}+\frac{b\int\cos(c+dx)\,dx}{d}+(ad)\int\frac{\cos(c+dx)}{x}\\
&= -\frac{bx\cos(c+dx)}{d}+\frac{b\sin(c+dx)}{d^2}-\frac{a\sin(c+dx)}{x}+(ad\cos(c))\int\frac{\cos(dx)}{x}\,dx\\
&= -\frac{bx\cos(c+dx)}{d}+ad\cos(c)\text{Ci}(dx)+\frac{b\sin(c+dx)}{d^2}-\frac{a\sin(c+dx)}{x}-ad\sin(
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 56, normalized size = 1.00

$$-\frac{bx\cos(c+dx)}{d}+ad\cos(c)\text{Ci}(dx)+\frac{b\sin(c+dx)}{d^2}-\frac{a\sin(c+dx)}{x}-ad\sin(c)\text{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b*x*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] + (b*Sin[c + d*x])/d^
2 - (a*Sin[c + d*x])/x - a*d*Sin[c]*SinIntegral[d*x]
```

Maple [A]
time = 0.08, size = 79, normalized size = 1.41

method	result
derivativedivides	$d\left(a\left(-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}\left(dx\right)\sin\left(c\right)+\text{cosineIntegral}\left(dx\right)\cos\left(c\right)\right)+\frac{3bc\cos(dx+c)}{d^3}+\frac{(2$
default	$d\left(a\left(-\frac{\sin(dx+c)}{dx}-\text{sinIntegral}\left(dx\right)\sin\left(c\right)+\text{cosineIntegral}\left(dx\right)\cos\left(c\right)\right)+\frac{3bc\cos(dx+c)}{d^3}+\frac{(2$
risch	$-\frac{d\cos(c)a\,\text{expIntegral}(1,-idx)}{2}-\frac{d\cos(c)a\,\text{expIntegral}(1,idx)}{2}-\frac{id\sin(c)a\,\text{expIntegral}(1,-idx)}{2}+\frac{id\sin(c)a\,\text{expInte}}{2}$
meijerg	$\frac{2b\sqrt{\pi}\,\sin(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b\sqrt{\pi}\,\cos(c)\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{a\sqrt{\pi}\,\sin(c)d^2\left(-\frac{4d^2}{x}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(a*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+3/d^3*b*c*cos(d*x+c)+(
2*c+1)/d^3*b*(sin(d*x+c)-(d*x+c)*cos(d*x+c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 0.69, size = 69, normalized size = 1.23

$$\frac{(a(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos(c)+a(-i\,\Gamma(-1,i\,dx)+i\,\Gamma(-1,-i\,dx))\sin(c))d^3-2\,bdx\cos(dx+c)+2\,b\sin(dx+c)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) + a*(-I*gamma(-1, I*d
*x) + I*gamma(-1, -I*d*x))*sin(c))*d^3 - 2*b*d*x*cos(d*x + c) + 2*b*sin(d*x
 + c))/d^2
```

Fricas [A]
time = 0.37, size = 79, normalized size = 1.41

$$-\frac{2\,ad^3x\sin(c)\,\mathrm{Si}\,(dx)+2\,bdx^2\cos(dx+c)-(ad^3x\,\mathrm{Ci}\,(dx)+ad^3x\,\mathrm{Ci}\,(-dx))\cos(c)+2\,(ad^2-bx)\sin(dx+c)}{2\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d^3*x*sin(c)*sin_integral(d*x) + 2*b*d*x^2*cos(d*x + c) - (a*d^3*
x*cos_integral(d*x) + a*d^3*x*cos_integral(-d*x))*cos(c) + 2*(a*d^2 - b*x)*
sin(d*x + c))/(d^2*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**3)*sin(c + d*x)/x**2, x)
```

Giac [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.68, size = 489, normalized size = 8.73

$$-\frac{ad^3x\Re(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2 + ad^3x\Re(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2 + 2ad^3x\Im(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right) - 2ad^3x\Im(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right) + 4ad^3x\,\mathrm{Si}(dx)\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right) - ad^3x\Re(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}dx\right)^2 - ad^3x\Re(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}dx\right)^2 + ad^3x\Re(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}c\right)^2 + ad^3x\Re(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}c\right)^2 + 2bdx^2\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2 + 2ad^3x\Im(\mathrm{Ci}(dx))\tan\left(\frac{1}{2}c\right) - 2ad^3x\Im(\mathrm{Ci}(-dx))\tan\left(\frac{1}{2}c\right) + 4ad^3x\,\mathrm{Si}(dx)\tan\left(\frac{1}{2}c\right) - ad^3x\Re(\mathrm{Ci}(dx)) - ad^3x\Re(\mathrm{Ci}(-dx)) - 2bdx^2\tan\left(\frac{1}{2}dx\right)^2 - 8bdx^2\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right) - 4ad^2\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right) - 2bdx^2\tan\left(\frac{1}{2}c\right)^2 - 4ad^2\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)^2 + 4bx\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right) + 4bx\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)^2 + 2bdx^2 + 4ad^2\tan\left(\frac{1}{2}dx\right) + 4ad^2\tan\left(\frac{1}{2}c\right) - 4bx\tan\left(\frac{1}{2}dx\right) - 4bx\tan\left(\frac{1}{2}c\right)}{2\left(d^2x\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2 + d^2x\tan\left(\frac{1}{2}dx\right)^2 + d^2x\tan\left(\frac{1}{2}c\right)^2 + d^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d^3*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*
d^3*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^3*x
*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x*imag_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x*sin_integral(d
*x)*tan(1/2*d*x)^2*tan(1/2*c) - a*d^3*x*real_part(cos_integral(d*x))*tan(1/
2*d*x)^2 - a*d^3*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + a*d^3*x*r
eal_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d^3*x*real_part(cos_integral(-
d*x))*tan(1/2*c)^2 + 2*b*d*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^3*x*imag
_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^3*x*imag_part(cos_integral(-d*x
))*tan(1/2*c) + 4*a*d^3*x*sin_integral(d*x)*tan(1/2*c) - a*d^3*x*real_part(
cos_integral(d*x)) - a*d^3*x*real_part(cos_integral(-d*x)) - 2*b*d*x^2*tan(
1/2*d*x)^2 - 8*b*d*x^2*tan(1/2*d*x)*tan(1/2*c) - 4*a*d^2*tan(1/2*d*x)^2*tan
(1/2*c) - 2*b*d*x^2*tan(1/2*c)^2 - 4*a*d^2*tan(1/2*d*x)*tan(1/2*c)^2 + 4*b*
x*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*x*tan(1/2*d*x)*tan(1/2*c)^2 + 2*b*d*x^2 +
 4*a*d^2*tan(1/2*d*x) + 4*a*d^2*tan(1/2*c) - 4*b*x*tan(1/2*d*x) - 4*b*x*tan
(1/2*c))/(d^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^2*x*tan(1/2*d*x)^2 + d^2*x*
tan(1/2*c)^2 + d^2*x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(c+d\,x)\,\left(b\,x^3+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^3))/x^2,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^3))/x^2, x)
```

3.85 $\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx$

Optimal. Leaf size=70

$$-\frac{b\cos(c+dx)}{d}-\frac{ad\cos(c+dx)}{2x}-\frac{1}{2}ad^2\text{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{2x^2}-\frac{1}{2}ad^2\cos(c)\text{Si}(dx)$$

```
[Out] -b*cos(d*x+c)/d-1/2*a*d*cos(d*x+c)/x-1/2*a*d^2*cos(c)*Si(d*x)-1/2*a*d^2*Ci(
d*x)*sin(c)-1/2*a*sin(d*x+c)/x^2
```

Rubi [A]
time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3420, 2718, 3378, 3384, 3380, 3383}

$$-\frac{1}{2}ad^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}ad^2\cos(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{2x^2}-\frac{ad\cos(c+dx)}{2x}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Sin[c + d*x])/x^3,x]
```

```
[Out] -((b*Cos[c + d*x])/d) - (a*d*Cos[c + d*x])/(2*x) - (a*d^2*CosIntegral[d*x]*
Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) - (a*d^2*Cos[c]*SinIntegral[d*x])/2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx &= \int \left(b\sin(c+dx)+\frac{a\sin(c+dx)}{x^3}\right)\,dx \\
&= a\int \frac{\sin(c+dx)}{x^3}\,dx + b\int \sin(c+dx)\,dx \\
&= -\frac{b\cos(c+dx)}{d} - \frac{a\sin(c+dx)}{2x^2} + \frac{1}{2}(ad)\int \frac{\cos(c+dx)}{x^2}\,dx \\
&= -\frac{b\cos(c+dx)}{d} - \frac{ad\cos(c+dx)}{2x} - \frac{a\sin(c+dx)}{2x^2} - \frac{1}{2}\left(ad^2\right)\int \frac{\sin(c+dx)}{x}\,dx \\
&= -\frac{b\cos(c+dx)}{d} - \frac{ad\cos(c+dx)}{2x} - \frac{a\sin(c+dx)}{2x^2} - \frac{1}{2}\left(ad^2\cos(c)\right)\int \frac{\sin(dx)}{x} \\
&= -\frac{b\cos(c+dx)}{d} - \frac{ad\cos(c+dx)}{2x} - \frac{1}{2}ad^2\mathrm{Ci}(dx)\sin(c) - \frac{a\sin(c+dx)}{2x^2} - \frac{1}{2}ad
\end{aligned}$$

Mathematica [**A**]
time = 0.11, size = 66, normalized size = 0.94

$$\frac{1}{2}\left(-\frac{2b\cos(c+dx)}{d} - \frac{ad\cos(c+dx)}{x} - ad^2\mathrm{Ci}(dx)\sin(c) - \frac{a\sin(c+dx)}{x^2} - ad^2\cos(c)\mathrm{Si}(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Sin[c + d*x])/x^3,x]
```

```
[Out] ((-2*b*Cos[c + d*x])/d - (a*d*Cos[c + d*x])/x - a*d^2*CosIntegral[d*x]*Sin[
c] - (a*Sin[c + d*x])/x^2 - a*d^2*Cos[c]*SinIntegral[d*x])/2
```

Maple [**A**]
time = 0.09, size = 65, normalized size = 0.93

method	result
derivativedivides	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\text{sinIntegral}(dx)\cos(c)}{2}-\frac{\text{cosineIntegral}(dx)\sin(c)}{2}\right)-\frac{b\cos(dx+c)}{d^3}\right)$
default	$d^2\left(a\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\text{sinIntegral}(dx)\cos(c)}{2}-\frac{\text{cosineIntegral}(dx)\sin(c)}{2}\right)-\frac{b\cos(dx+c)}{d^3}\right)$
risch	$\frac{i\cos(c)\,\text{expIntegral}(1,idx)a\,d^2}{4}-\frac{i\cos(c)\,\text{expIntegral}(1,-idx)a\,d^2}{4}+\frac{\sin(c)\,\text{expIntegral}(1,idx)a\,d^2}{4}+\frac{\sin(c)\,\text{expIntegr}}{4}$
meijerg	$\frac{b\sin(c)\sin(dx)}{d}+\frac{b\sqrt{\pi}\,\cos(c)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}+\frac{a\sqrt{\pi}\,\sin(c)d^2\left(-\frac{4}{\sqrt{\pi}\,x^2d^2}-\frac{2\left(2\gamma-3+2\ln(x)+\ln\left(d^2\right)\right)}{\sqrt{\pi}}+\frac{-6d}{\sqrt{\pi}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(a*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*C
i(d*x)*sin(c))-b*cos(d*x+c)/d^3)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.54, size = 1146, normalized size = 16.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(((I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c)^3 +
 (I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c)*sin(c)^2
 + (exp_integral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*sin(c)^3 + (I*exp
_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c) + ((exp_integra
l_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*cos(c)^2 + exp_integral_e(3, I*d
*x) + exp_integral_e(3, -I*d*x))*sin(c))*b*c^3/((d*x + c)^2*(cos(c)^2 + sin
(c)^2)*d^3 - 2*(c*cos(c)^2 + c*sin(c)^2)*(d*x + c)*d^3 + (c^2*cos(c)^2 + c^
2*sin(c)^2)*d^3) - ((I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*
x))*cos(c)^3 + (I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*c
os(c)*sin(c)^2 + (exp_integral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*sin
(c)^3 + (I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c) +
 ((exp_integral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*cos(c)^2 + exp_int
egral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*sin(c))*a/(c^2*cos(c)^2 + c^
2*sin(c)^2 + (d*x + c)^2*(cos(c)^2 + sin(c)^2) - 2*(c*cos(c)^2 + c*sin(c)^2
)*(d*x + c)) - (2*((b*cos(c)^2 + b*sin(c)^2)*(d*x + c)^3 - 3*(b*c*cos(c)^2
+ b*c*sin(c)^2)*(d*x + c)^2 + 3*(b*c^2*cos(c)^2 + b*c^2*sin(c)^2)*(d*x + c)
)*cos(d*x + c)^3 - 3*(b*c^3*(exp_integral_e(4, I*d*x) + exp_integral_e(4, -
I*d*x))*cos(c)^3 + b*c^3*(exp_integral_e(4, I*d*x) + exp_integral_e(4, -I*d
*x))*cos(c)*sin(c)^2 + b*c^3*(-I*exp_integral_e(4, I*d*x) + I*exp_integral_
e(4, -I*d*x))*sin(c)^3 + b*c^3*(exp_integral_e(4, I*d*x) + exp_integral_e(4
```

```
, -I*d*x))*cos(c) + (b*c^3*(-I*exp_integral_e(4, I*d*x) + I*exp_integral_e(
4, -I*d*x))*cos(c)^2 + b*c^3*(-I*exp_integral_e(4, I*d*x) + I*exp_integral_
e(4, -I*d*x)))*sin(c))*cos(d*x + c)^2 - (3*b*c^3*(exp_integral_e(4, I*d*x)
+ exp_integral_e(4, -I*d*x))*cos(c)^3 + 3*b*c^3*(exp_integral_e(4, I*d*x) +
 exp_integral_e(4, -I*d*x))*cos(c)*sin(c)^2 + 3*b*c^3*(-I*exp_integral_e(4,
 I*d*x) + I*exp_integral_e(4, -I*d*x))*sin(c)^3 + 3*b*c^3*(exp_integral_e(4
, I*d*x) + exp_integral_e(4, -I*d*x))*cos(c) - 2*((b*cos(c)^2 + b*sin(c)^2)
*(d*x + c)^3 - 3*(b*c*cos(c)^2 + b*c*sin(c)^2)*(d*x + c)^2 + 3*(b*c^2*cos(c
)^2 + b*c^2*sin(c)^2)*(d*x + c))*cos(d*x + c) + 3*(b*c^3*(-I*exp_integral_e
(4, I*d*x) + I*exp_integral_e(4, -I*d*x))*cos(c)^2 + b*c^3*(-I*exp_integral
_e(4, I*d*x) + I*exp_integral_e(4, -I*d*x)))*sin(c))*sin(d*x + c)^2 + 2*((b
*cos(c)^2 + b*sin(c)^2)*(d*x + c)^3 - 3*(b*c*cos(c)^2 + b*c*sin(c)^2)*(d*x
+ c)^2 + 3*(b*c^2*cos(c)^2 + b*c^2*sin(c)^2)*(d*x + c))*cos(d*x + c))/(((d*
x + c)^3*(cos(c)^2 + sin(c)^2)*d^3 - 3*(c*cos(c)^2 + c*sin(c)^2)*(d*x + c)^
2*d^3 + 3*(c^2*cos(c)^2 + c^2*sin(c)^2)*(d*x + c)*d^3 - (c^3*cos(c)^2 + c^3
*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((d*x + c)^3*(cos(c)^2 + sin(c)^2)*d^3 - 3
*(c*cos(c)^2 + c*sin(c)^2)*(d*x + c)^2*d^3 + 3*(c^2*cos(c)^2 + c^2*sin(c)^2
)*(d*x + c)*d^3 - (c^3*cos(c)^2 + c^3*sin(c)^2)*d^3)*sin(d*x + c)^2))*d^2
```

Fricas [A]
time = 0.37, size = 84, normalized size = 1.20

$$-\frac{2\,ad^3x^2\cos\left(c\right)\operatorname{Si}\left(dx\right)+2\,ad\sin\left(dx+c\right)+2\,\left(ad^2x+2\,bx^2\right)\cos\left(dx+c\right)+\left(ad^3x^2\operatorname{Ci}\left(dx\right)+ad^3x^2\operatorname{Ci}\left(-dx\right)\right)\sin\left(c\right)}{4\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d^3*x^2*cos(c)*sin_integral(d*x) + 2*a*d*sin(d*x + c) + 2*(a*d^2*
x + 2*b*x^2)*cos(d*x + c) + (a*d^3*x^2*cos_integral(d*x) + a*d^3*x^2*cos_in
tegral(-d*x))*sin(c))/(d*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^3\right)\sin\left(c+dx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**3)*sin(c + d*x)/x**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.13, size = 564, normalized size = 8.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^3*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a
*d^3*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^
3*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^3*x^2*real_part
(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^2*real_part(cos_i
ntegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a*d^3*x^2*imag_part(cos_integral
(d*x))*tan(1/2*d*x)^2 + a*d^3*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x
)^2 - 2*a*d^3*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + a*d^3*x^2*imag_part(co
s_integral(d*x))*tan(1/2*c)^2 - a*d^3*x^2*imag_part(cos_integral(-d*x))*tan
(1/2*c)^2 + 2*a*d^3*x^2*sin_integral(d*x)*tan(1/2*c)^2 - 2*a*d^3*x^2*real_p
art(cos_integral(d*x))*tan(1/2*c) - 2*a*d^3*x^2*real_part(cos_integral(-d*x
))*tan(1/2*c) - 2*a*d^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^3*x^2*imag_part
(cos_integral(d*x)) + a*d^3*x^2*imag_part(cos_integral(-d*x)) - 2*a*d^3*x^2
*sin_integral(d*x) - 4*b*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^2*x*tan(1/
2*d*x)^2 + 8*a*d^2*x*tan(1/2*d*x)*tan(1/2*c) + 2*a*d^2*x*tan(1/2*c)^2 + 4*b
*x^2*tan(1/2*d*x)^2 + 16*b*x^2*tan(1/2*d*x)*tan(1/2*c) + 4*a*d*tan(1/2*d*x)
^2*tan(1/2*c) + 4*b*x^2*tan(1/2*c)^2 + 4*a*d*tan(1/2*d*x)*tan(1/2*c)^2 - 2*
a*d^2*x - 4*b*x^2 - 4*a*d*tan(1/2*d*x) - 4*a*d*tan(1/2*c))/(d*x^2*tan(1/2*d
*x)^2*tan(1/2*c)^2 + d*x^2*tan(1/2*d*x)^2 + d*x^2*tan(1/2*c)^2 + d*x^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)\,\left(b\,x^3+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^3))/x^3,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^3))/x^3, x)
```

3.86 $\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx$

Optimal. Leaf size=91

$$-\frac{ad\cos(c+dx)}{6x^2}-\frac{1}{6}ad^3\cos(c)\mathrm{Ci}(dx)+b\mathrm{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3}+\frac{ad^2\sin(c+dx)}{6x}+b\cos(c)\mathrm{Si}(dx)+\frac{1}{6}ad^3$$

```
[Out] -1/6*a*d^3*Ci(d*x)*cos(c)-1/6*a*d*cos(d*x+c)/x^2+b*cos(c)*Si(d*x)+b*Ci(d*x)
*sin(c)+1/6*a*d^3*Si(d*x)*sin(c)-1/3*a*sin(d*x+c)/x^3+1/6*a*d^2*sin(d*x+c)/
x
```

Rubi [A]
time = 0.13, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3420, 3378, 3384, 3380, 3383}

$$-\frac{1}{6}ad^3\cos(c)\mathrm{CosIntegral}(dx)+\frac{1}{6}ad^3\sin(c)\mathrm{Si}(dx)+\frac{ad^2\sin(c+dx)}{6x}-\frac{a\sin(c+dx)}{3x^3}-\frac{ad\cos(c+dx)}{6x^2}+b\sin(c)\mathrm{CosIntegral}(dx)+b\cos(c)\mathrm{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Sin[c + d*x])/x^4,x]
```

```
[Out] -1/6*(a*d*Cos[c + d*x])/x^2 - (a*d^3*Cos[c]*CosIntegral[d*x])/6 + b*CosInte
gral[d*x]*Sin[c] - (a*Sin[c + d*x])/(3*x^3) + (a*d^2*Sin[c + d*x])/(6*x) +
b*Cos[c]*SinIntegral[d*x] + (a*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^4}+\frac{b\sin(c+dx)}{x}\right)dx\\ &= a\int \frac{\sin(c+dx)}{x^4}\,dx+b\int \frac{\sin(c+dx)}{x}\,dx\\ &= -\frac{a\sin(c+dx)}{3x^3}+\frac{1}{3}(ad)\int \frac{\cos(c+dx)}{x^3}\,dx+(b\cos(c))\int \frac{\sin(dx)}{x}\,dx+(b\sin(c\\ &= -\frac{ad\cos(c+dx)}{6x^2}+b\mathrm{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3}+b\cos(c)\mathrm{Si}(dx)-\frac{1}{6}\left(ad^2\right)\int\\ &= -\frac{ad\cos(c+dx)}{6x^2}+b\mathrm{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3}+\frac{ad^2\sin(c+dx)}{6x}+b\cos(c)\\ &= -\frac{ad\cos(c+dx)}{6x^2}+b\mathrm{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3}+\frac{ad^2\sin(c+dx)}{6x}+b\cos(c)\\ &= -\frac{ad\cos(c+dx)}{6x^2}-\frac{1}{6}ad^3\cos(c)\mathrm{Ci}(dx)+b\mathrm{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3}+\frac{ad^2\,\mathrm{s}}{}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 104, normalized size = 1.14

$$b\mathrm{Ci}(dx)\sin(c)+\frac{a\cos(dx)\left(-dx\cos(c)-2\sin(c)+d^2x^2\sin(c)\right)}{6x^3}+\frac{a\left(-2\cos(c)+d^2x^2\cos(c)+dx\sin(c)\right)\sin(dx)}{6x^3}+b\cos(c)\mathrm{Si}(dx)-\frac{1}{6}ad^3(\cos(c)\mathrm{Ci}(dx)-\sin(c)\mathrm{Si}(dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Sin[c + d*x])/x^4,x]
```

```
[Out] b*CosIntegral[d*x]*Sin[c] + (a*Cos[d*x]*(-(d*x*Cos[c]) - 2*Sin[c] + d^2*x^2
*Sin[c]))/(6*x^3) + (a*(-2*Cos[c] + d^2*x^2*Cos[c] + d*x*Sin[c])*Sin[d*x])/
(6*x^3) + b*Cos[c]*SinIntegral[d*x] - (a*d^3*(Cos[c]*CosIntegral[d*x] - Sin
[c]*SinIntegral[d*x]))/6
```

Maple [A]
time = 0.08, size = 87, normalized size = 0.96

method	result
derivativedivides	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\text{sinIntegral}(dx)\sin(c)}{6}-\frac{\text{cosineIntegral}(dx)\cos(c)}{6}\right)+\frac{b(\text{sinI}}{}\right.$
default	$d^3\left(a\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\text{sinIntegral}(dx)\sin(c)}{6}-\frac{\text{cosineIntegral}(dx)\cos(c)}{6}\right)+\frac{b(\text{sinI}}{}\right.$
risch	$-\frac{i\cos(c)\,\text{expIntegral}(1,idx)b}{2}+\frac{\text{expIntegral}(1,idx)\cos(c)a\,d^3}{12}+\frac{\text{expIntegral}(1,-idx)\cos(c)a\,d^3}{12}+\frac{i\cos(c)\,\text{expInte}}{2}$
meijerg	$\frac{b\sqrt{\pi}\ \sin(c)\left(\frac{2\gamma+2\ln(x)+\ln(d^2)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{dx}{2}\right)}{\sqrt{\pi}}+\frac{2\,\text{cosineIntegral}(dx)}{\sqrt{\pi}}\right)}{2}+b\cos\left(c\right)\text{sinIntegral}\left(dx\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(a*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x+1
/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+1/d^3*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c
)))
```

Maxima [C] Result contains complex when optimal does not.
time = 1.09, size = 132, normalized size = 1.45

$$-\frac{((a(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos(c)+a(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin(c))d^6-6\,(b(i\,\Gamma(-3,i\,dx)-i\,\Gamma(-3,-i\,dx))\cos(c)+b(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\sin(c))d^3)x^3+2\,bdx\sin(dx+c)+2\,(bd^2x^2-2\,b)\cos(dx+c)}{2\,d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a*(-I*gamma(-3, I
*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^6 - 6*(b*(I*gamma(-3, I*d*x) - I*gam
ma(-3, -I*d*x))*cos(c) + b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*sin(c))*d
^3)*x^3 + 2*b*d*x*sin(d*x + c) + 2*(b*d^2*x^2 - 2*b)*cos(d*x + c))/(d^3*x^3
)
```

Fricas [A]
time = 0.38, size = 114, normalized size = 1.25

$$-\frac{2\,adx\cos(dx+c)+(ad^3x^3\,\mathrm{Ci}(dx)+ad^3x^3\,\mathrm{Ci}(-dx)-12\,bx^3\,\mathrm{Si}(dx))\cos(c)-2\,(ad^2x^2-2\,a)\sin(dx+c)-2\,(ad^3x^3\,\mathrm{Si}(dx)+3\,bx^3\,\mathrm{Ci}(dx)+3\,bx^3\,\mathrm{Ci}(-dx))\sin(c)}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*a*d*x*cos(d*x + c) + (a*d^3*x^3*cos_integral(d*x) + a*d^3*x^3*cos_
integral(-d*x) - 12*b*x^3*sin_integral(d*x))*cos(c) - 2*(a*d^2*x^2 - 2*a)*s
in(d*x + c) - 2*(a*d^3*x^3*sin_integral(d*x) + 3*b*x^3*cos_integral(d*x) +
3*b*x^3*cos_integral(-d*x))*sin(c))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**3)*sin(c + d*x)/x**4, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.28, size = 796, normalized size = 8.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d
^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^3
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x^3*sin_
integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a*d^3*x^3*real_part(cos_integral(
d*x))*tan(1/2*d*x)^2 - a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)
^2 + a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d^3*x^3*real_p
art(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^3*x^3*imag_part(cos_integral(d
*x))*tan(1/2*c) - 2*a*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*
a*d^3*x^3*sin_integral(d*x)*tan(1/2*c) - 6*b*x^3*imag_part(cos_integral(d*x
))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*b*x^3*imag_part(cos_integral(-d*x))*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 12*b*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2
*c)^2 - a*d^3*x^3*real_part(cos_integral(d*x)) - a*d^3*x^3*real_part(cos_in
tegral(-d*x)) - 4*a*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 12*b*x^3*real_part(
cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 12*b*x^3*real_part(cos_integ
ral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^
2 + 6*b*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 6*b*x^3*imag_part
(cos_integral(-d*x))*tan(1/2*d*x)^2 + 12*b*x^3*sin_integral(d*x)*tan(1/2*d*
x)^2 - 6*b*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 6*b*x^3*imag_par
t(cos_integral(-d*x))*tan(1/2*c)^2 - 12*b*x^3*sin_integral(d*x)*tan(1/2*c)^
2 - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*d^2*x^2*tan(1/2*d*x) + 4*a*d^
2*x^2*tan(1/2*c) + 12*b*x^3*real_part(cos_integral(d*x))*tan(1/2*c) + 12*b*
x^3*real_part(cos_integral(-d*x))*tan(1/2*c) + 6*b*x^3*imag_part(cos_integr
al(d*x)) - 6*b*x^3*imag_part(cos_integral(-d*x)) + 12*b*x^3*sin_integral(d*
x) + 2*a*d*x*tan(1/2*d*x)^2 + 8*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 2*a*d*x*tan
(1/2*c)^2 + 8*a*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*tan(1/2*d*x)*tan(1/2*c)^2 -
```

```
 2*a*d*x - 8*a*tan(1/2*d*x) - 8*a*tan(1/2*c))/(x^3*tan(1/2*d*x)^2*tan(1/2*c
)^2 + x^3*tan(1/2*d*x)^2 + x^3*tan(1/2*c)^2 + x^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (c+d\,x)\ \left(b\,x^3+a\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^3))/x^4,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^3))/x^4, x)
```

3.87 $\int x(a+bx^3)^2 \sin(c+dx)\,dx$

Optimal. Leaf size=235

$$-\frac{48ab\cos(c+dx)}{d^5}+\frac{5040b^2x\cos(c+dx)}{d^7}-\frac{a^2x\cos(c+dx)}{d}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{840b^2x^3\cos(c+dx)}{d^5}-2abx$$

```
[Out] -48*a*b*cos(d*x+c)/d^5+5040*b^2*x*cos(d*x+c)/d^7-a^2*x*cos(d*x+c)/d+24*a*b*
x^2*cos(d*x+c)/d^3-840*b^2*x^3*cos(d*x+c)/d^5-2*a*b*x^4*cos(d*x+c)/d+42*b^2
*x^5*cos(d*x+c)/d^3-b^2*x^7*cos(d*x+c)/d-5040*b^2*sin(d*x+c)/d^8+a^2*sin(d*
x+c)/d^2-48*a*b*x*sin(d*x+c)/d^4+2520*b^2*x^2*sin(d*x+c)/d^6+8*a*b*x^3*sin(
d*x+c)/d^2-210*b^2*x^4*sin(d*x+c)/d^4+7*b^2*x^6*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.22, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3420, 3377, 2717, 2718}

$$\frac{a^2\sin(c+dx)}{d^2}-\frac{a^2x\cos(c+dx)}{d}-\frac{48ab\cos(c+dx)}{d^5}-\frac{48abx\sin(c+dx)}{d^4}+\frac{24abx^2\cos(c+dx)}{d^3}+\frac{8abx^3\sin(c+dx)}{d^2}-\frac{2abx^4\cos(c+dx)}{d}-\frac{5040b^2\sin(c+dx)}{d^8}+\frac{5040b^2x\cos(c+dx)}{d^7}+\frac{2520b^2x^2\sin(c+dx)}{d^6}-\frac{840b^2x^3\cos(c+dx)}{d^5}-\frac{210b^2x^4\sin(c+dx)}{d^4}+\frac{42b^2x^5\cos(c+dx)}{d^3}+\frac{7b^2x^6\sin(c+dx)}{d^2}-\frac{b^2x^7\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^2*Sin[c + d*x],x]
```

```
[Out] (-48*a*b*Cos[c + d*x])/d^5 + (5040*b^2*x*Cos[c + d*x])/d^7 - (a^2*x*Cos[c +
 d*x])/d + (24*a*b*x^2*Cos[c + d*x])/d^3 - (840*b^2*x^3*Cos[c + d*x])/d^5 -
 (2*a*b*x^4*Cos[c + d*x])/d + (42*b^2*x^5*Cos[c + d*x])/d^3 - (b^2*x^7*Cos[
c + d*x])/d - (5040*b^2*Sin[c + d*x])/d^8 + (a^2*Sin[c + d*x])/d^2 - (48*a*
b*x*Sin[c + d*x])/d^4 + (2520*b^2*x^2*Sin[c + d*x])/d^6 + (8*a*b*x^3*Sin[c
+ d*x])/d^2 - (210*b^2*x^4*Sin[c + d*x])/d^4 + (7*b^2*x^6*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+bx^3\right)^2\sin(c+dx)\,dx &= \int \left(a^2x\sin(c+dx)+2abx^4\sin(c+dx)+b^2x^7\sin(c+dx)\right)\,dx \\
&= a^2\int x\sin(c+dx)\,dx+(2ab)\int x^4\sin(c+dx)\,dx+b^2\int x^7\sin(c+dx)\,dx \\
&= -\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^4\cos(c+dx)}{d}-\frac{b^2x^7\cos(c+dx)}{d}+\frac{a^2\int\cos(c+\cdots}{d} \\
&= -\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^4\cos(c+dx)}{d}-\frac{b^2x^7\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cdots}{d^2} \\
&= -\frac{a^2x\cos(c+dx)}{d}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{2abx^4\cos(c+dx)}{d}+\frac{42b^2x^5\cos\cdots}{d^3} \\
&= -\frac{a^2x\cos(c+dx)}{d}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{2abx^4\cos(c+dx)}{d}+\frac{42b^2x^5\cos\cdots}{d^3} \\
&= -\frac{48ab\cos(c+dx)}{d^5}-\frac{a^2x\cos(c+dx)}{d}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{840b^2x^3\cos\cdots}{d^5} \\
&= -\frac{48ab\cos(c+dx)}{d^5}-\frac{a^2x\cos(c+dx)}{d}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{840b^2x^3\cos\cdots}{d^5} \\
&= -\frac{48ab\cos(c+dx)}{d^5}+\frac{5040b^2x\cos(c+dx)}{d^7}-\frac{a^2x\cos(c+dx)}{d}+\frac{24abx^2\cos\cdots}{d^3} \\
&= -\frac{48ab\cos(c+dx)}{d^5}+\frac{5040b^2x\cos(c+dx)}{d^7}-\frac{a^2x\cos(c+dx)}{d}+\frac{24abx^2\cos\cdots}{d^3}
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 139, normalized size = 0.59

$$\frac{-d(a^2d^6x+2abd^2(24-12d^2x^2+d^4x^4)+b^2x(-5040+840d^2x^2-42d^4x^4+d^6x^6))\cos(c+dx)+(a^2d^6+8abd^4x(-6+d^2x^2)+7b^2(-720+360d^2x^2-30d^4x^4+d^6x^6))\sin(c+dx)}{d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^2*Sin[c + d*x],x]
```

```
[Out] (-(d*(a^2*d^6*x + 2*a*b*d^2*(24 - 12*d^2*x^2 + d^4*x^4) + b^2*x*(-5040 + 84
0*d^2*x^2 - 42*d^4*x^4 + d^6*x^6))*Cos[c + d*x]) + (a^2*d^6 + 8*a*b*d^4*x*(
-6 + d^2*x^2) + 7*b^2*(-720 + 360*d^2*x^2 - 30*d^4*x^4 + d^6*x^6))*Sin[c +
d*x])/d^8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 821 vs. $2(235) = 470$.
time = 0.09, size = 822, normalized size = 3.50 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(a^2*c*cos(d*x+c)+a^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-2/d^3*a*b*c^4*c
os(d*x+c)-8/d^3*a*b*c^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+12/d^3*a*b*c^2*(-(d
*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-8/d^3*a*b*c*(-(d*x+c)
^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+2/d
^3*a*b*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c
)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))+1/d^6*b^2*c^7*cos(d*x+c)+7/d^6*b^2*c
^6*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-21/d^6*b^2*c^5*(-(d*x+c)^2*cos(d*x+c)+2*
cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+35/d^6*b^2*c^4*(-(d*x+c)^3*cos(d*x+c)+3*(d
*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))-35/d^6*b^2*c^3*(-(d*x
+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+
c)-24*(d*x+c)*sin(d*x+c))+21/d^6*b^2*c^2*(-(d*x+c)^5*cos(d*x+c)+5*(d*x+c)^4
*sin(d*x+c)+20*(d*x+c)^3*cos(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(d*x+c)-
120*(d*x+c)*cos(d*x+c))-7/d^6*b^2*c*(-(d*x+c)^6*cos(d*x+c)+6*(d*x+c)^5*sin(
d*x+c)+30*(d*x+c)^4*cos(d*x+c)-120*(d*x+c)^3*sin(d*x+c)-360*(d*x+c)^2*cos(d
*x+c)+720*cos(d*x+c)+720*(d*x+c)*sin(d*x+c))+1/d^6*b^2*(-(d*x+c)^7*cos(d*x+
c)+7*(d*x+c)^6*sin(d*x+c)+42*(d*x+c)^5*cos(d*x+c)-210*(d*x+c)^4*sin(d*x+c)-
840*(d*x+c)^3*cos(d*x+c)+2520*(d*x+c)^2*sin(d*x+c)-5040*sin(d*x+c)+5040*(d*
x+c)*cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(235) = 470$.
time = 0.34, size = 662, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a^2*c*cos(d*x + c) + b^2*c^7*cos(d*x + c)/d^6 - 2*a*b*c^4*cos(d*x + c)/d^3
 - ((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2 - 7*((d*x + c)*cos(d*x + c)
- sin(d*x + c))*b^2*c^6/d^6 + 8*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a*b
*c^3/d^3 + 21*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b
^2*c^5/d^6 - 12*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))
*a*b*c^2/d^3 - 35*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^
2 - 2)*sin(d*x + c))*b^2*c^4/d^6 + 8*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x +
 c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b*c/d^3 + 35*(((d*x + c)^4 - 12*(
d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))
*b^2*c^3/d^6 - 2*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*
x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*a*b/d^3 - 21*(((d*x + c)^5 - 20*(d*x
```

```
+ c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 +
24)*sin(d*x + c))*b^2*c^2/d^6 + 7*(((d*x + c)^6 - 30*(d*x + c)^4 + 360*(d*x
 + c)^2 - 720)*cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 1
20*c)*sin(d*x + c))*b^2*c/d^6 - (((d*x + c)^7 - 42*(d*x + c)^5 + 840*(d*x +
 c)^3 - 5040*d*x - 5040*c)*cos(d*x + c) - 7*((d*x + c)^6 - 30*(d*x + c)^4 +
 360*(d*x + c)^2 - 720)*sin(d*x + c))*b^2/d^6)/d^2
```

Fricas [A]
time = 0.35, size = 161, normalized size = 0.69

$$-\frac{(b^2d^7x^7 + 2\,abd^7x^4 - 42\,b^2d^5x^5 - 24\,abd^5x^2 + 840\,b^2d^3x^3 + 48\,abd^3 + (a^2d^7 - 5040\,b^2d)x)\cos(dx+c) - (7\,b^2d^6x^6 + 8\,abd^6x^3 - 210\,b^2d^4x^4 + a^2d^6 - 48\,abd^4x + 2520\,b^2d^2x^2 - 5040\,b^2)\sin(dx+c)}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^7*x^7 + 2*a*b*d^7*x^4 - 42*b^2*d^5*x^5 - 24*a*b*d^5*x^2 + 840*b^2*
d^3*x^3 + 48*a*b*d^3 + (a^2*d^7 - 5040*b^2*d)*x)*cos(d*x + c) - (7*b^2*d^6*
x^6 + 8*a*b*d^6*x^3 - 210*b^2*d^4*x^4 + a^2*d^6 - 48*a*b*d^4*x + 2520*b^2*d
^2*x^2 - 5040*b^2)*sin(d*x + c))/d^8
```

Sympy [A]
time = 0.94, size = 284, normalized size = 1.21

$$\begin{cases} -\frac{a^2x\cos(c+dx)}{d} + \frac{a^2\sin(c+dx)}{d^2} - \frac{2abx^4\cos(c+dx)}{d} + \frac{8abx^3\sin(c+dx)}{d^2} + \frac{24abx^2\cos(c+dx)}{d^3} - \frac{48abx\sin(c+dx)}{d^4} - \frac{48ab\cos(c+dx)}{d^5} - \frac{b^2x^7\cos(c+dx)}{d} + \frac{7b^2x^6\sin(c+dx)}{d^2} + \frac{42b^2x^5\cos(c+dx)}{d^3} - \frac{210b^2x^4\sin(c+dx)}{d^4} - \frac{840b^2x^3\cos(c+dx)}{d^5} + \frac{2520b^2x^2\sin(c+dx)}{d^6} + \frac{5040b^2x\cos(c+dx)}{d^7} - \frac{5040b^2\sin(c+dx)}{d^8} & \text{for } d \neq 0 \\ \left(\frac{a^2x^2}{2} + \frac{2abx^5}{5} + \frac{b^2x^8}{8}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*x*cos(c + d*x)/d + a**2*sin(c + d*x)/d**2 - 2*a*b*x**4*cos
(c + d*x)/d + 8*a*b*x**3*sin(c + d*x)/d**2 + 24*a*b*x**2*cos(c + d*x)/d**3
- 48*a*b*x*sin(c + d*x)/d**4 - 48*a*b*cos(c + d*x)/d**5 - b**2*x**7*cos(c +
 d*x)/d + 7*b**2*x**6*sin(c + d*x)/d**2 + 42*b**2*x**5*cos(c + d*x)/d**3 -
210*b**2*x**4*sin(c + d*x)/d**4 - 840*b**2*x**3*cos(c + d*x)/d**5 + 2520*b*
*2*x**2*sin(c + d*x)/d**6 + 5040*b**2*x*cos(c + d*x)/d**7 - 5040*b**2*sin(c
 + d*x)/d**8, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x**5/5 + b**2*x**8/8)*sin(c)
, True))
```

Giac [A]
time = 5.48, size = 161, normalized size = 0.69

$$-\frac{(b^2d^7x^7 + 2\,abd^7x^4 - 42\,b^2d^5x^5 + a^2d^7x - 24\,abd^5x^2 + 840\,b^2d^3x^3 + 48\,abd^3 - 5040\,b^2dx)\cos(dx+c)}{d^8} + \frac{(7\,b^2d^6x^6 + 8\,abd^6x^3 - 210\,b^2d^4x^4 + a^2d^6 - 48\,abd^4x + 2520\,b^2d^2x^2 - 5040\,b^2)\sin(dx+c)}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^7*x^7 + 2*a*b*d^7*x^4 - 42*b^2*d^5*x^5 + a^2*d^7*x - 24*a*b*d^5*x^2
 + 840*b^2*d^3*x^3 + 48*a*b*d^3 - 5040*b^2*d*x)*cos(d*x + c)/d^8 + (7*b^2*d
^6*x^6 + 8*a*b*d^6*x^3 - 210*b^2*d^4*x^4 + a^2*d^6 - 48*a*b*d^4*x + 2520*b^
2*d^2*x^2 - 5040*b^2)*sin(d*x + c)/d^8
```

Mupad [B]
time = 5.09, size = 225, normalized size = 0.96

$$\frac{42\,b^2\,x^5\,\cos\left(c+d\,x\right)+24\,a\,b\,x^2\,\cos\left(c+d\,x\right)}{d^3}-\frac{b^2\,x^7\,\cos\left(c+d\,x\right)+a^2\,x\,\cos\left(c+d\,x\right)+2\,a\,b\,x^4\,\cos\left(c+d\,x\right)}{d}-\frac{840\,b^2\,x^3\,\cos\left(c+d\,x\right)+48\,a\,b\,\cos\left(c+d\,x\right)}{d^5}+\frac{a^2\,\sin\left(c+d\,x\right)+7\,b^2\,x^6\,\sin\left(c+d\,x\right)+8\,a\,b\,x^3\,\sin\left(c+d\,x\right)}{d^2}-\frac{210\,b^2\,x^4\,\sin\left(c+d\,x\right)+48\,a\,b\,x\,\sin\left(c+d\,x\right)}{d^4}-\frac{5040\,b^2\,\sin\left(c+d\,x\right)}{d^8}+\frac{2520\,b^2\,x^2\,\sin\left(c+d\,x\right)}{d^6}+\frac{5040\,b^2\,x\,\cos\left(c+d\,x\right)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(c + d*x)*(a + b*x^3)^2,x)
```

```
[Out] (42*b^2*x^5*cos(c + d*x) + 24*a*b*x^2*cos(c + d*x))/d^3 - (b^2*x^7*cos(c +
d*x) + a^2*x*cos(c + d*x) + 2*a*b*x^4*cos(c + d*x))/d - (840*b^2*x^3*cos(c
+ d*x) + 48*a*b*cos(c + d*x))/d^5 + (a^2*sin(c + d*x) + 7*b^2*x^6*sin(c + d
*x) + 8*a*b*x^3*sin(c + d*x))/d^2 - (210*b^2*x^4*sin(c + d*x) + 48*a*b*x*si
n(c + d*x))/d^4 - (5040*b^2*sin(c + d*x))/d^8 + (2520*b^2*x^2*sin(c + d*x))
/d^6 + (5040*b^2*x*cos(c + d*x))/d^7
```

3.88 $\int (a+bx^3)^2 \sin(c+dx)\,dx$

Optimal. Leaf size=188

$$\frac{720b^2\cos(c+dx)}{d^7}-\frac{a^2\cos(c+dx)}{d}+\frac{12abx\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+dx)}{d^5}-\frac{2abx^3\cos(c+dx)}{d}+\frac{30b^2x^4\cos(c+dx)}{d^3}$$

```
[Out] 720*b^2*cos(d*x+c)/d^7-a^2*cos(d*x+c)/d+12*a*b*x*cos(d*x+c)/d^3-360*b^2*x^2
*cos(d*x+c)/d^5-2*a*b*x^3*cos(d*x+c)/d+30*b^2*x^4*cos(d*x+c)/d^3-b^2*x^6*co
s(d*x+c)/d-12*a*b*sin(d*x+c)/d^4+720*b^2*x*sin(d*x+c)/d^6+6*a*b*x^2*sin(d*x
+c)/d^2-120*b^2*x^3*sin(d*x+c)/d^4+6*b^2*x^5*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.17, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3410, 2718, 3377, 2717}

$$-\frac{a^2\cos(c+dx)}{d}-\frac{12ab\sin(c+dx)}{d^4}+\frac{12abx\cos(c+dx)}{d^3}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{2abx^3\cos(c+dx)}{d}+\frac{720b^2\cos(c+dx)}{d^7}+\frac{720b^2x\sin(c+dx)}{d^6}-\frac{360b^2x^2\cos(c+dx)}{d^5}-\frac{120b^2x^3\sin(c+dx)}{d^4}+\frac{30b^2x^4\cos(c+dx)}{d^3}+\frac{6b^2x^5\sin(c+dx)}{d^2}-\frac{b^2x^6\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*Sin[c + d*x],x]
```

```
[Out] (720*b^2*Cos[c + d*x])/d^7 - (a^2*Cos[c + d*x])/d + (12*a*b*x*Cos[c + d*x])
/d^3 - (360*b^2*x^2*Cos[c + d*x])/d^5 - (2*a*b*x^3*Cos[c + d*x])/d + (30*b^
2*x^4*Cos[c + d*x])/d^3 - (b^2*x^6*Cos[c + d*x])/d - (12*a*b*Sin[c + d*x])/
d^4 + (720*b^2*x*Sin[c + d*x])/d^6 + (6*a*b*x^2*Sin[c + d*x])/d^2 - (120*b^
2*x^3*Sin[c + d*x])/d^4 + (6*b^2*x^5*Sin[c + d*x])/d^2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3410

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
```

```
n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int (a+bx^3)^2 \sin(c+dx)\, dx = \int \left(a^2\sin(c+dx)+2abx^3\sin(c+dx)+b^2x^6\sin(c+dx)\right)\, dx$$
$$= a^2\int \sin(c+dx)\, dx+(2ab)\int x^3\sin(c+dx)\, dx+b^2\int x^6\sin(c+dx)\, dx$$
$$= -\frac{a^2\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^6\cos(c+dx)}{d}+\frac{(6ab)\int x^2\cos(c+}{d}$$
$$= -\frac{a^2\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^6\cos(c+dx)}{d}+\frac{6abx^2\sin(c+dx)}{d^2}$$
$$= -\frac{a^2\cos(c+dx)}{d}+\frac{12abx\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+dx)}{d}+\frac{30b^2x^4\cos(c+d}{d^3}$$
$$= -\frac{a^2\cos(c+dx)}{d}+\frac{12abx\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+dx)}{d}+\frac{30b^2x^4\cos(c+d}{d^3}$$
$$= -\frac{a^2\cos(c+dx)}{d}+\frac{12abx\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+dx)}{d^5}-\frac{2abx^3\cos(c+}{d}$$
$$= -\frac{a^2\cos(c+dx)}{d}+\frac{12abx\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+dx)}{d^5}-\frac{2abx^3\cos(c+}{d}$$
$$= \frac{720b^2\cos(c+dx)}{d^7}-\frac{a^2\cos(c+dx)}{d}+\frac{12abx\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+d}{d^5}$$

Mathematica [A]
time = 0.20, size = 112, normalized size = 0.60

$$\frac{-((a^2d^6+2abd^4x(-6+d^2x^2)+b^2(-720+360d^2x^2-30d^4x^4+d^6x^6))\cos(c+dx))+6bd(ad^2(-2+d^2x^2)+bx(120-20d^2x^2+d^4x^4))\sin(c+dx)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*Sin[c + d*x],x]
```

```
[Out] (-((a^2*d^6 + 2*a*b*d^4*x*(-6 + d^2*x^2) + b^2*(-720 + 360*d^2*x^2 - 30*d^4
*x^4 + d^6*x^6))*Cos[c + d*x]) + 6*b*d*(a*d^2*(-2 + d^2*x^2) + b*x*(120 - 2
0*d^2*x^2 + d^4*x^4))*Sin[c + d*x])/d^7
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 598 vs. 2(188) = 376.
time = 0.08, size = 599, normalized size = 3.19 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*cos(d*x+c)+2/d^3*a*b*c^3*cos(d*x+c)+6/d^3*a*b*c^2*(sin(d*x+c)-(d*
x+c)*cos(d*x+c))-6/d^3*a*b*c*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*
sin(d*x+c))+2/d^3*a*b*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d
*x+c)+6*(d*x+c)*cos(d*x+c))-1/d^6*b^2*c^6*cos(d*x+c)-6/d^6*b^2*c^5*(sin(d*x
+c)-(d*x+c)*cos(d*x+c))+15/d^6*b^2*c^4*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+
2*(d*x+c)*sin(d*x+c))-20/d^6*b^2*c^3*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin
(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+15/d^6*b^2*c^2*(-(d*x+c)^4*cos(d
*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)-24*(d*x+
c)*sin(d*x+c))-6/d^6*b^2*c*(-(d*x+c)^5*cos(d*x+c)+5*(d*x+c)^4*sin(d*x+c)+20
*(d*x+c)^3*cos(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(d*x+c)-120*(d*x+c)*co
s(d*x+c))+1/d^6*b^2*(-(d*x+c)^6*cos(d*x+c)+6*(d*x+c)^5*sin(d*x+c)+30*(d*x+c
)^4*cos(d*x+c)-120*(d*x+c)^3*sin(d*x+c)-360*(d*x+c)^2*cos(d*x+c)+720*cos(d*
x+c)+720*(d*x+c)*sin(d*x+c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(188) = 376$.
time = 0.34, size = 489, normalized size = 2.60

$$-\frac{a^2\cos(dx+c)+\frac{b^2c^6\cos(dx+c)}{d^6}-\frac{2abc^3\cos(dx+c)}{d^3}-\frac{6((dx+c)\cos(dx+c)-\sin(dx+c))b^2c^5}{d^6}+\frac{6((dx+c)\cos(dx+c)-\sin(dx+c))abc^2}{d^3}+\frac{15\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)b^2c^4}{d^6}-\frac{6\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)abc}{d^3}-\frac{20\left(\left((dx+c)^3-6dx-6c\right)\cos(dx+c)-3\left((dx+c)^2-2\right)\sin(dx+c)\right)b^2c^3}{d^6}+\frac{2\left(\left((dx+c)^3-6dx-6c\right)\cos(dx+c)-3\left((dx+c)^2-2\right)\sin(dx+c)\right)ab}{d^3}+\frac{15\left(\left((dx+c)^4-12(dx+c)^2+24\right)\cos(dx+c)-4\left((dx+c)^3-6dx-6c\right)\sin(dx+c)\right)b^2c^2}{d^6}-\frac{6\left(\left((dx+c)^5-20(dx+c)^3+120dx+120c\right)\cos(dx+c)-5\left((dx+c)^4-12(dx+c)^2+24\right)\sin(dx+c)\right)b^2c}{d^6}+\frac{\left(\left((dx+c)^6-30(dx+c)^4+360(dx+c)^2-720\right)\cos(dx+c)-6\left((dx+c)^5-20(dx+c)^3+120dx+120c\right)\sin(dx+c)\right)b^2}{d^6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*cos(d*x + c) + b^2*c^6*cos(d*x + c)/d^6 - 2*a*b*c^3*cos(d*x + c)/d^3
- 6*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b^2*c^5/d^6 + 6*((d*x + c)*cos(
d*x + c) - sin(d*x + c))*a*b*c^2/d^3 + 15*(((d*x + c)^2 - 2)*cos(d*x + c) -
 2*(d*x + c)*sin(d*x + c))*b^2*c^4/d^6 - 6*(((d*x + c)^2 - 2)*cos(d*x + c)
- 2*(d*x + c)*sin(d*x + c))*a*b*c/d^3 - 20*(((d*x + c)^3 - 6*d*x - 6*c)*cos
(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b^2*c^3/d^6 + 2*(((d*x + c)^3
 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b/d^3 +
15*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d
*x - 6*c)*sin(d*x + c))*b^2*c^2/d^6 - 6*(((d*x + c)^5 - 20*(d*x + c)^3 + 12
0*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 + 24)*sin(d*x
 + c))*b^2*c/d^6 + (((d*x + c)^6 - 30*(d*x + c)^4 + 360*(d*x + c)^2 - 720)*
cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 120*c)*sin(d*x +
 c))*b^2/d^6)/d
```

Fricas [**A**]
time = 0.35, size = 129, normalized size = 0.69

$$-\frac{\left(b^2d^6x^6+2\,abd^6x^3-30\,b^2d^4x^4+a^2d^6-12\,abd^4x+360\,b^2d^2x^2-720\,b^2\right)\cos\left(dx+c\right)-6\left(b^2d^5x^5+abd^5x^2-20\,b^2d^3x^3-2\,abd^3+120\,b^2dx\right)\sin\left(dx+c\right)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^6*x^6 + 2*a*b*d^6*x^3 - 30*b^2*d^4*x^4 + a^2*d^6 - 12*a*b*d^4*x +
360*b^2*d^2*x^2 - 720*b^2)*cos(d*x + c) - 6*(b^2*d^5*x^5 + a*b*d^5*x^2 - 20
*b^2*d^3*x^3 - 2*a*b*d^3 + 120*b^2*d*x)*sin(d*x + c))/d^7
```

Sympy [A]
time = 0.64, size = 226, normalized size = 1.20

$$\begin{cases} -\frac{a^2\cos(c+dx)}{d} - \frac{2abx^3\cos(c+dx)}{d} + \frac{6abx^2\sin(c+dx)}{d^2} + \frac{12abx\cos(c+dx)}{d^3} - \frac{12ab\sin(c+dx)}{d^4} - \frac{b^2x^6\cos(c+dx)}{d} + \frac{6b^2x^5\sin(c+dx)}{d^2} + \frac{30b^2x^4\cos(c+dx)}{d^3} - \frac{120b^2x^3\sin(c+dx)}{d^4} - \frac{360b^2x^2\cos(c+dx)}{d^5} + \frac{720b^2x\sin(c+dx)}{d^6} + \frac{720b^2\cos(c+dx)}{d^7} & \text{for } d \neq 0 \\ \left(a^2x + \frac{abx^4}{2} + \frac{b^2x^7}{7}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)/d - 2*a*b*x**3*cos(c + d*x)/d + 6*a*b*x**2*si
n(c + d*x)/d**2 + 12*a*b*x*cos(c + d*x)/d**3 - 12*a*b*sin(c + d*x)/d**4 - b
**2*x**6*cos(c + d*x)/d + 6*b**2*x**5*sin(c + d*x)/d**2 + 30*b**2*x**4*cos(
c + d*x)/d**3 - 120*b**2*x**3*sin(c + d*x)/d**4 - 360*b**2*x**2*cos(c + d*x
)/d**5 + 720*b**2*x*sin(c + d*x)/d**6 + 720*b**2*cos(c + d*x)/d**7, Ne(d, 0
)), ((a**2*x + a*b*x**4/2 + b**2*x**7/7)*sin(c), True))
```

Giac [A]
time = 4.84, size = 131, normalized size = 0.70

$$-\frac{(b^2d^6x^6 + 2\,abd^6x^3 - 30\,b^2d^4x^4 + a^2d^6 - 12\,abd^4x + 360\,b^2d^2x^2 - 720\,b^2)\cos(dx+c)}{d^7} + \frac{6\,(b^2d^5x^5 + abd^5x^2 - 20\,b^2d^3x^3 - 2\,abd^3 + 120\,b^2dx)\sin(dx+c)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^6*x^6 + 2*a*b*d^6*x^3 - 30*b^2*d^4*x^4 + a^2*d^6 - 12*a*b*d^4*x + 3
60*b^2*d^2*x^2 - 720*b^2)*cos(d*x + c)/d^7 + 6*(b^2*d^5*x^5 + a*b*d^5*x^2 -
 20*b^2*d^3*x^3 - 2*a*b*d^3 + 120*b^2*d*x)*sin(d*x + c)/d^7
```

Mupad [B]
time = 0.62, size = 184, normalized size = 0.98

$$\frac{\cos(c+d\,x)\,(720\,b^2 - a^2\,d^6)}{d^7} - \frac{b^2\,x^6\,\cos(c+d\,x)}{d} + \frac{30\,b^2\,x^4\,\cos(c+d\,x)}{d^3} - \frac{360\,b^2\,x^2\,\cos(c+d\,x)}{d^5} + \frac{6\,b^2\,x^5\,\sin(c+d\,x)}{d^2} - \frac{120\,b^2\,x^3\,\sin(c+d\,x)}{d^4} - \frac{12\,a\,b\,\sin(c+d\,x)}{d^4} + \frac{720\,b^2\,x\,\sin(c+d\,x)}{d^6} - \frac{2\,a\,b\,x^3\,\cos(c+d\,x)}{d} + \frac{6\,a\,b\,x^2\,\sin(c+d\,x)}{d^2} + \frac{12\,a\,b\,x\,\cos(c+d\,x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*x^3)^2,x)
```

```
[Out] (cos(c + d*x)*(720*b^2 - a^2*d^6))/d^7 - (b^2*x^6*cos(c + d*x))/d + (30*b^2
*x^4*cos(c + d*x))/d^3 - (360*b^2*x^2*cos(c + d*x))/d^5 + (6*b^2*x^5*sin(c
+ d*x))/d^2 - (120*b^2*x^3*sin(c + d*x))/d^4 - (12*a*b*sin(c + d*x))/d^4 +
(720*b^2*x*sin(c + d*x))/d^6 - (2*a*b*x^3*cos(c + d*x))/d + (6*a*b*x^2*sin(
c + d*x))/d^2 + (12*a*b*x*cos(c + d*x))/d^3
```

3.89 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x} \, dx$

Optimal. Leaf size=161

$$\frac{4ab\cos(c+dx)}{d^3}-\frac{120b^2x\cos(c+dx)}{d^5}-\frac{2abx^2\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+dx)}{d^3}-\frac{b^2x^5\cos(c+dx)}{d}+a^2\mathrm{Ci}(dx)$$

```
[Out] 4*a*b*cos(d*x+c)/d^3-120*b^2*x*cos(d*x+c)/d^5-2*a*b*x^2*cos(d*x+c)/d+20*b^2
*x^3*cos(d*x+c)/d^3-b^2*x^5*cos(d*x+c)/d+a^2*cos(c)*Si(d*x)+a^2*Ci(d*x)*sin
(c)+120*b^2*sin(d*x+c)/d^6+4*a*b*x*sin(d*x+c)/d^2-60*b^2*x^2*sin(d*x+c)/d^4
+5*b^2*x^4*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.17, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3420, 3384, 3380, 3383, 3377, 2718, 2717}

$$a^2\sin(c)\mathrm{CosIntegral}(dx)+a^2\cos(c)\mathrm{Si}(dx)+\frac{4ab\cos(c+dx)}{d^3}+\frac{4abx\sin(c+dx)}{d^2}-\frac{2abx^2\cos(c+dx)}{d}+\frac{120b^2\sin(c+dx)}{d^6}-\frac{120b^2x\cos(c+dx)}{d^5}-\frac{60b^2x^2\sin(c+dx)}{d^4}+\frac{20b^2x^3\cos(c+dx)}{d^3}+\frac{5b^2x^4\sin(c+dx)}{d^2}-\frac{b^2x^5\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x,x]
```

```
[Out] (4*a*b*Cos[c + d*x])/d^3 - (120*b^2*x*Cos[c + d*x])/d^5 - (2*a*b*x^2*Cos[c
+ d*x])/d + (20*b^2*x^3*Cos[c + d*x])/d^3 - (b^2*x^5*Cos[c + d*x])/d + a^2*
CosIntegral[d*x]*Sin[c] + (120*b^2*Sin[c + d*x])/d^6 + (4*a*b*x*Sin[c + d*x
])/d^2 - (60*b^2*x^2*Sin[c + d*x])/d^4 + (5*b^2*x^4*Sin[c + d*x])/d^2 + a^2
*Cos[c]*SinIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^2\sin(c+dx)}{x}\,dx &= \int \left(\frac{a^2\sin(c+dx)}{x}+2abx^2\sin(c+dx)+b^2x^5\sin(c+dx)\right)dx \\
&= a^2\int \frac{\sin(c+dx)}{x}\,dx+(2ab)\int x^2\sin(c+dx)\,dx+b^2\int x^5\sin(c+dx)\,dx \\
&= -\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^5\cos(c+dx)}{d}+\frac{(4ab)\int x\cos(c+dx)\,dx}{d}+\frac{(5b^2)\int
\\
&= -\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^5\cos(c+dx)}{d}+a^2\mathrm{Ci}(dx)\sin(c)+\frac{4abx\sin(c+dx)}{d^2} \\
&= \frac{4ab\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+dx)}{d^3}-\frac{b^2x^5\cos(c+dx)}{d} \\
&= \frac{4ab\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+dx)}{d^3}-\frac{b^2x^5\cos(c+dx)}{d} \\
&= \frac{4ab\cos(c+dx)}{d^3}-\frac{120b^2x\cos(c+dx)}{d^5}-\frac{2abx^2\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+}{d^3} \\
&= \frac{4ab\cos(c+dx)}{d^3}-\frac{120b^2x\cos(c+dx)}{d^5}-\frac{2abx^2\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+}{d^3}
\end{aligned}$$

Mathematica [A]

time = 0.33, size = 108, normalized size = 0.67

$$-\frac{b(2ad^2(-2+d^2x^2)+bx(120-20d^2x^2+d^4x^4))\cos(c+dx)}{d^5}+a^2\mathrm{Ci}(dx)\sin(c)+\frac{b(4ad^4x+5b(24-12d^2x^2+d^4x^4))\sin(c+dx)}{d^6}+a^2\cos(c)\mathrm{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x,x]
```

```
[Out] -((b*(2*a*d^2*(-2 + d^2*x^2) + b*x*(120 - 20*d^2*x^2 + d^4*x^4))*Cos[c + d*
x])/d^5) + a^2*CosIntegral[d*x]*Sin[c] + (b*(4*a*d^4*x + 5*b*(24 - 12*d^2*x
^2 + d^4*x^4))*Sin[c + d*x])/d^6 + a^2*Cos[c]*SinIntegral[d*x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(161) = 322$.
time = 0.18, size = 487, normalized size = 3.02 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-6/d^3*a*b*c^2*cos(d*x+c)-6*a*b*c*(c+1)/
d^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+2*(c^2+c+1)/d^3*a*b*(-(d*x+c)^2*cos(d*x
+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+6/d^6*b^2*c^5*cos(d*x+c)+15*(c+1)/d^
6*b^2*c^4*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-20*b^2*c^3*(c^2+c+1)/d^6*(-(d*x+c
)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+15*(c^3+c^2+c+1)/d^6*b^2*
c^2*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*co
s(d*x+c))-6*b^2*c*(c^4+c^3+c^2+c+1)/d^6*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*
sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))+(c^
5+c^4+c^3+c^2+c+1)/d^6*b^2*(-(d*x+c)^5*cos(d*x+c)+5*(d*x+c)^4*sin(d*x+c)+20
*(d*x+c)^3*cos(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(d*x+c)-120*(d*x+c)*co
s(d*x+c))
```

Maxima [C] Result contains complex when optimal does not.
time = 6.41, size = 147, normalized size = 0.91

$$\frac{(a^2(-i\,\mathrm{Ei}(i\,dx)+i\,\mathrm{Ei}(-i\,dx))\cos(c)+a^2(\mathrm{Ei}(i\,dx)+\mathrm{Ei}(-i\,dx))\sin(c))d^6-2\,(b^2d^5x^5+2\,abd^5x^2-20\,b^2d^3x^3-4\,abd^3+120\,b^2dx)\cos(dx+c)+2\,(5\,b^2d^4x^4+4\,abd^4x-60\,b^2d^2x^2+120\,b^2)\sin(dx+c)}{2\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a^2*(Ei(I*d*x) + Ei(-I*d*x
))*sin(c))*d^6 - 2*(b^2*d^5*x^5 + 2*a*b*d^5*x^2 - 20*b^2*d^3*x^3 - 4*a*b*d^
3 + 120*b^2*d*x)*cos(d*x + c) + 2*(5*b^2*d^4*x^4 + 4*a*b*d^4*x - 60*b^2*d^2
*x^2 + 120*b^2)*sin(d*x + c))/d^6
```

Fricas [A]
time = 0.37, size = 145, normalized size = 0.90

$$\frac{2\,a^2d^6\cos(c)\,\mathrm{Si}(dx)-2\,(b^2d^5x^5+2\,abd^5x^2-20\,b^2d^3x^3-4\,abd^3+120\,b^2dx)\cos(dx+c)+2\,(5\,b^2d^4x^4+4\,abd^4x-60\,b^2d^2x^2+120\,b^2)\sin(dx+c)+(a^2d^6\,\mathrm{Ci}(dx)+a^2d^6\,\mathrm{Ci}(-dx))\sin(c)}{2\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*d^6*cos(c)*sin_integral(d*x) - 2*(b^2*d^5*x^5 + 2*a*b*d^5*x^2 -
20*b^2*d^3*x^3 - 4*a*b*d^3 + 120*b^2*d*x)*cos(d*x + c) + 2*(5*b^2*d^4*x^4 +
 4*a*b*d^4*x - 60*b^2*d^2*x^2 + 120*b^2)*sin(d*x + c) + (a^2*d^6*cos_integr
al(d*x) + a^2*d^6*cos_integral(-d*x))*sin(c))/d^6
```

Sympy [**A**]
time = 4.16, size = 207, normalized size = 1.29

$$a^2 \sin(c) \operatorname{Ci}(dx) + a^2 \cos(c) \operatorname{Si}(dx) + 2abx^2 \left(\begin{cases} x \sin(c) & \text{for } d = 0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise} \end{cases} \right) - 4ab \left(\begin{cases} \frac{x^3 \sin(c)}{3} & \text{for } d = 0 \\ -\frac{\begin{cases} \frac{x \sin(c+dx)}{d} + \frac{\cos(c+dx)}{d^2} & \text{for } d \neq 0 \\ \frac{x^2 \cos(c)}{2} & \text{otherwise} \end{cases}}{d} & \text{otherwise} \end{cases} \right) + b^2 x^5 \left(\begin{cases} x \sin(c) & \text{for } d = 0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise} \end{cases} \right) - 5b^2 \left(\begin{cases} \frac{x^6 \sin(c)}{6} & \text{for } d = 0 \\ -\frac{\begin{cases} \frac{x^4 \sin(c+dx)}{d} + \frac{4x^3 \cos(c+dx)}{d^2} - \frac{12x^2 \sin(c+dx)}{d^3} - \frac{24x \cos(c+dx)}{d^4} + \frac{24 \sin(c+dx)}{d^5} & \text{for } d \neq 0 \\ \frac{x^5 \cos(c)}{5} & \text{otherwise} \end{cases}}{d} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*sin(d*x+c)/x,x)
```

```
[Out] a**2*sin(c)*Ci(d*x) + a**2*cos(c)*Si(d*x) + 2*a*b*x**2*Piecewise((x*sin(c),
 Eq(d, 0)), (-cos(c + d*x)/d, True)) - 4*a*b*Piecewise((x**3*sin(c)/3, Eq(d
, 0)), (-Piecewise((x*sin(c + d*x)/d + cos(c + d*x)/d**2, Ne(d, 0)), (x**2*
cos(c)/2, True))/d, True)) + b**2*x**5*Piecewise((x*sin(c), Eq(d, 0)), (-co
s(c + d*x)/d, True)) - 5*b**2*Piecewise((x**6*sin(c)/6, Eq(d, 0)), (-Piecew
ise((x**4*sin(c + d*x)/d + 4*x**3*cos(c + d*x)/d**2 - 12*x**2*sin(c + d*x)/
d**3 - 24*x*cos(c + d*x)/d**4 + 24*sin(c + d*x)/d**5, Ne(d, 0)), (x**5*cos(
c)/5, True))/d, True))
```

Giac [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.59, size = 921, normalized size = 5.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*d^5*x^5*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 2*b^2*d^5*x^5*tan(
1/2*d*x + 1/2*c)^2 - 2*b^2*d^5*x^5*tan(1/2*c)^2 + 20*b^2*d^4*x^4*tan(1/2*d*
x + 1/2*c)*tan(1/2*c)^2 + 4*a*b*d^5*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2
 - a^2*d^6*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2
 + a^2*d^6*imag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^
2 - 2*a^2*d^6*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*b^2
*d^5*x^5 + 2*a^2*d^6*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*c) + 2*a^2*d^6*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*t
an(1/2*c) - 40*b^2*d^3*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 20*b^2*d^4
*x^4*tan(1/2*d*x + 1/2*c) + 4*a*b*d^5*x^2*tan(1/2*d*x + 1/2*c)^2 + a^2*d^6*
imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2 - a^2*d^6*imag_part(cos
_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2 + 2*a^2*d^6*sin_integral(d*x)*tan(1
```

```
/2*d*x + 1/2*c)^2 - 4*a*b*d^5*x^2*tan(1/2*c)^2 - a^2*d^6*imag_part(cos_inte
gral(d*x))*tan(1/2*c)^2 + a^2*d^6*imag_part(cos_integral(-d*x))*tan(1/2*c)^
2 - 2*a^2*d^6*sin_integral(d*x)*tan(1/2*c)^2 - 40*b^2*d^3*x^3*tan(1/2*d*x +
 1/2*c)^2 + 2*a^2*d^6*real_part(cos_integral(d*x))*tan(1/2*c) + 2*a^2*d^6*r
eal_part(cos_integral(-d*x))*tan(1/2*c) + 40*b^2*d^3*x^3*tan(1/2*c)^2 + 16*
a*b*d^4*x*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 - 4*a*b*d^5*x^2 + a^2*d^6*imag_
part(cos_integral(d*x)) - a^2*d^6*imag_part(cos_integral(-d*x)) + 2*a^2*d^6
*sin_integral(d*x) - 240*b^2*d^2*x^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 - 8*
a*b*d^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 40*b^2*d^3*x^3 + 16*a*b*d^4*x
*tan(1/2*d*x + 1/2*c) + 240*b^2*d*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2
40*b^2*d^2*x^2*tan(1/2*d*x + 1/2*c) - 8*a*b*d^3*tan(1/2*d*x + 1/2*c)^2 + 8*
a*b*d^3*tan(1/2*c)^2 + 240*b^2*d*x*tan(1/2*d*x + 1/2*c)^2 - 240*b^2*d*x*tan
(1/2*c)^2 + 8*a*b*d^3 + 480*b^2*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 - 240*b^2
*d*x + 480*b^2*tan(1/2*d*x + 1/2*c))/(d^6*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)
^2 + d^6*tan(1/2*d*x + 1/2*c)^2 + d^6*tan(1/2*c)^2 + d^6)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)\,{\left(b\,x^3+a\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^3)^2)/x,x)
[Out] int((sin(c + d*x)*(a + b*x^3)^2)/x, x)
```

3.90 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^2} \, dx$

Optimal. Leaf size=145

$$-\frac{24b^2 \cos(c+dx)}{d^5} - \frac{2abx\cos(c+dx)}{d} + \frac{12b^2x^2\cos(c+dx)}{d^3} - \frac{b^2x^4\cos(c+dx)}{d} + a^2 d\cos(c)\text{Ci}(dx) + \frac{2ab\sin(c+}{d^2}$$

```
[Out] a^2*d*Ci(d*x)*cos(c)-24*b^2*cos(d*x+c)/d^5-2*a*b*x*cos(d*x+c)/d+12*b^2*x^2*
cos(d*x+c)/d^3-b^2*x^4*cos(d*x+c)/d-a^2*d*Si(d*x)*sin(c)+2*a*b*sin(d*x+c)/d
^2-a^2*sin(d*x+c)/x-24*b^2*x*sin(d*x+c)/d^4+4*b^2*x^3*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.15, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3420, 3378, 3384, 3380, 3383, 3377, 2717, 2718}

$$a^2 d\cos(c)\text{CosIntegral}(dx) - a^2 d\sin(c)\text{Si}(dx) - \frac{a^2\sin(c+dx)}{x} + \frac{2ab\sin(c+dx)}{d^2} - \frac{2abx\cos(c+dx)}{d} - \frac{24b^2\cos(c+dx)}{d^5} - \frac{24b^2x\sin(c+dx)}{d^4} + \frac{12b^2x^2\cos(c+dx)}{d^3} + \frac{4b^2x^3\sin(c+dx)}{d^2} - \frac{b^2x^4\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] (-24*b^2*Cos[c + d*x])/d^5 - (2*a*b*x*Cos[c + d*x])/d + (12*b^2*x^2*Cos[c +
 d*x])/d^3 - (b^2*x^4*Cos[c + d*x])/d + a^2*d*Cos[c]*CosIntegral[d*x] + (2*
a*b*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/x - (24*b^2*x*Sin[c + d*x])/d^4
+ (4*b^2*x^3*Sin[c + d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^2\sin(c+dx)}{x^2}\,dx &= \int \left(\frac{a^2\sin(c+dx)}{x^2}+2abx\sin(c+dx)+b^2x^4\sin(c+dx)\right)dx \\
&= a^2\int \frac{\sin(c+dx)}{x^2}\,dx+(2ab)\int x\sin(c+dx)\,dx+b^2\int x^4\sin(c+dx)\,dx \\
&= -\frac{2abx\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{x}+\frac{(2ab)\int\cos(c+d\ldots}{d} \\
&= -\frac{2abx\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}+\frac{2ab\sin(c+dx)}{d^2}-\frac{a^2\sin(c+dx)}{x}+\ldots \\
&= -\frac{2abx\cos(c+dx)}{d}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{b^2x^4\cos(c+dx)}{d}+a^2d\cos(c)\mathrm{Ci}(\ldots \\
&= -\frac{2abx\cos(c+dx)}{d}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{b^2x^4\cos(c+dx)}{d}+a^2d\cos(c)\mathrm{Ci}(\ldots \\
&= -\frac{24b^2\cos(c+dx)}{d^5}-\frac{2abx\cos(c+dx)}{d}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{b^2x^4\cos(c+\ldots}{d}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 145, normalized size = 1.00

$$-\frac{24b^2\cos(c+dx)}{d^5}-\frac{2abx\cos(c+dx)}{d}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{b^2x^4\cos(c+dx)}{d}+a^2d\cos(c)\mathrm{Ci}(dx)+\frac{2ab\sin(c+dx)}{d^2}-\frac{a^2\sin(c+dx)}{x}-\frac{24b^2x\sin(c+dx)}{d^4}+\frac{4b^2x^3\sin(c+dx)}{d^2}-a^2d\sin(c)\mathrm{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] (-24*b^2*Cos[c + d*x])/d^5 - (2*a*b*x*Cos[c + d*x])/d + (12*b^2*x^2*Cos[c + 
 d*x])/d^3 - (b^2*x^4*Cos[c + d*x])/d + a^2*d*Cos[c]*CosIntegral[d*x] + (2*
a*b*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/x - (24*b^2*x*Sin[c + d*x])/d^4
+ (4*b^2*x^3*Sin[c + d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(145) = 290.
time = 0.22, size = 365, normalized size = 2.52 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-6*c*b^2*(4*c^3+3*c^2+2*c+1)/d^6*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(
d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+a^2*(-sin(d*x+c)/d/x-Si(d*x)*sin(
c)+Ci(d*x)*cos(c))+15*(3*c^2+2*c+1)/d^6*b^2*c^2*(-(d*x+c)^2*cos(d*x+c)+2*co
s(d*x+c)+2*(d*x+c)*sin(d*x+c))+6/d^3*a*b*c*cos(d*x+c)+2*(2*c+1)/d^3*a*b*(si
n(d*x+c)-(d*x+c)*cos(d*x+c))+(5*c^4+4*c^3+3*c^2+2*c+1)/d^6*b^2*(-(d*x+c)^4*
cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)-24*
(d*x+c)*sin(d*x+c))-15/d^6*b^2*c^4*cos(d*x+c)-20*b^2*c^3*(2*c+1)/d^6*(sin(d
*x+c)-(d*x+c)*cos(d*x+c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 7.43, size = 129, normalized size = 0.89

$$\frac{\left(a^2(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos(c)+a^2(-i\,\Gamma(-1,i\,dx)+i\,\Gamma(-1,-i\,dx))\sin(c)\right)d^6-2\,(b^2d^4x^4+2\,abd^4x-12\,b^2d^2x^2+24\,b^2)\cos(dx+c)+4\,(2\,b^2d^3x^3+abd^3-12\,b^2dx)\sin(dx+c)}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) + a^2*(-I*gamma(-1,
 I*d*x) + I*gamma(-1, -I*d*x))*sin(c))*d^6 - 2*(b^2*d^4*x^4 + 2*a*b*d^4*x -
 12*b^2*d^2*x^2 + 24*b^2)*cos(d*x + c) + 4*(2*b^2*d^3*x^3 + a*b*d^3 - 12*b^
2*d*x)*sin(d*x + c))/d^5
```

Fricas [A]
time = 0.35, size = 145, normalized size = 1.00

$$-\frac{2\,a^2d^6x\sin(c)\,\mathrm{Si}(dx)+2\,(b^2d^4x^5+2\,abd^4x^2-12\,b^2d^2x^3+24\,b^2x)\cos(dx+c)-(a^2d^6x\,\mathrm{Ci}(dx)+a^2d^6x\,\mathrm{Ci}(-dx))\cos(c)-2\,(4\,b^2d^3x^4-a^2d^5+2\,abd^3x-24\,b^2dx^2)\sin(dx+c)}{2\,d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d^6*x*sin(c)*sin_integral(d*x) + 2*(b^2*d^4*x^5 + 2*a*b*d^4*x^2
 - 12*b^2*d^2*x^3 + 24*b^2*x)*cos(d*x + c) - (a^2*d^6*x*cos_integral(d*x) +
 a^2*d^6*x*cos_integral(-d*x))*cos(c) - 2*(4*b^2*d^3*x^4 - a^2*d^5 + 2*a*b*
d^3*x - 24*b^2*d*x^2)*sin(d*x + c))/(d^5*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b x^{3}\right)^{2} \sin{\left(c + d x \right)}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**3)**2*sin(c + d*x)/x**2, x)
```

Giac [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.97, size = 2038, normalized size = 14.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*d^4*x^5*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2
*d^6*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*t
an(1/2*c)^2 - a^2*d^6*x*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^
2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*d^4*x^5*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*d*x)^2 - 2*a^2*d^6*x*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*
tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d^6*x*imag_part(cos_integral(-d*x))*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*d^6*x*sin_integral(d*x)
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) + 2*b^2*d^4*x^5*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*c)^2 - 2*b^2*d^4*x^5*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^
2*d^6*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2
+ a^2*d^6*x*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*
x)^2 - a^2*d^6*x*real_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*c)^2 - a^2*d^6*x*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan
(1/2*c)^2 - a^2*d^6*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c
)^2 - a^2*d^6*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 16*b^2*d^3*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b*d^
4*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*d^4*x^5*ta
n(1/2*d*x + 1/2*c)^2 - 2*b^2*d^4*x^5*tan(1/2*d*x)^2 - 2*a^2*d^6*x*imag_part
```

```
(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) + 2*a^2*d^6*x*imag_pa
rt(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 4*a^2*d^6*x*sin_
integral(d*x)*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*a^2*d^6*x*imag_part(cos
_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d^6*x*imag_part(cos_integ
ral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*d^6*x*sin_integral(d*x)*tan(1/
2*d*x)^2*tan(1/2*c) - 2*b^2*d^4*x^5*tan(1/2*c)^2 - 24*b^2*d^2*x^3*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^2*d^6*x*real_part(cos_integra
l(d*x))*tan(1/2*d*x + 1/2*c)^2 + a^2*d^6*x*real_part(cos_integral(-d*x))*ta
n(1/2*d*x + 1/2*c)^2 + a^2*d^6*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^
2 + a^2*d^6*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 16*b^2*d^3*x^4
*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x)^2 + 4*a*b*d^4*x^2*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x)^2 + 4*a^2*d^5*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c
) - a^2*d^6*x*real_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^6*x*real_pa
rt(cos_integral(-d*x))*tan(1/2*c)^2 + 16*b^2*d^3*x^4*tan(1/2*d*x + 1/2*c)*t
an(1/2*c)^2 + 4*a*b*d^4*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 4*a^2*d^5
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)*tan(1/2*c)^2 - 4*a*b*d^4*x^2*tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*b^2*d^4*x^5 - 24*b^2*d^2*x^3*tan(1/2*d*x + 1/2*c)^2*
tan(1/2*d*x)^2 - 2*a^2*d^6*x*imag_part(cos_integral(d*x))*tan(1/2*c) + 2*a^
2*d^6*x*imag_part(cos_integral(-d*x))*tan(1/2*c) - 4*a^2*d^6*x*sin_integral
(d*x)*tan(1/2*c) - 24*b^2*d^2*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + 24*
b^2*d^2*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a*b*d^3*x*tan(1/2*d*x + 1/2*c)*
tan(1/2*d*x)^2*tan(1/2*c)^2 + a^2*d^6*x*real_part(cos_integral(d*x)) + a^2*
d^6*x*real_part(cos_integral(-d*x)) + 16*b^2*d^3*x^4*tan(1/2*d*x + 1/2*c) +
 4*a*b*d^4*x^2*tan(1/2*d*x + 1/2*c)^2 - 4*a^2*d^5*tan(1/2*d*x + 1/2*c)^2*ta
n(1/2*d*x) - 4*a*b*d^4*x^2*tan(1/2*d*x)^2 - 4*a^2*d^5*tan(1/2*d*x + 1/2*c)^
2*tan(1/2*c) + 4*a^2*d^5*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b*d^4*x^2*tan(1/2*
c)^2 + 4*a^2*d^5*tan(1/2*d*x)*tan(1/2*c)^2 - 96*b^2*d*x^2*tan(1/2*d*x + 1/2
*c)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*b^2*d^2*x^3*tan(1/2*d*x + 1/2*c)^2 + 2
4*b^2*d^2*x^3*tan(1/2*d*x)^2 + 8*a*b*d^3*x*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x
)^2 + 24*b^2*d^2*x^3*tan(1/2*c)^2 + 8*a*b*d^3*x*tan(1/2*d*x + 1/2*c)*tan(1/
2*c)^2 + 48*b^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*
b*d^4*x^2 - 4*a^2*d^5*tan(1/2*d*x) - 96*b^2*d*x^2*tan(1/2*d*x + 1/2*c)*tan(
1/2*d*x)^2 - 4*a^2*d^5*tan(1/2*c) - 96*b^2*d*x^2*tan(1/2*d*x + 1/2*c)*tan(1
/2*c)^2 + 24*b^2*d^2*x^3 + 8*a*b*d^3*x*tan(1/2*d*x + 1/2*c) + 48*b^2*x*tan(
1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2 + 48*b^2*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2
*c)^2 - 48*b^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 96*b^2*d*x^2*tan(1/2*d*x + 1
/2*c) + 48*b^2*x*tan(1/2*d*x + 1/2*c)^2 - 48*b^2*x*tan(1/2*d*x)^2 - 48*b^2*
x*tan(1/2*c)^2 - 48*b^2*x)/(d^5*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan
(1/2*c)^2 + d^5*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2 + d^5*x*tan(1/2*d*x
 + 1/2*c)^2*tan(1/2*c)^2 + d^5*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^5*x*tan(1/
2*d*x + 1/2*c)^2 + d^5*x*tan(1/2*d*x)^2 + d^5*x*tan(1/2*c)^2 + d^5*x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)\,\left(b\,x^3+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^3)^2)/x^2,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^3)^2)/x^2, x)
```

3.91 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3} \, dx$

Optimal. Leaf size=142

$$-\frac{2ab\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{2x}+\frac{6b^2x\cos(c+dx)}{d^3}-\frac{b^2x^3\cos(c+dx)}{d}-\frac{1}{2}a^2d^2\text{Ci}(dx)\sin(c)-\frac{6b^2\sin(c+dx)}{d^4}$$

```
[Out] -2*a*b*cos(d*x+c)/d-1/2*a^2*d*cos(d*x+c)/x+6*b^2*x*cos(d*x+c)/d^3-b^2*x^3*c
os(d*x+c)/d-1/2*a^2*d^2*cos(c)*Si(d*x)-1/2*a^2*d^2*Ci(d*x)*sin(c)-6*b^2*sin
(d*x+c)/d^4-1/2*a^2*sin(d*x+c)/x^2+3*b^2*x^2*sin(d*x+c)/d^2
```

Rubi [A]
time = 0.14, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3420, 2718, 3378, 3384, 3380, 3383, 3377, 2717}

$$-\frac{1}{2}a^2d^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)-\frac{a^2\sin(c+dx)}{2x^2}-\frac{a^2d\cos(c+dx)}{2x}-\frac{2ab\cos(c+dx)}{d}-\frac{6b^2\sin(c+dx)}{d^4}+\frac{6b^2x\cos(c+dx)}{d^3}+\frac{3b^2x^2\sin(c+dx)}{d^2}-\frac{b^2x^3\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] (-2*a*b*Cos[c + d*x])/d - (a^2*d*Cos[c + d*x])/(2*x) + (6*b^2*x*Cos[c + d*x
])/d^3 - (b^2*x^3*Cos[c + d*x])/d - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 - (
6*b^2*Sin[c + d*x])/d^4 - (a^2*Sin[c + d*x])/(2*x^2) + (3*b^2*x^2*Sin[c + d
*x])/d^2 - (a^2*d^2*Cos[c]*SinIntegral[d*x])/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3}\,dx &= \int \left(2ab\sin(c+dx)+\frac{a^2\sin(c+dx)}{x^3}+b^2x^3\sin(c+dx)\right)\,dx \\
&= a^2\int \frac{\sin(c+dx)}{x^3}\,dx+(2ab)\int \sin(c+dx)\,dx+b^2\int x^3\sin(c+dx)\,dx \\
&= -\frac{2ab\cos(c+dx)}{d}-\frac{b^2x^3\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{2x^2}+\frac{(3b^2)\int x^2\cos(c+
\\
&= -\frac{2ab\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{2x}-\frac{b^2x^3\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{2x^2}+
\\
&= -\frac{2ab\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{2x}+\frac{6b^2x\cos(c+dx)}{d^3}-\frac{b^2x^3\cos(c+dx)}{d}
\\
&= -\frac{2ab\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{2x}+\frac{6b^2x\cos(c+dx)}{d^3}-\frac{b^2x^3\cos(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 138, normalized size = 0.97

$$\frac{1}{2}\left(-\frac{4ab\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{x}+\frac{12b^2x\cos(c+dx)}{d^3}-\frac{2b^2x^3\cos(c+dx)}{d}-a^2d^2\mathrm{Ci}(dx)\sin(c)-\frac{12b^2\sin(c+dx)}{d^4}-\frac{a^2\sin(c+dx)}{x^2}+\frac{6b^2x^2\sin(c+dx)}{d^2}-a^2d^2\cos(c)\mathrm{Si}(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] ((-4*a*b*Cos[c + d*x])/d - (a^2*d*Cos[c + d*x])/x + (12*b^2*x*Cos[c + d*x])
/d^3 - (2*b^2*x^3*Cos[c + d*x])/d - a^2*d^2*CosIntegral[d*x]*Sin[c] - (12*b
^2*Sin[c + d*x])/d^4 - (a^2*Sin[c + d*x])/x^2 + (6*b^2*x^2*Sin[c + d*x])/d^
2 - a^2*d^2*Cos[c]*SinIntegral[d*x])/2
```

Maple [A]
time = 0.25, size = 251, normalized size = 1.77 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(a^2*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2
*Ci(d*x)*sin(c))-2*a*b*cos(d*x+c)/d^3+(10*c^3+6*c^2+3*c+1)/d^6*b^2*(-(d*x+c
)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+20
/d^6*b^2*c^3*cos(d*x+c)+15*(3*c+1)/d^6*b^2*c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+
c))-6*c*b^2*(6*c^2+3*c+1)/d^6*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)
*sin(d*x+c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 2.46, size = 110, normalized size = 0.77

$$\frac{\left(a^2\left(i\,\Gamma(-2,i\,dx)-i\,\Gamma(-2,-i\,dx)\right)\cos(c)+a^2\left(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx)\right)\sin(c)\right)d^6-2\left(b^2d^3x^3+2\,abd^3-6\,b^2dx\right)\cos(dx+c)+6\left(b^2d^2x^2-2\,b^2\right)\sin(dx+c)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(I*gamma(-2, I*d*x) - I*gamma(-2, -I*d*x))*cos(c) + a^2*(gamma(-2
, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^6 - 2*(b^2*d^3*x^3 + 2*a*b*d^3 - 6*
b^2*d*x)*cos(d*x + c) + 6*(b^2*d^2*x^2 - 2*b^2)*sin(d*x + c))/d^4
```

Fricas [A]
time = 0.36, size = 142, normalized size = 1.00

$$-\frac{2\,a^2d^6x^2\cos(c)\,\mathrm{Si}(dx)+2\left(2\,b^2d^3x^5+a^2d^5x+4\,abd^3x^2-12\,b^2dx^3\right)\cos(dx+c)-2\left(6\,b^2d^2x^4-a^2d^4-12\,b^2x^2\right)\sin(dx+c)+\left(a^2d^6x^2\,\mathrm{Ci}(dx)+a^2d^6x^2\,\mathrm{Ci}(-dx)\right)\sin(c)}{4\,d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*d^6*x^2*cos(c)*sin_integral(d*x) + 2*(2*b^2*d^3*x^5 + a^2*d^5*x
 + 4*a*b*d^3*x^2 - 12*b^2*d*x^3)*cos(d*x + c) - 2*(6*b^2*d^2*x^4 - a^2*d^4
- 12*b^2*x^2)*sin(d*x + c) + (a^2*d^6*x^2*cos_integral(d*x) + a^2*d^6*x^2*c
os_integral(-d*x))*sin(c))/(d^4*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**3)**2*sin(c + d*x)/x**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.86, size = 2171, normalized size = 15.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^6*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/
2*d*x)^2*tan(1/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^6*x^2*sin_integral(d*x)
*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^6*x^2*real_pa
rt(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) - 2*
a^2*d^6*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*
x)^2*tan(1/2*c) + 4*b^2*d^3*x^5*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1
/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*t
an(1/2*d*x)^2 + a^2*d^6*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2
*c)^2*tan(1/2*d*x)^2 - 2*a^2*d^6*x^2*sin_integral(d*x)*tan(1/2*d*x + 1/2*c)
^2*tan(1/2*d*x)^2 + a^2*d^6*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x +
1/2*c)^2*tan(1/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d
*x + 1/2*c)^2*tan(1/2*c)^2 + 2*a^2*d^6*x^2*sin_integral(d*x)*tan(1/2*d*x +
1/2*c)^2*tan(1/2*c)^2 + a^2*d^6*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*
x)^2*tan(1/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*a^2*d^6*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^
2 + 4*b^2*d^3*x^5*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2 - 2*a^2*d^6*x^2*rea
l_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*a^2*d^6*x^2
*real_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c) - 2*a^2*d^
6*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^6*x^
2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b^2*d^3*x^5*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 4*b^2*d^3*x^5*tan(1/2*d*x)^2*tan(1/2*c
```

```
)^2 - 2*a^2*d^5*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*
d^6*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x + 1/2*c)^2 + a^2*d^6*x^2*i
mag_part(cos_integral(-d*x))*tan(1/2*d*x + 1/2*c)^2 - 2*a^2*d^6*x^2*sin_int
egral(d*x)*tan(1/2*d*x + 1/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(d*x)
)*tan(1/2*d*x)^2 + a^2*d^6*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2
 - 2*a^2*d^6*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + a^2*d^6*x^2*imag_part(c
os_integral(d*x))*tan(1/2*c)^2 - a^2*d^6*x^2*imag_part(cos_integral(-d*x))*
tan(1/2*c)^2 + 2*a^2*d^6*x^2*sin_integral(d*x)*tan(1/2*c)^2 + 24*b^2*d^2*x^
4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a*b*d^3*x^2*tan(1/2*
d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*b^2*d^3*x^5*tan(1/2*d*x + 1/
2*c)^2 - 4*b^2*d^3*x^5*tan(1/2*d*x)^2 + 2*a^2*d^5*x*tan(1/2*d*x + 1/2*c)^2*
tan(1/2*d*x)^2 - 2*a^2*d^6*x^2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*
a^2*d^6*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a^2*d^5*x*tan(1/2*
d*x + 1/2*c)^2*tan(1/2*d*x)*tan(1/2*c) - 4*b^2*d^3*x^5*tan(1/2*c)^2 + 2*a^2
*d^5*x*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 - 2*a^2*d^5*x*tan(1/2*d*x)^2*tan
(1/2*c)^2 - 24*b^2*d*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2
 - a^2*d^6*x^2*imag_part(cos_integral(d*x)) + a^2*d^6*x^2*imag_part(cos_int
egral(-d*x)) - 2*a^2*d^6*x^2*sin_integral(d*x) + 24*b^2*d^2*x^4*tan(1/2*d*x
 + 1/2*c)*tan(1/2*d*x)^2 + 8*a*b*d^3*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x
)^2 + 4*a^2*d^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c) + 24*b^2*d
^2*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*c)^2 + 8*a*b*d^3*x^2*tan(1/2*d*x + 1/2*
c)^2*tan(1/2*c)^2 + 4*a^2*d^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x)*tan(1/2*c
)^2 - 8*a*b*d^3*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^2*d^3*x^5 - 2*a^2*d^5
*x*tan(1/2*d*x + 1/2*c)^2 + 2*a^2*d^5*x*tan(1/2*d*x)^2 - 24*b^2*d*x^3*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x)^2 + 8*a^2*d^5*x*tan(1/2*d*x)*tan(1/2*c) + 2*
a^2*d^5*x*tan(1/2*c)^2 - 24*b^2*d*x^3*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 +
 24*b^2*d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 24*b^2*d^2*x^4*tan(1/2*d*x + 1/
2*c) + 8*a*b*d^3*x^2*tan(1/2*d*x + 1/2*c)^2 - 4*a^2*d^4*tan(1/2*d*x + 1/2*c
)^2*tan(1/2*d*x) - 8*a*b*d^3*x^2*tan(1/2*d*x)^2 - 4*a^2*d^4*tan(1/2*d*x + 1
/2*c)^2*tan(1/2*c) + 4*a^2*d^4*tan(1/2*d*x)^2*tan(1/2*c) - 8*a*b*d^3*x^2*ta
n(1/2*c)^2 + 4*a^2*d^4*tan(1/2*d*x)*tan(1/2*c)^2 - 48*b^2*x^2*tan(1/2*d*x +
 1/2*c)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^5*x - 24*b^2*d*x^3*tan(1/2*d*
x + 1/2*c)^2 + 24*b^2*d*x^3*tan(1/2*d*x)^2 + 24*b^2*d*x^3*tan(1/2*c)^2 - 8*
a*b*d^3*x^2 - 4*a^2*d^4*tan(1/2*d*x) - 48*b^2*x^2*tan(1/2*d*x + 1/2*c)*tan(
1/2*d*x)^2 - 4*a^2*d^4*tan(1/2*c) - 48*b^2*x^2*tan(1/2*d*x + 1/2*c)*tan(1/2
*c)^2 + 24*b^2*d*x^3 - 48*b^2*x^2*tan(1/2*d*x + 1/2*c))/(d^4*x^2*tan(1/2*d*
x + 1/2*c)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^4*x^2*tan(1/2*d*x + 1/2*c)^2*t
an(1/2*d*x)^2 + d^4*x^2*tan(1/2*d*x + 1/2*c)^2*tan(1/2*c)^2 + d^4*x^2*tan(1
/2*d*x)^2*tan(1/2*c)^2 + d^4*x^2*tan(1/2*d*x + 1/2*c)^2 + d^4*x^2*tan(1/2*d
*x)^2 + d^4*x^2*tan(1/2*c)^2 + d^4*x^2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)\,\left(b\,x^3+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^3)^2)/x^3,x)
[Out] int((sin(c + d*x)*(a + b*x^3)^2)/x^3, x)
```

3.92 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4} \, dx$

Optimal. Leaf size=151

$$\frac{2b^2\cos(c+dx)}{d^3}-\frac{a^2d\cos(c+dx)}{6x^2}-\frac{b^2x^2\cos(c+dx)}{d}-\frac{1}{6}a^2d^3\cos(c)\text{Ci}(dx)+2ab\text{Ci}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{3x^3}+$$

```
[Out] -1/6*a^2*d^3*Ci(d*x)*cos(c)+2*b^2*cos(d*x+c)/d^3-1/6*a^2*d*cos(d*x+c)/x^2-b
^2*x^2*cos(d*x+c)/d+2*a*b*cos(c)*Si(d*x)+2*a*b*Ci(d*x)*sin(c)+1/6*a^2*d^3*S
i(d*x)*sin(c)-1/3*a^2*sin(d*x+c)/x^3+1/6*a^2*d^2*sin(d*x+c)/x+2*b^2*x*sin(d
*x+c)/d^2
```

Rubi [A]
time = 0.17, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3420, 3378, 3384, 3380, 3383, 3377, 2718}

$$-\frac{1}{6}a^2d^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{6x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{a^2d\cos(c+dx)}{6x^2}+2ab\sin(c)\text{CosIntegral}(dx)+2ab\cos(c)\text{Si}(dx)+\frac{2b^2\cos(c+dx)}{d^3}+\frac{2b^2x\sin(c+dx)}{d^2}-\frac{b^2x^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] (2*b^2*Cos[c + d*x])/d^3 - (a^2*d*Cos[c + d*x])/(6*x^2) - (b^2*x^2*Cos[c +
d*x])/d - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 + 2*a*b*CosIntegral[d*x]*Sin[
c] - (a^2*Sin[c + d*x])/(3*x^3) + (a^2*d^2*Sin[c + d*x])/(6*x) + (2*b^2*x*S
in[c + d*x])/d^2 + 2*a*b*Cos[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinInteg
ral[d*x])/6
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4}\,dx &= \int \left(\frac{a^2\sin(c+dx)}{x^4}+\frac{2ab\sin(c+dx)}{x}+b^2x^2\sin(c+dx)\right)dx \\
&= a^2\int \frac{\sin(c+dx)}{x^4}\,dx+(2ab)\int \frac{\sin(c+dx)}{x}\,dx+b^2\int x^2\sin(c+dx)\,dx \\
&= -\frac{b^2x^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{3x^3}+\frac{(2b^2)\int x\cos(c+dx)\,dx}{d}+\frac{1}{3}(a^2d)\int \\
&= -\frac{a^2d\cos(c+dx)}{6x^2}-\frac{b^2x^2\cos(c+dx)}{d}+2ab\text{Ci}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{3x^3}+ \\
&= \frac{2b^2\cos(c+dx)}{d^3}-\frac{a^2d\cos(c+dx)}{6x^2}-\frac{b^2x^2\cos(c+dx)}{d}+2ab\text{Ci}(dx)\sin(c)- \\
&= \frac{2b^2\cos(c+dx)}{d^3}-\frac{a^2d\cos(c+dx)}{6x^2}-\frac{b^2x^2\cos(c+dx)}{d}+2ab\text{Ci}(dx)\sin(c)- \\
&= \frac{2b^2\cos(c+dx)}{d^3}-\frac{a^2d\cos(c+dx)}{6x^2}-\frac{b^2x^2\cos(c+dx)}{d}-\frac{1}{6}a^2d^3\cos(c)\text{Ci}(dx)
\end{aligned}$$

Mathematica [A]

time = 0.35, size = 135, normalized size = 0.89

$$\frac{1}{6}\left(\frac{12b^2\cos(c+dx)}{d^3}-\frac{a^2d\cos(c+dx)}{x^2}-\frac{6b^2x^2\cos(c+dx)}{d}-a\mathrm{Ci}(dx)\left(ad^3\cos(c)-12b\sin(c)\right)-\frac{2a^2\sin(c+dx)}{x^3}+\frac{a^2d^2\sin(c+dx)}{x}+\frac{12b^2x\sin(c+dx)}{d^2}+a\left(12b\cos(c)+ad^3\sin(c)\right)\mathrm{Si}(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] ((12*b^2*Cos[c + d*x])/d^3 - (a^2*d*Cos[c + d*x])/x^2 - (6*b^2*x^2*Cos[c +
d*x])/d - a*CosIntegral[d*x]*(a*d^3*Cos[c] - 12*b*Sin[c]) - (2*a^2*Sin[c +
d*x])/x^3 + (a^2*d^2*Sin[c + d*x])/x + (12*b^2*x*Sin[c + d*x])/d^2 + a*(12*
b*Cos[c] + a*d^3*Sin[c])*SinIntegral[d*x])/6
```

Maple [A]
time = 0.24, size = 196, normalized size = 1.30

method	result
derivativedivides	$d^3\left(a^2\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{sinIntegral}(dx)\sin(c)}{6}-\frac{\mathrm{cosineIntegral}(dx)\cos(c)}{6}\right)+\frac{2ab(\mathrm{si}\right.$
default	$d^3\left(a^2\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(dx+c)}{6dx}+\frac{\mathrm{sinIntegral}(dx)\sin(c)}{6}-\frac{\mathrm{cosineIntegral}(dx)\cos(c)}{6}\right)+\frac{2ab(\mathrm{si}\right.$
risch	$\frac{\mathrm{expIntegral}(1,-idx)\cos(c)a^2d^3}{12}+\frac{\mathrm{expIntegral}(1,idx)\cos(c)a^2d^3}{12}+i\cos(c)\,\mathrm{expIntegral}(1,-idx)\,ab-i\,c$
meijerg	$\frac{4b^2\sqrt{\pi}\,\sin(c)\left(\frac{x(d^2)^{\frac{3}{2}}\cos(dx)}{2\sqrt{\pi}\,d^2}-\frac{(d^2)^{\frac{3}{2}}\left(-\frac{3d^2x^2}{2}+3\right)\sin(dx)}{6\sqrt{\pi}\,d^3}\right)}{d^2\sqrt{d^2}}+\frac{4b^2\sqrt{\pi}\,\cos(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{d^2x^2}{2}+1\right)\cos(dx)}{2\sqrt{\pi}}+\frac{d}{}\right.}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(a^2*(-1/3*sin(d*x+c)/d^3/x^3-1/6*cos(d*x+c)/d^2/x^2+1/6*sin(d*x+c)/d/x
+1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+2/d^3*a*b*(Si(d*x)*cos(c)+Ci(d*x)*s
in(c))+(10*c^2+4*c+1)/d^6*b^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)
*sin(d*x+c))-15/d^6*b^2*c^2*cos(d*x+c)-6*c*b^2*(4*c+1)/d^6*(sin(d*x+c)-(d*x
+c)*cos(d*x+c)))
```

Maxima [C] Result contains complex when optimal does not.
time = 8.16, size = 173, normalized size = 1.15

$$-\frac{\left(\left(a^2(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos(c)-a^2(i\,\Gamma(-3,i\,dx)-i\,\Gamma(-3,-i\,dx))\sin(c)\right)d^6+12\left(ab(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\cos(c)-ab(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\sin(c)\right)d^3\right)x^3+2\left(b^2d^2x^5+2\,abd^2x^2-2\,b^2x^3-4\,ab\right)\cos(dx+c)-4\left(b^2dx^4-abdx\right)\sin(dx+c)}{2\,d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) - a^2*(I*gamma(-3
, I*d*x) - I*gamma(-3, -I*d*x))*sin(c))*d^6 + 12*(a*b*(-I*gamma(-3, I*d*x)
+ I*gamma(-3, -I*d*x))*cos(c) - a*b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*
sin(c))*d^3)*x^3 + 2*(b^2*d^2*x^5 + 2*a*b*d^2*x^2 - 2*b^2*x^3 - 4*a*b)*cos(
d*x + c) - 4*(b^2*d*x^4 - a*b*d*x)*sin(d*x + c))/(d^3*x^3)
```

Fricas [A]
time = 0.36, size = 176, normalized size = 1.17

$$-\frac{2\,(6\,b^2d^2x^5+a^2d^4x-12\,b^2x^3)\cos\left(dx+c\right)+(a^2d^6x^3\operatorname{Ci}\left(dx\right)+a^2d^6x^3\operatorname{Ci}\left(-dx\right)-24\,abd^3x^3\operatorname{Si}\left(dx\right))\cos\left(c\right)-2\,(a^2d^5x^2+12\,b^2dx^4-2\,a^2d^3)\sin\left(dx+c\right)-2\,(a^2d^6x^3\operatorname{Si}\left(dx\right)+6\,abd^3x^3\operatorname{Ci}\left(dx\right)+6\,abd^3x^3\operatorname{Ci}\left(-dx\right))\sin\left(c\right)}{12\,d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(6*b^2*d^2*x^5 + a^2*d^4*x - 12*b^2*x^3)*cos(d*x + c) + (a^2*d^6*x
^3*cos_integral(d*x) + a^2*d^6*x^3*cos_integral(-d*x) - 24*a*b*d^3*x^3*sin_
integral(d*x))*cos(c) - 2*(a^2*d^5*x^2 + 12*b^2*d*x^4 - 2*a^2*d^3)*sin(d*x
+ c) - 2*(a^2*d^6*x^3*sin_integral(d*x) + 6*a*b*d^3*x^3*cos_integral(d*x) +
 6*a*b*d^3*x^3*cos_integral(-d*x))*sin(c))/(d^3*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^2\sin\left(c+dx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**3)**2*sin(c + d*x)/x**4, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.54, size = 1181, normalized size = 7.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a^2*d^6*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
+ a^2*d^6*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*a^2*d^6*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2
*d^6*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^
6*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^6*x^3*real_part(c
os_integral(d*x))*tan(1/2*d*x)^2 - a^2*d^6*x^3*real_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2 + a^2*d^6*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 +
```

```
a^2*d^6*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^6*x^3*imag
_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^6*x^3*imag_part(cos_integral(
-d*x))*tan(1/2*c) + 4*a^2*d^6*x^3*sin_integral(d*x)*tan(1/2*c) - 12*b^2*d^2
*x^5*tan(1/2*d*x)^2*tan(1/2*c)^2 - 12*a*b*d^3*x^3*imag_part(cos_integral(d*
x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a*b*d^3*x^3*imag_part(cos_integral(-d*
x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*a*b*d^3*x^3*sin_integral(d*x)*tan(1/2*
d*x)^2*tan(1/2*c)^2 - a^2*d^6*x^3*real_part(cos_integral(d*x)) - a^2*d^6*x^
3*real_part(cos_integral(-d*x)) - 4*a^2*d^5*x^2*tan(1/2*d*x)^2*tan(1/2*c) +
 24*a*b*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 24
*a*b*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^
2*d^5*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 12*b^2*d^2*x^5*tan(1/2*d*x)^2 + 12*a*
b*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 12*a*b*d^3*x^3*imag
_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 24*a*b*d^3*x^3*sin_integral(d*x)
*tan(1/2*d*x)^2 + 48*b^2*d^2*x^5*tan(1/2*d*x)*tan(1/2*c) + 12*b^2*d^2*x^5*t
an(1/2*c)^2 - 12*a*b*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 12
*a*b*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 24*a*b*d^3*x^3*si
n_integral(d*x)*tan(1/2*c)^2 - 2*a^2*d^4*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*
a^2*d^5*x^2*tan(1/2*d*x) + 4*a^2*d^5*x^2*tan(1/2*c) + 24*a*b*d^3*x^3*real_p
art(cos_integral(d*x))*tan(1/2*c) + 24*a*b*d^3*x^3*real_part(cos_integral(-
d*x))*tan(1/2*c) - 48*b^2*d*x^4*tan(1/2*d*x)^2*tan(1/2*c) - 48*b^2*d*x^4*ta
n(1/2*d*x)*tan(1/2*c)^2 - 12*b^2*d^2*x^5 + 12*a*b*d^3*x^3*imag_part(cos_int
egral(d*x)) - 12*a*b*d^3*x^3*imag_part(cos_integral(-d*x)) + 24*a*b*d^3*x^3
*sin_integral(d*x) + 2*a^2*d^4*x*tan(1/2*d*x)^2 + 8*a^2*d^4*x*tan(1/2*d*x)*
tan(1/2*c) + 2*a^2*d^4*x*tan(1/2*c)^2 + 24*b^2*x^3*tan(1/2*d*x)^2*tan(1/2*c
)^2 + 48*b^2*d*x^4*tan(1/2*d*x) + 48*b^2*d*x^4*tan(1/2*c) + 8*a^2*d^3*tan(1
/2*d*x)^2*tan(1/2*c) + 8*a^2*d^3*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a^2*d^4*x -
24*b^2*x^3*tan(1/2*d*x)^2 - 96*b^2*x^3*tan(1/2*d*x)*tan(1/2*c) - 24*b^2*x^3
*tan(1/2*c)^2 - 8*a^2*d^3*tan(1/2*d*x) - 8*a^2*d^3*tan(1/2*c) + 24*b^2*x^3)
/(d^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^3*x^3*tan(1/2*d*x)^2 + d^3*x^3*ta
n(1/2*c)^2 + d^3*x^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (c+d x)\ (b x^3+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^3)^2)/x^4,x)
[Out] int((sin(c + d*x)*(a + b*x^3)^2)/x^4, x)
```

3.93 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^5} \, dx$

Optimal. Leaf size=167

$$-\frac{a^2 d\cos(c+dx)}{12x^3}+\frac{a^2 d^3\cos(c+dx)}{24x}-\frac{b^2 x\cos(c+dx)}{d}+2abd\cos(c)\text{Ci}(dx)+\frac{1}{24}a^2 d^4\text{Ci}(dx)\sin(c)+\frac{b^2\sin(c+$$

```
[Out] 2*a*b*d*Ci(d*x)*cos(c)-1/12*a^2*d*cos(d*x+c)/x^3+1/24*a^2*d^3*cos(d*x+c)/x-
b^2*x*cos(d*x+c)/d+1/24*a^2*d^4*cos(c)*Si(d*x)+1/24*a^2*d^4*Ci(d*x)*sin(c)-
2*a*b*d*Si(d*x)*sin(c)+b^2*sin(d*x+c)/d^2-1/4*a^2*sin(d*x+c)/x^4+1/24*a^2*d
^2*sin(d*x+c)/x^2-2*a*b*sin(d*x+c)/x
```

Rubi [A]

time = 0.20, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3420, 3378, 3384, 3380, 3383, 3377, 2717}

$$\frac{1}{24}a^2 d^4\sin(c)\text{CosIntegral}(dx)+\frac{1}{24}a^2 d^4\cos(c)\text{Si}(dx)+\frac{a^2 d^3\cos(c+dx)}{24x}+\frac{a^2 d^2\sin(c+dx)}{24x^2}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{a^2 d\cos(c+dx)}{12x^3}+2abd\cos(c)\text{CosIntegral}(dx)-2abd\sin(c)\text{Si}(dx)-\frac{2ab\sin(c+dx)}{x}+\frac{b^2\sin(c+dx)}{d^2}-\frac{b^2 x\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] -1/12*(a^2*d*Cos[c + d*x])/x^3 + (a^2*d^3*Cos[c + d*x])/(24*x) - (b^2*x*Cos
[c + d*x])/d + 2*a*b*d*Cos[c]*CosIntegral[d*x] + (a^2*d^4*CosIntegral[d*x]*
Sin[c])/24 + (b^2*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/(4*x^4) + (a^2*d^2
*Sin[c + d*x])/(24*x^2) - (2*a*b*Sin[c + d*x])/x + (a^2*d^4*Cos[c]*SinInteg
ral[d*x])/24 - 2*a*b*d*Sin[c]*SinIntegral[d*x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3420

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^2\sin(c+dx)}{x^5}\,dx &= \int \left(\frac{a^2\sin(c+dx)}{x^5}+\frac{2ab\sin(c+dx)}{x^2}+b^2x\sin(c+dx)\right)dx \\
&= a^2\int \frac{\sin(c+dx)}{x^5}\,dx+(2ab)\int \frac{\sin(c+dx)}{x^2}\,dx+b^2\int x\sin(c+dx)\,dx \\
&= -\frac{b^2x\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{2ab\sin(c+dx)}{x}+\frac{b^2\int\cos(c+dx)\,dx}{d}+ \\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{b^2x\cos(c+dx)}{d}+\frac{b^2\sin(c+dx)}{d^2}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{2ab}{} \\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{b^2x\cos(c+dx)}{d}+2abd\cos(c)\text{Ci}(dx)+\frac{b^2\sin(c+dx)}{d^2}- \\
&= -\frac{a^2d\cos(c+dx)}{12x^3}+\frac{a^2d^3\cos(c+dx)}{24x}-\frac{b^2x\cos(c+dx)}{d}+2abd\cos(c)\text{Ci}(dx) \\
&= -\frac{a^2d\cos(c+dx)}{12x^3}+\frac{a^2d^3\cos(c+dx)}{24x}-\frac{b^2x\cos(c+dx)}{d}+2abd\cos(c)\text{Ci}(dx) \\
&= -\frac{a^2d\cos(c+dx)}{12x^3}+\frac{a^2d^3\cos(c+dx)}{24x}-\frac{b^2x\cos(c+dx)}{d}+2abd\cos(c)\text{Ci}(dx)
\end{aligned}$$

Mathematica [A]
time = 0.35, size = 148, normalized size = 0.89

$$\frac{1}{24}\left(-\frac{2a^2d\cos(c+dx)}{x^3}+\frac{a^2d^3\cos(c+dx)}{x}-\frac{24b^2x\cos(c+dx)}{d}+ad\mathrm{Ci}(dx)\left(48b\cos(c)+ad^3\sin(c)\right)+\frac{24b^2\sin(c+dx)}{d^2}-\frac{6a^2\sin(c+dx)}{x^4}+\frac{a^2d^2\sin(c+dx)}{x^2}-\frac{48ab\sin(c+dx)}{x}+ad\left(ad^3\cos(c)-48b\sin(c)\right)\mathrm{Si}(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] ((-2*a^2*d*Cos[c + d*x])/x^3 + (a^2*d^3*Cos[c + d*x])/x - (24*b^2*x*Cos[c +
 d*x])/d + a*d*CosIntegral[d*x]*(48*b*Cos[c] + a*d^3*Sin[c]) + (24*b^2*Sin[
c + d*x])/d^2 - (6*a^2*Sin[c + d*x])/x^4 + (a^2*d^2*Sin[c + d*x])/x^2 - (48
*a*b*Sin[c + d*x])/x + a*d*(a*d^3*Cos[c] - 48*b*Sin[c])*SinIntegral[d*x])/2
4
```

Maple [A]
time = 0.34, size = 167, normalized size = 1.00

method	result
derivativedivides	$d^4\left(a^2\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\mathrm{sinIntegral}(dx)\cos(c)}{24}+\frac{\mathrm{cosineIntegral}(dx)\,\mathrm{s}}{24}\right.\right.$
default	$d^4\left(a^2\left(-\frac{\sin(dx+c)}{4d^4x^4}-\frac{\cos(dx+c)}{12d^3x^3}+\frac{\sin(dx+c)}{24d^2x^2}+\frac{\cos(dx+c)}{24dx}+\frac{\mathrm{sinIntegral}(dx)\cos(c)}{24}+\frac{\mathrm{cosineIntegral}(dx)\,\mathrm{s}}{24}\right.\right.$
risch	$-\frac{i\cos(c)\,\mathrm{expIntegral}(1,idx)a^2d^4}{48}+\frac{i\cos(c)\,\mathrm{expIntegral}(1,-idx)a^2d^4}{48}-\cos(c)\,\mathrm{expIntegral}(1,idx)\,abd\,-$
meijerg	$\frac{2b^2\sqrt{\pi}\,\sin(c)\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(dx)}{2\sqrt{\pi}}+\frac{dx\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{2b^2\sqrt{\pi}\,\cos(c)\left(-\frac{dx\cos(dx)}{2\sqrt{\pi}}+\frac{\sin(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{d^2ab\sqrt{\pi}\,\sin(c)\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] d^4*(a^2*(-1/4*sin(d*x+c)/d^4/x^4-1/12*cos(d*x+c)/d^3/x^3+1/24*sin(d*x+c)/d
^2/x^2+1/24*cos(d*x+c)/d/x+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c))+2/d^3*a
*b*(-sin(d*x+c)/d/x-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+(5*c+1)/d^6*b^2*(sin(d*x
+c)-(d*x+c)*cos(d*x+c))+6/d^6*b^2*c*cos(d*x+c))
```

Maxima [C] Result contains complex when optimal does not.
time = 6.92, size = 164, normalized size = 0.98

$$\frac{\left(\left(a^2\left(-i\,\Gamma(-4,i\,dx)+i\,\Gamma(-4,-i\,dx)\right)\cos(c)-a^2\left(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx)\right)\sin(c)\right)d^7-48\left(ab\left(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx)\right)\cos(c)+ab\left(-i\,\Gamma(-4,i\,dx)+i\,\Gamma(-4,-i\,dx)\right)\sin(c)\right)d^4\right)x^4-2\left(b^2d^2x^5+2\,abd^2x^2-12\,ab\right)\cos(dx+c)+2\left(b^2dx^4-4\,abdx\right)\sin(dx+c)}{2\,d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*(((a^2*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x))*cos(c) - a^2*(gamma(
-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^7 - 48*(a*b*(gamma(-4, I*d*x) + g
amma(-4, -I*d*x))*cos(c) + a*b*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x))*
sin(c))*d^4)*x^4 - 2*(b^2*d^2*x^5 + 2*a*b*d^2*x^2 - 12*a*b)*cos(d*x + c) +
2*(b^2*d*x^4 - 4*a*b*d*x)*sin(d*x + c))/(d^3*x^4)
```

Fricas [A]
time = 0.37, size = 186, normalized size = 1.11

$$\frac{2\left(a^{2}d^{5}x^{3}-24\,b^{2}dx^{5}-2\,a^{2}d^{3}x\right)\cos\left(dx+c\right)+2\left(a^{2}d^{6}x^{4}\operatorname{Si}\left(dx\right)+24\,abd^{3}x^{4}\operatorname{Ci}\left(dx\right)+24\,abd^{3}x^{4}\operatorname{Ci}\left(-dx\right)\right)\cos\left(c\right)+2\left(a^{2}d^{4}x^{2}-48\,abd^{2}x^{3}+24\,b^{2}x^{4}-6\,a^{2}d^{2}\right)\sin\left(dx+c\right)+\left(a^{2}d^{6}x^{4}\operatorname{Ci}\left(dx\right)+a^{2}d^{6}x^{4}\operatorname{Ci}\left(-dx\right)-96\,abd^{3}x^{4}\operatorname{Si}\left(dx\right)\right)\sin\left(c\right)}{48\,d^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(a^2*d^5*x^3 - 24*b^2*d*x^5 - 2*a^2*d^3*x)*cos(d*x + c) + 2*(a^2*d^
6*x^4*sin_integral(d*x) + 24*a*b*d^3*x^4*cos_integral(d*x) + 24*a*b*d^3*x^4
*cos_integral(-d*x))*cos(c) + 2*(a^2*d^4*x^2 - 48*a*b*d^2*x^3 + 24*b^2*x^4
- 6*a^2*d^2)*sin(d*x + c) + (a^2*d^6*x^4*cos_integral(d*x) + a^2*d^6*x^4*co
s_integral(-d*x) - 96*a*b*d^3*x^4*sin_integral(d*x))*sin(c))/(d^2*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{3}\right)^{2}\sin\left(c + dx\right)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*sin(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**3)**2*sin(c + d*x)/x**5, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.38, size = 1255, normalized size = 7.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(a^2*d^6*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
 - a^2*d^6*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*a^2*d^6*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^6*x^4
*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^6*x^4*rea
l_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^6*x^4*imag_par
t(cos_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^6*x^4*imag_part(cos_integral(-d
*x))*tan(1/2*d*x)^2 - 2*a^2*d^6*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 + a^2*
```

```
d^6*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^6*x^4*imag_part(c
os_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^6*x^4*sin_integral(d*x)*tan(1/2*c
)^2 - 2*a^2*d^6*x^4*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^6*x^4
*real_part(cos_integral(-d*x))*tan(1/2*c) - 2*a^2*d^5*x^3*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 48*a*b*d^3*x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan
(1/2*c)^2 + 48*a*b*d^3*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan
(1/2*c)^2 - a^2*d^6*x^4*imag_part(cos_integral(d*x)) + a^2*d^6*x^4*imag_par
t(cos_integral(-d*x)) - 2*a^2*d^6*x^4*sin_integral(d*x) + 96*a*b*d^3*x^4*im
ag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 96*a*b*d^3*x^4*imag_
part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 192*a*b*d^3*x^4*sin_in
tegral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d^5*x^3*tan(1/2*d*x)^2 - 48*a
*b*d^3*x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 48*a*b*d^3*x^4*rea
l_part(cos_integral(-d*x))*tan(1/2*d*x)^2 + 8*a^2*d^5*x^3*tan(1/2*d*x)*tan(
1/2*c) + 2*a^2*d^5*x^3*tan(1/2*c)^2 + 48*a*b*d^3*x^4*real_part(cos_integral
(d*x))*tan(1/2*c)^2 + 48*a*b*d^3*x^4*real_part(cos_integral(-d*x))*tan(1/2*
c)^2 + 48*b^2*d*x^5*tan(1/2*d*x)^2*tan(1/2*c)^2 + 96*a*b*d^3*x^4*imag_part(
cos_integral(d*x))*tan(1/2*c) - 96*a*b*d^3*x^4*imag_part(cos_integral(-d*x)
)*tan(1/2*c) + 192*a*b*d^3*x^4*sin_integral(d*x)*tan(1/2*c) + 4*a^2*d^4*x^2
*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^4*x^2*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a^
2*d^5*x^3 - 48*a*b*d^3*x^4*real_part(cos_integral(d*x)) - 48*a*b*d^3*x^4*re
al_part(cos_integral(-d*x)) - 48*b^2*d*x^5*tan(1/2*d*x)^2 - 192*b^2*d*x^5*t
an(1/2*d*x)*tan(1/2*c) - 192*a*b*d^2*x^3*tan(1/2*d*x)^2*tan(1/2*c) - 48*b^2
*d*x^5*tan(1/2*c)^2 - 192*a*b*d^2*x^3*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a^2*d^3
*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^2*d^4*x^2*tan(1/2*d*x) - 4*a^2*d^4*x^2
*tan(1/2*c) + 96*b^2*x^4*tan(1/2*d*x)^2*tan(1/2*c) + 96*b^2*x^4*tan(1/2*d*x
)*tan(1/2*c)^2 + 48*b^2*d*x^5 + 192*a*b*d^2*x^3*tan(1/2*d*x) - 4*a^2*d^3*x*
tan(1/2*d*x)^2 + 192*a*b*d^2*x^3*tan(1/2*c) - 16*a^2*d^3*x*tan(1/2*d*x)*tan
(1/2*c) - 4*a^2*d^3*x*tan(1/2*c)^2 - 96*b^2*x^4*tan(1/2*d*x) - 96*b^2*x^4*t
an(1/2*c) - 24*a^2*d^2*tan(1/2*d*x)^2*tan(1/2*c) - 24*a^2*d^2*tan(1/2*d*x)*
tan(1/2*c)^2 + 4*a^2*d^3*x + 24*a^2*d^2*tan(1/2*d*x) + 24*a^2*d^2*tan(1/2*c
))/(d^2*x^4*tan(1/2*d*x)^2*tan(1/2*c)^2 + d^2*x^4*tan(1/2*d*x)^2 + d^2*x^4*
tan(1/2*c)^2 + d^2*x^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+dx)\,(b\,x^3+a)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*x^3)^2)/x^5,x)
```

```
[Out] int((sin(c + d*x)*(a + b*x^3)^2)/x^5, x)
```

3.94 $\int \frac{x^4 \sin(c+dx)}{a+bx^3}\,dx$

Optimal. Leaf size=371

$$-\frac{x\cos(c+dx)}{bd}+\frac{a^{2/3}\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{5/3}}+\frac{(-1)^{2/3}a^{2/3}\text{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{}}{\sqrt[3]{b}}\right.}{3b^{5/3}}$$

```
[Out] -x*cos(d*x+c)/b/d+1/3*(-1)^(2/3)*a^(2/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3)
)*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/b^(5/3)+1/3*a^(2/3)*cos(c-a^(1/3)*d
/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/b^(5/3)-1/3*(-1)^(1/3)*a^(2/3)*cos(c-(-
1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/b^(5/3)+1/
3*a^(2/3)*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/b^(5/3)+1/3*(-
1)^(2/3)*a^(2/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1
/3)*d/b^(1/3))/b^(5/3)-1/3*(-1)^(1/3)*a^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/
3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b^(5/3)+sin(d*x+c)/b/d^2
```

Rubi [A]
time = 0.61, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3426, 3377, 2717, 3384, 3380, 3383}

$$\frac{a^{2/3}\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}}+\frac{(-1)^{2/3}a^{2/3}\sin\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}}-\frac{\sqrt[3]{-1}\,a^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}}-\frac{(-1)^{2/3}a^{2/3}\cos\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}}+\frac{a^{2/3}\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{5/3}}-\frac{\sqrt[3]{-1}\,a^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{5/3}}+\frac{\sin(c+dx)}{bd^2}-\frac{x\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] -((x*Cos[c + d*x])/(b*d)) + (a^(2/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]
*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*b^(5/3)) + ((-1)^(2/3)*a^(2/3)*CosIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/
b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b^(5/3)) + Sin[c
 + d*x]/(b*d^2) - ((-1)^(2/3)*a^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3
)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(5/3)) + (a^(2/3
)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b
^(5/3)) - ((-1)^(1/3)*a^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIn
tegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(5/3))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4 \sin(c+dx)}{a+bx^3}\,dx = \int \left(\frac{x\sin(c+dx)}{b} - \frac{ax\sin(c+dx)}{b\,(a+bx^3)}\right)\,dx$$

$$= \frac{\int x\sin(c+dx)\,dx}{b} - \frac{a\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{b}$$

$$= -\frac{x\cos(c+dx)}{bd} - \frac{a\int \left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)} + \frac{}{3\sqrt[3]{}}\right.}{b}$$

$$= -\frac{x\cos(c+dx)}{bd} + \frac{\sin(c+dx)}{bd^2} + \frac{a^{2/3}\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}\,a^{2/3}\right)\int \frac{\sin(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}{3b^{4/3}}$$

$$= -\frac{x\cos(c+dx)}{bd} + \frac{\sin(c+dx)}{bd^2} + \frac{\left(a^{2/3}\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{4/3}} + \frac{\left(\sqrt[3]{-}\right.}{}$$

$$= -\frac{x\cos(c+dx)}{bd} + \frac{a^{2/3}\mathrm{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{5/3}} + \frac{(-1)^{2/3}a^{2/3}\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}}{\sqrt[3]{b}}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.33, size = 231, normalized size = 0.62

$$\frac{-iad^2\mathrm{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\mathrm{Ci}(d(x-\#1))-i\mathrm{Ci}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\mathrm{Si}(d(x-\#1))-\sin(c+d\#1)\mathrm{Si}(d(x-\#1))}{\#1}\&\right]+iad^2\mathrm{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\mathrm{Ci}(d(x-\#1))+i\mathrm{Ci}(d(x-\#1))\sin(c+d\#1)+i\cos(c+d\#1)\mathrm{Si}(d(x-\#1))-\sin(c+d\#1)\mathrm{Si}(d(x-\#1))}{\#1}\&\right]+6b(-dx\cos(c+dx)+\sin(c+dx))}{6b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] ((-I)*a*d^2*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] -
 I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x
 - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1 & ] + I*a*d^2*RootSum[a
 + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x -
 #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#
1]*SinIntegral[d*(x - #1)])/#1 & ] + 6*b*(-(d*x*Cos[c + d*x]) + Sin[c + d*x
]))/(6*b^2*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 558, normalized size = 1.50 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^5*(1/3*d^3*c^4/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci
(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
-4/3*d^3*c^3/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x
-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2*d
^3*c^2/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1
+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+4*d^3*c
/b*cos(d*x+c)+4/3/b^2*d^3*c*sum((-3*_R1^2*b*c+3*_R1*b*c^2+a*d^3-b*c^3)/(_R1
^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootO
f(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+(-3*cos(d*x+c)*d^3*c+d^3*(sin(
d*x+c)-(d*x+c)*cos(d*x+c)))/b-1/3/b^2*d^3*sum((-6*_R1^2*b*c^2+_R1*a*d^3+8*_
R1*b*c^3+3*a*c*d^3-3*b*c^4)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+C
i(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)
))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^4*cos(d*x + c) - (cos(c)^2 + sin(c)^2)*x^3*
sin(d*x + c) + ((d*x^4*cos(c) + x^3*sin(c))*cos(d*x + c)^2 + (d*x^4*cos(c)
+ x^3*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c)^2
)*d^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 +
b*sin(c)^2)*d^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integr
ate(-3/2*(a*d*x^3*cos(d*x + c) - a*x^2*sin(d*x + c))/(b^2*d^2*x^6 + 2*a*b*d
^2*x^3 + a^2*d^2), x) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^3 + (a*cos(c)^2
 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d^2*x^3 + (
a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-3/2*(a*d*x^3*cos(d
*x + c) - a*x^2*sin(d*x + c))/((b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2*d^2)*cos(
d*x + c)^2 + (b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2*d^2)*sin(d*x + c)^2), x) +
((d*x^4*sin(c) - x^3*cos(c))*cos(d*x + c)^2 + (d*x^4*sin(c) - x^3*cos(c))*s
in(d*x + c)^2)*sin(d*x + 2*c))/(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^3 + (a*cos
(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d^2*x^
3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Fricas [**C**] Result contains complex when optimal does not.
time = 0.42, size = 397, normalized size = 1.07

$$\frac{\left(\frac{i a d^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}+i\right)\mathrm{Ei}\left(-i d x+\frac{1}{2}\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i c\right)}-\left(-\frac{i a d^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}+i\right)\mathrm{Ei}\left(i d x+\frac{1}{2}\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)+i c\right)}-\left(\frac{i a d^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}-i\right)\mathrm{Ei}\left(-i d x+\frac{1}{2}\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)-i c\right)}+\left(-\frac{i a d^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}-i\right)\mathrm{Ei}\left(i d x+\frac{1}{2}\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)+i c\right)}-12 d x\cos\left(d x+c\right)+2 i\left(-\frac{i a d^3}{b}\right)^{\frac{2}{3}}\mathrm{Ei}\left(i d x+\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(i c-\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\right)}-2 i\left(\frac{i a d^3}{b}\right)^{\frac{2}{3}}\mathrm{Ei}\left(-i d x+\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(-i c-\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\right)}+12\sin\left(d x+c\right)}{12 b d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((I*a*d^3/b)^(2/3)*(sqrt(3) + I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I
*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - (-I*a*d^3/
b)^(2/3)*(sqrt(3) + I)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*
e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (I*a*d^3/b)^(2/3)*(sqrt(
3) - I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/
b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(2/3)*(sqrt(3) - I)*Ei(I*d*
x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*s
qrt(3) + 1) + I*c) - 12*d*x*cos(d*x + c) + 2*I*(-I*a*d^3/b)^(2/3)*Ei(I*d*x
+ (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*I*(I*a*d^3/b)^(2/3)*
Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) + 12*sin(d*x +
c))/(b*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sin(c+dx)}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sin(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x**4*sin(c + d*x)/(a + b*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^4*sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sin(c+d\,x)}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*sin(c + d*x))/(a + b*x^3),x)
```

```
[Out] int((x^4*sin(c + d*x))/(a + b*x^3), x)
```

3.95 $\int \frac{x^3 \sin(c+dx)}{a+bx^3} dx$

Optimal. Leaf size=357

$$-\frac{\cos(c+dx)}{bd}-\frac{\sqrt[3]{a}\,\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{4/3}}+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,\text{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}}{\sqrt[3]{b}}\right.$$

```
[Out] -cos(d*x+c)/b/d+1/3*(-1)^(1/3)*a^(1/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*
Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/b^(4/3)-1/3*a^(1/3)*cos(c-a^(1/3)*d/b
^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*cos(c-(-1)
^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/b^(4/3)-1/3*
a^(1/3)*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/b^(4/3)+1/3*(-1)
^(1/3)*a^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)
+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b^(4/3)
```

Rubi [A]
time = 0.42, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3426, 2718, 3414, 3384, 3380, 3383}

$$-\frac{\sqrt[3]{a}\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}}+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\sin\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}}-\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\cos\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}-\frac{\sqrt[3]{a}\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{4/3}}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] -(Cos[c + d*x]/(b*d)) - (a^(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - (a^(1/3)*d)/b^(1/3)])/(3*b^(4/3)) + ((-1)^(1/3)*a^(1/3)*CosIntegral[((
-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]
)/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1
/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b^(4/3)) - ((-1)^(1/
3)*a^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - (a^(1/3)*Cos[c - (a^(1/3)*d)/b^(1/
3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/
3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*
d)/b^(1/3) + d*x])/(3*b^(4/3))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3}\,dx = \int \left(\frac{\sin(c+dx)}{b} - \frac{a\sin(c+dx)}{b\,(a+bx^3)}\right) dx$$

$$= \frac{\int \sin(c+dx)\,dx}{b} - \frac{a\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{a\int\left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)} - \frac{\sin}{3a^{2/3}\left(-\sqrt[3]{a}\right.}\right.}{b}$$

$$= -\frac{\cos(c+dx)}{bd} + \frac{\sqrt[3]{a}\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{3b} + \frac{\sqrt[3]{a}\int}{-\sqrt[3]{a}}$$

$$= -\frac{\cos(c+dx)}{bd} + \frac{\left(\sqrt[3]{a}\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3b} - \frac{\left(\sqrt[3]{a}\cos\left(c+\frac{\sqrt[3]{-}}{}\right.\right.}{}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{\sqrt[3]{a}\,\mathrm{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b^{4/3}} + \frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right.}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.21, size = 216, normalized size = 0.61

$$-\frac{6b\cos(c+dx) + iad\,\mathrm{RootSum}\left[a+b\#1^3\&, \frac{\cos(c+d\#1)\mathrm{Ci}(d(x-\#1)) - i\,\mathrm{Ci}(d(x-\#1))\sin(c+d\#1) - i\cos(c+d\#1)\mathrm{Si}(d(x-\#1)) - \sin(c+d\#1)\mathrm{Si}(d(x-\#1))}{\#1^2}\&\right] - iad\,\mathrm{RootSum}\left[a+b\#1^3\&, \frac{\cos(c+d\#1)\mathrm{Ci}(d(x-\#1)) + i\,\mathrm{Ci}(d(x-\#1))\sin(c+d\#1) + i\cos(c+d\#1)\mathrm{Si}(d(x-\#1)) - \sin(c+d\#1)\mathrm{Si}(d(x-\#1))}{\#1^2}\&\right]}{6b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] -1/6*(6*b*Cos[c + d*x] + I*a*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosInt
egral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#
1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 &
] - I*a*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I
*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x -
 #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ])/(b^2*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.06, size = 392, normalized size = 1.10 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-1/3*d^3*c^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+C
i(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)
```

```
)+d^3*c^2/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R
1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-d^3*c/
b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin
(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-d^3/b*cos(d*x+
c)-1/3/b^2*d^3*sum((-3*_R1^2*b*c+3*_R1*b*c^2+a*d^3-b*c^3)/(_R1^2-2*_R1*c+c^
2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z
^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x^3*cos(d*x + c) + (x^3*cos(d*x + c)^2*cos(c) +
 x^3*cos(c)*sin(d*x + c)^2)*cos(d*x + 2*c) - 6*(((a*b*cos(c)^2 + a*b*sin(c)
^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((a*b*cos(c)^
2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)*
integrate(1/2*x^2*cos(d*x + c)/(b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d), x) - 6*((
(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(
d*x + c)^2 + ((a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin
(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*x^2*cos(d*x + c)/((b^2*d*x^6 + 2*a*
b*d*x^3 + a^2*d)*cos(d*x + c)^2 + (b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d)*sin(d*x
 + c)^2), x) + (x^3*cos(d*x + c)^2*sin(c) + x^3*sin(d*x + c)^2*sin(c))*sin(
d*x + 2*c))/(((b*cos(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d
)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c
)^2)*d)*sin(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.39, size = 393, normalized size = 1.10

$$\frac{\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\mathrm{Ei}\left(-i dx+\frac{1}{2}\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i c\right)}+\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\mathrm{Ei}\left(i dx+\frac{1}{2}\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)+i c\right)}+\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\mathrm{Ei}\left(-i dx+\frac{1}{2}\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)-i c\right)}+\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\mathrm{Ei}\left(i dx+\frac{1}{2}\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)+i c\right)}+2\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\mathrm{Ei}\left(i dx+\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(i c-\left(-\frac{i a d^3}{b}\right)^{\frac{1}{3}}\right)}+2\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\mathrm{Ei}\left(-i dx+\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(-i c-\left(\frac{i a d^3}{b}\right)^{\frac{1}{3}}\right)}-12\cos\left(dx+c\right)}{12\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*
(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (-I*a*d
^3/b)^(1/3)*(-I*sqrt(3) - 1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3)
- 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (I*a*d^3/b)^(1/3)*
(I*sqrt(3) - 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(
I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(1/3)*(I*sqrt(3) -
1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(
1/3)*(-I*sqrt(3) + 1) + I*c) + 2*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + (-I*a*d^3/b)
```

```
^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + 2*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + (I*a
*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) - 12*cos(d*x + c))/(b*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x**3+a),x)
[Out] Integral(x**3*sin(c + d*x)/(a + b*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(x^3*sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sin\left(c+d\,x\right)}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sin(c + d*x))/(a + b*x^3),x)
[Out] int((x^3*sin(c + d*x))/(a + b*x^3), x)
```

3.96 $\int \frac{x^2 \sin(c+dx)}{a+bx^3} dx$

Optimal. Leaf size=281

$$\frac{\operatorname{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b}+\frac{\operatorname{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b}+\frac{\operatorname{Ci}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin}{3b}$$

[Out] 1/3*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/b+1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/b+1/3*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/b+1/3*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/b+1/3*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/b+1/3*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/b

Rubi [A]
time = 0.29, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3426, 3384, 3380, 3383}

$$\frac{\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3b}+\frac{\sin\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3b}-\frac{\cos\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b}+\frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b}$$

Antiderivative was successfully verified.

[In] Int[(x^2*Sin[c + d*x])/(a + b*x^3),x]

[Out] (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*b) + (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*b) + (CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b) - (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b) + (Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b)

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosIntegral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) - c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2\sin(c+dx)}{a+bx^3}\,dx = \int \left(\frac{\sin(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sin(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sin(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{\right.}}$$

$$=\frac{\int\frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}}+\frac{\int\frac{\sin(c+dx)}{-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}}+\frac{\int\frac{\sin(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}}$$

$$=\frac{\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int\frac{\sin\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}}-\frac{\cos\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int\frac{\sin\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x}}{3b^{2/3}}$$

$$=\frac{\operatorname{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b}+\frac{\operatorname{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3b}+$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.17, size = 186, normalized size = 0.66

$$\frac{i\left(\text{RootSum}\left[a+b\#1^3\&,\cos(c+d\#1)\operatorname{Ci}(d(x-\#1))-i\operatorname{Ci}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\operatorname{Si}(d(x-\#1))-\sin(c+d\#1)\operatorname{Si}(d(x-\#1))\&\right]-\text{RootSum}\left[a+b\#1^3\&,\cos(c+d\#1)\operatorname{Ci}(d(x-\#1))+i\operatorname{Ci}(d(x-\#1))\sin(c+d\#1)+i\cos(c+d\#1)\operatorname{Si}(d(x-\#1))-\sin(c+d\#1)\operatorname{Si}(d(x-\#1))\&\right]\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] ((I/6)*(RootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*Co
sIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #1
)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)] & ] - RootSum[a + b*#1^3 & , Cos
[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c + d*#1
```

```
] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(
x - #1)] & ]))/b
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.05, size = 266, normalized size = 0.95

method	result
derivativedivides	$\frac{d^3c^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\frac{-\mathrm{sinIntegral}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{cosineIntegral}\left(dx-\_R\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
default	$\frac{d^3c^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\frac{-\mathrm{sinIntegral}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{cosineIntegral}\left(dx-\_R\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
risch	$\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-iad^3+ibc^3+b\_Z^3-3bc^2\_Z\right)}\frac{\_R1^2e^{\_R1}\mathrm{expIntegral}\left(1,-idx-ic+\_R1\right)}{-2ic\_R1+\_R1^2-c^2}\right)}{6b}-\frac{ic^2\left(\sum_{\_R1=\mathrm{Ro}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(1/3*d^3*c^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci
(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
-2/3*d^3*c/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_
R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/3*d
^3/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*
sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + (cos(c)^2 + sin(c)^2)*x*si
n(d*x + c) + ((d*x^2*cos(c) - x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c) - x*
sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d^2*
x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(
c)^2)*d^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-1
/2*(3*a*d*x*cos(d*x + c) - (2*b*x^3 - a)*sin(d*x + c))/(b^2*d^2*x^6 + 2*a*b
*d^2*x^3 + a^2*d^2), x) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^3 + (a*cos(c)
^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*d^2*x^3 +
 (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-1/2*(3*a*d*x*cos
```

```
(d*x + c) - (2*b*x^3 - a)*sin(d*x + c))/((b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2
*d^2)*cos(d*x + c)^2 + (b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2*d^2)*sin(d*x + c)
^2), x) + ((d*x^2*sin(c) + x*cos(c))*cos(d*x + c)^2 + (d*x^2*sin(c) + x*cos
(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b*cos(c)^2 + b*sin(c)^2)*d^2*x^3 +
(a*cos(c)^2 + a*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*
d^2*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

<u>Fricas</u> [C] Result contains complex when optimal does not.
time = 0.42, size = 292, normalized size = 1.04

$$\frac{i\,\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}+1\right)-i\,c\right)}-i\,\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}+1\right)+i\,c\right)}+i\,\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}+1\right)-i\,c\right)}-i\,\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}+1\right)+i\,c\right)}-i\,\mathrm{Ei}\left(i\,dx+\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(i\,c-\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)}+i\,\mathrm{Ei}\left(-i\,dx+\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(-i\,c-\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(I*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/
b)^(1/3)*(I*sqrt(3) + 1) - I*c) - I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*s
qrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + I*Ei(-I*d*x
 + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt
(3) + 1) - I*c) - I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1
/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - I*Ei(I*d*x + (-I*a*d^3/b)^(
1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + I*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I
*c - (I*a*d^3/b)^(1/3)))/b
```

<u>Sympy</u> [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sin(c + dx)}{a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x**3), x)
```

<u>Giac</u> [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sin\left(c+d\,x\right)}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sin(c + d*x))/(a + b*x^3),x)
```

```
[Out] int((x^2*sin(c + d*x))/(a + b*x^3), x)
```

3.97 $\int \frac{x\sin(c+dx)}{a+bx^3}\,dx$

Optimal. Leaf size=343

$$-\frac{\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{(-1)^{2/3}\text{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{\sqrt[3]{-1}\,\text{Ci}\left(\frac{(-1)^{2/3}\ldots}{\ldots}\right.}{}$$

```
[Out] -1/3*(-1)^(2/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*
d/b^(1/3)+d*x)/a^(1/3)/b^(2/3)-1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^
(1/3)+d*x)/a^(1/3)/b^(2/3)+1/3*(-1)^(1/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3
))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(2/3)-1/3*Ci(a^(1/3)*d/b^
(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(1/3)/b^(2/3)-1/3*(-1)^(2/3)*Ci((-1)^
(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^
(2/3)+1/3*(-1)^(1/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*
a^(1/3)*d/b^(1/3))/a^(1/3)/b^(2/3)
```

Rubi [A]
time = 0.26, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3426, 3384, 3380, 3383}

$$-\frac{\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{(-1)^{2/3}\sin\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{\sqrt[3]{-1}\,\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{(-1)^{2/3}\cos\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}\,b^{2/3}}+\frac{\sqrt[3]{-1}\,\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] -1/3*(CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/
(a^(1/3)*b^(2/3)) - ((-1)^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
- d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(1/3)*b^(2/3)) + ((-1)
^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3
)*a^(1/3)*d)/b^(1/3)])/(3*a^(1/3)*b^(2/3)) + ((-1)^(2/3)*Cos[c + ((-1)^(1/3
)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3
*a^(1/3)*b^(2/3)) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b
^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3
)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)
*b^(2/3))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(c+dx)}{a+bx^3}\,dx &= \int \left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\left(\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)}+\frac{\sqrt[3]{-1}\,\sin\cdots}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\left(\sqrt[3]{a}\cdots\right.}\right.\\ &= -\frac{\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}}+\frac{\sqrt[3]{-1}\,\int \frac{\sin(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}}-\frac{(-1)^{2/3}\int \frac{\sin(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}}\\ &= -\frac{\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}}-\frac{\left(\sqrt[3]{-1}\,\cos\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\cdots}{\sqrt[3]{b}}\right.}{\sqrt[3]{a}+(-1)\cdots}}{3\sqrt[3]{a}\,\sqrt[3]{b}}\\ &= -\frac{\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{(-1)^{2/3}\text{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{\cdots}}{\sqrt[3]{b}}\right.}{3\sqrt[3]{a}\,b^{2/3}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.16, size = 196, normalized size = 0.57

$$\frac{i\left(\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\text{Ci}(d(x-\#1))-i\text{Ci}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-\#1))-\sin(c+d\#1)\text{Si}(d(x-\#1))}{\#1}\&\right]-\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\text{Ci}(d(x-\#1))+i\text{Ci}(d(x-\#1))\sin(c+d\#1)+i\cos(c+d\#1)\text{Si}(d(x-\#1))-\sin(c+d\#1)\text{Si}(d(x-\#1))}{\#1}\&\right]\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] ((I/6)*(RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*C
osIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #
1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1 & ] - RootSum[a + b*#1^3 &
, (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c
+ d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegr
al[d*(x - #1)])/#1 & ]))/b
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.06, size = 176, normalized size = 0.51

method	result
derivativedivides	$\frac{d^3\left(\sum_{\_\_R1=\text{RootOf}\left(b\_\_Z^3-3cb\_\_Z^2+3b\,c^2\_\_Z+a\,d^3-b\,c^3\right)}\frac{\_\_R1\left(-\text{sinIntegral}\left(-dx+\_\_R1-c\right)\cos\left(\_\_R1\right)+\text{cosineIntegral}\left(\right.\right.}{\_\_R1^2-2\_\_R1c+c^2}}{3b}$
default	$\frac{d^3\left(\sum_{\_\_R1=\text{RootOf}\left(b\_\_Z^3-3cb\_\_Z^2+3b\,c^2\_\_Z+a\,d^3-b\,c^3\right)}\frac{\_\_R1\left(-\text{sinIntegral}\left(-dx+\_\_R1-c\right)\cos\left(\_\_R1\right)+\text{cosineIntegral}\left(\right.\right.}{\_\_R1^2-2\_\_R1c+c^2}}{3b}$
risch	$-\frac{d\left(\sum_{\_\_R1=\text{RootOf}\left(-3i\_\_Z^2bc-ia\,d^3+ib\,c^3+b\_\_Z^3-3b\,c^2\_\_Z\right)}\frac{\_\_R1\,e^{-\_\_R1}\,\text{expIntegral}\left(1,-idx-ic+\_\_R1\right)}{-2ic\_\_R1+\_\_R1^2-c^2}\right)}{6b}+\frac{idc\left(\_\_R\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(1/3*d^3/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d
*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1
/3*d^3*c/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c
)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*c
os(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b*cos(c)^2 + b*sin(c)^2)*d*x^3
+ (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*
d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(2*b*x^3
 - a)*cos(d*x + c)/(b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d), x) + 2*(((b*cos(c)^2
+ b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b*cos
(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^2)*in
```

```
tegrate(1/2*(2*b*x^3 - a)*cos(d*x + c)/((b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d)*c
os(d*x + c)^2 + (b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d)*sin(d*x + c)^2), x) + (x*
cos(d*x + c)^2*sin(c) + x*sin(d*x + c)^2*sin(c))*sin(d*x + 2*c))/(((b*cos(c
)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*cos(d*x + c)^2 + ((b
*cos(c)^2 + b*sin(c)^2)*d*x^3 + (a*cos(c)^2 + a*sin(c)^2)*d)*sin(d*x + c)^2
)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.40, size = 379, normalized size = 1.10

$$-\frac{\left(\frac{iad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}+i\right)\mathrm{Ei}\left(-idx+\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-ic\right)}-\left(-\frac{iad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}+i\right)\mathrm{Ei}\left(idx+\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)+ic\right)}-\left(\frac{iad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}-i\right)\mathrm{Ei}\left(-idx+\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)-ic\right)}+\left(-\frac{iad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}-i\right)\mathrm{Ei}\left(idx+\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)+ic\right)}+2i\left(-\frac{iad^3}{b}\right)^{\frac{2}{3}}\mathrm{Ei}\left(idx+\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(ic-\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)}-2i\left(\frac{iad^3}{b}\right)^{\frac{2}{3}}\mathrm{Ei}\left(-idx+\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(-ic-\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)}}{12\,ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*((I*a*d^3/b)^(2/3)*(sqrt(3) + I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-
I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - (-I*a*d^3
/b)^(2/3)*(sqrt(3) + I)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))
*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (I*a*d^3/b)^(2/3)*(sqrt
(3) - I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3
/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(2/3)*(sqrt(3) - I)*Ei(I*d
*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*
sqrt(3) + 1) + I*c) + 2*I*(-I*a*d^3/b)^(2/3)*Ei(I*d*x + (-I*a*d^3/b)^(1/3))
*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*I*(I*a*d^3/b)^(2/3)*Ei(-I*d*x + (I*a*d^3/
b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin\left(c+dx\right)}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x*sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sin\left(c+d\,x\right)}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(c + d*x))/(a + b*x^3),x)
[Out] int((x*sin(c + d*x))/(a + b*x^3), x)
```

3.98 $\int \frac{\sin(c+dx)}{a+bx^3}\,dx$

Optimal. Leaf size=343

$$\frac{\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\,\text{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\text{Ci}\left(\frac{(-1)^{2/3}\sqrt[3]{}}{\sqrt[3]{b}}\right.}{}$$

```
[Out] -1/3*(-1)^(1/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*
d/b^(1/3)+d*x)/a^(2/3)/b^(1/3)+1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^
(1/3)+d*x)/a^(2/3)/b^(1/3)+1/3*(-1)^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3
))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(1/3)+1/3*Ci(a^(1/3)*d/b^
(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(1/3)-1/3*(-1)^(1/3)*Ci((-1)^
(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^
(1/3)+1/3*(-1)^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*
a^(1/3)*d/b^(1/3))/a^(2/3)/b^(1/3)
```

Rubi [A]
time = 0.26, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3414, 3384, 3380, 3383}

$$\frac{\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\,\sin\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\sqrt[3]{-1}\,\cos\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x^3),x]
```

```
[Out] (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^
(2/3)*b^(1/3)) - ((-1)^(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d
*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2
/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a
^(1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) + ((-1)^(1/3)*Cos[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^
(2/3)*b^(1/3)) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1
/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d
)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^
(1/3))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)}-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2\ldots}\right.}\right.$$

$$=-\frac{\int\frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3a^{2/3}}-\frac{\int\frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{3a^{2/3}}-\frac{\int\frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}\,dx}{3a^{2/3}}$$

$$=-\frac{\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int\frac{\sin\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3a^{2/3}}+\frac{\cos\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int\frac{\sin\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}\ldots}{3a^{2/3}}$$

$$=\frac{\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\,\text{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\ldots\right.}{3a^{2/3}\sqrt[3]{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.10, size = 196, normalized size = 0.57

$$\frac{i\left(\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\text{Ci}(d(x-\#1))-i\text{Ci}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-\#1))-\sin(c+d\#1)\text{Si}(d(x-\#1))}{\#1^2}\&\right]-\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\text{Ci}(d(x-\#1))+i\text{Ci}(d(x-\#1))\sin(c+d\#1)+i\cos(c+d\#1)\text{Si}(d(x-\#1))-\sin(c+d\#1)\text{Si}(d(x-\#1))}{\#1^2}\&\right]\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*x^3),x]
```

```
[Out] ((I/6)*(RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*C
osIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #
```

```
1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] - RootSum[a + b*#1^3
& , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[
c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinInte
gral[d*(x - #1)])/#1^2 & ]))/b
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.05, size = 85, normalized size = 0.25

method	result
derivativedivides	$\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{-\mathrm{sinIntegral}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{cosineIntegral}\left(dx-\_R1\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
default	$\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{-\mathrm{sinIntegral}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{cosineIntegral}\left(dx-\_R1\right.}{\_R1^2-2\_R1c+c^2}\right.}{3b}$
risch	$-\frac{id^2\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-ia\,d^3+ib\,c^3+b\_Z^3-3b\,c^2\_Z\right)}\frac{e^{\_R1}\,\mathrm{expIntegral}\left(1,-idx-ic+\_R1\right)}{-2ic\_R1+\_R1^2-c^2}\right)}{6b}+\frac{id^2\left(\sum_{\_R1=\mathrm{Root}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*d^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)
*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*x^3 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.41, size = 385, normalized size = 1.12

$$\frac{\left(\frac{iad^3}{b}\right)^{\frac13}\left(i\sqrt{3}+1\right)\mathrm{Ei}\left(-idx+\frac12\left(\frac{iad^3}{b}\right)^{\frac13}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac12\left(\frac{iad^3}{b}\right)^{\frac13}\left(i\sqrt{3}+1\right)-ic\right)}+\left(-\frac{iad^3}{b}\right)^{\frac13}\left(i\sqrt{3}+1\right)\mathrm{Ei}\left(idx+\frac12\left(-\frac{iad^3}{b}\right)^{\frac13}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac12\left(-\frac{iad^3}{b}\right)^{\frac13}\left(i\sqrt{3}+1\right)+ic\right)}+\left(\frac{iad^3}{b}\right)^{\frac13}\left(-i\sqrt{3}+1\right)\mathrm{Ei}\left(-idx+\frac12\left(\frac{iad^3}{b}\right)^{\frac13}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac12\left(\frac{iad^3}{b}\right)^{\frac13}\left(-i\sqrt{3}+1\right)-ic\right)}+\left(-\frac{iad^3}{b}\right)^{\frac13}\left(-i\sqrt{3}+1\right)\mathrm{Ei}\left(idx+\frac12\left(-\frac{iad^3}{b}\right)^{\frac13}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac12\left(-\frac{iad^3}{b}\right)^{\frac13}\left(-i\sqrt{3}+1\right)+ic\right)}-2\left(-\frac{iad^3}{b}\right)^{\frac13}\mathrm{Ei}\left(idx+\left(-\frac{iad^3}{b}\right)^{\frac13}\right)e^{\left(ic-\left(-\frac{iad^3}{b}\right)^{\frac13}\right)}-2\left(\frac{iad^3}{b}\right)^{\frac13}\mathrm{Ei}\left(-idx+\left(\frac{iad^3}{b}\right)^{\frac13}\right)e^{\left(-ic-\left(\frac{iad^3}{b}\right)^{\frac13}\right)}}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(
-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (-I*a*d^
```

```
3/b)^(1/3)*(I*sqrt(3) + 1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) -
1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (I*a*d^3/b)^(1/3)*(-
I*sqrt(3) + 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I
*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(1/3)*(-I*sqrt(3) +
1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(
1/3)*(-I*sqrt(3) + 1) + I*c) - 2*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + (-I*a*d^3/b)
^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + (I*a
*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c + dx)}{a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x**3+a),x)
[Out] Integral(sin(c + d*x)/(a + b*x**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/(b*x^3 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c + d\,x\right)}{b\,x^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*x^3),x)
[Out] int(sin(c + d*x)/(a + b*x^3), x)
```

3.99 $\int \frac{\sin(c+dx)}{x(a+bx^3)}\,dx$

Optimal. Leaf size=301

$$\frac{\operatorname{Ci}(dx)\sin(c)}{a}-\frac{\operatorname{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a}-\frac{\operatorname{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a}-\frac{\operatorname{Ci}\left(\frac{(-1)^{2/3}}{\sqrt[3]{}}\right.}{}$$

```
[Out] cos(c)*Si(d*x)/a-1/3*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(
1/3)*d/b^(1/3)+d*x)/a-1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x
)/a-1/3*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)
+d*x)/a+Ci(d*x)*sin(c)/a-1/3*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1
/3))/a-1/3*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/
b^(1/3))/a-1/3*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3
)*d/b^(1/3))/a
```

Rubi [A]
time = 0.34, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3426, 3384, 3380, 3383}

$$-\frac{\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3a}-\frac{\sin\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3a}+\frac{\cos\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a}-\frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a}+\frac{\sin(c)\text{CosIntegral}(dx)}{a}+\frac{\cos(c)\text{Si}(dx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^3)),x]
```

```
[Out] (CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c
 - (a^(1/3)*d)/b^(1/3)])/(3*a) - (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3
) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a) - (CosIntegral[((-1
)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/
(3*a) + (Cos[c]*SinIntegral[d*x])/a + (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/
3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) - (Cos[c - (a^
(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a) - (Cos[c -
((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
+ d*x])/(3*a)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)}\,dx = \int \left(\frac{\sin(c+dx)}{ax} - \frac{bx^2\sin(c+dx)}{a\left(a+bx^3\right)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{x^2\sin(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{b\int \left(\frac{\sin(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\right)dx}{a} + \dots$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a} + \frac{\cos(c)\text{Si}(dx)}{a} - \frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a} - \frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a} - \dots$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a} + \frac{\cos(c)\text{Si}(dx)}{a} - \frac{\left(\sqrt[3]{b}\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a} + \frac{\left(\sqrt[3]{b}\,\text{co}\dots\right.}{\dots}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a} - \frac{\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a} - \frac{\text{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\dots\right.}{3a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.22, size = 206, normalized size = 0.68

$$\frac{-i\text{RootSum}\left[a+b\#1^3\&,\cos(c+d\#1)\text{Ci}(d(x-\#1))-i\text{Ci}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-\#1))-\sin(c+d\#1)\text{Si}(d(x-\#1))\&\right]+i\text{RootSum}\left[a+b\#1^3\&,\cos(c+d\#1)\text{Ci}(d(x-\#1))+i\text{Ci}(d(x-\#1))\sin(c+d\#1)+i\cos(c+d\#1)\text{Si}(d(x-\#1))-\sin(c+d\#1)\text{Si}(d(x-\#1))\&\right]+6\text{Ci}(dx)\sin(c)+6\cos(c)\text{Si}(dx)}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^3)),x]
```

```
[Out] ((-I)*RootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*CosI
ntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #1)]
 - Sin[c + d*#1]*SinIntegral[d*(x - #1)] & ] + I*RootSum[a + b*#1^3 & , Cos
[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c + d*#1
] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(
x - #1)] & ] + 6*CosIntegral[d*x]*Sin[c] + 6*Cos[c]*SinIntegral[d*x])/(6*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 88, normalized size = 0.29

method	result
derivativedivides	$-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\left(-\mathrm{sinIntegral}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{cosineIntegral}\left(dx-\_\right.\right.}{3a}$
default	$-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+ad^3-bc^3\right)}\left(-\mathrm{sinIntegral}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{cosineIntegral}\left(dx-\_\right.\right.}{3a}$
risch	$-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-iad^3+ibc^3+b\_Z^3-3bc^2\_Z\right)}\mathrm{e}^{\_R1}\,\mathrm{expIntegral}\left(1,-idx-ic+\_R1\right)\right)}{6a}+\frac{ie^{ic}\,\mathrm{expIntegral}}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a*sum(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1),_R1=RootOf(_Z^3*
b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.40, size = 314, normalized size = 1.04

$$\frac{-i\,\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-ic\right)}+i\,\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)+ic\right)}-i\,\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)-ic\right)}+i\,\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)+ic\right)}-3i\,\mathrm{Ei}(i\,dx)\,e^{(ic)}+3i\,\mathrm{Ei}(-i\,dx)\,e^{(-ic)}+i\,\mathrm{Ei}\left(i\,dx+\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(ic-\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)}-i\,\mathrm{Ei}\left(-i\,dx+\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(-ic-\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)}}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(-I*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3
/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*
sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - I*Ei(-I*d*
x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqr
t(3) + 1) - I*c) + I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(
1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 3*I*Ei(I*d*x)*e^(I*c) + 3*
I*Ei(-I*d*x)*e^(-I*c) + I*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3
/b)^(1/3)) - I*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))
/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin (c+dx)}{x\,(a+bx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin (c+d\,x)}{x\,\left(b\,x^3+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x*(a + b*x^3)),x)
```

```
[Out] int(sin(c + d*x)/(x*(a + b*x^3)), x)
```

3.100 $\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=380

$$\frac{d\cos(c)\mathrm{Ci}(dx)}{a}+\frac{\sqrt[3]{b}\,\mathrm{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\,\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right.}{3a^{4/3}}$$

```
[Out] d*Ci(d*x)*cos(c)/a+1/3*(-1)^(2/3)*b^(1/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3
))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)+1/3*b^(1/3)*cos(c-a^(1/3)*
d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)-1/3*(-1)^(1/3)*b^(1/3)*cos(c-(
-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)-d
*Si(d*x)*sin(c)/a+1/3*b^(1/3)*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(
1/3))/a^(4/3)+1/3*(-1)^(2/3)*b^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*s
in(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)-1/3*(-1)^(1/3)*b^(1/3)*Ci((-1)^(
2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)-sin
(d*x+c)/a/x
```

Rubi [A]
time = 0.39, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3426, 3378, 3384, 3380, 3383}

$$\frac{\sqrt[3]{b}\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\,\sqrt[3]{b}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{b}\cos\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\,\sqrt[3]{b}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{d\cos(c)\text{CosIntegral}(dx)}{a}-\frac{d\sin(c)\text{Si}(dx)}{a}-\frac{\sin(c+dx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^3)),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a + (b^(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) +
d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) + ((-1)^(2/3)*b^(1/3)*CosInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/
b^(1/3)])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)
*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) - S
in[c + d*x]/(a*x) - (d*Sin[c]*SinIntegral[d*x])/a - ((-1)^(2/3)*b^(1/3)*Cos
[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(
1/3) - d*x])/(3*a^(4/3)) + (b^(1/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegra
l[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*Cos[c - ((-
1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x])/(3*a^(4/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

]

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)}\,dx = \int \left(\frac{\sin(c+dx)}{ax^2} - \frac{bx\sin(c+dx)}{a\left(a+bx^3\right)}\right) dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{\sin(c+dx)}{ax} - \frac{b\int \left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\cdots}{3\sqrt[3]{a}\sqrt[3]{\cdots}}\right.}{a}$$

$$= -\frac{\sin(c+dx)}{ax} + \frac{b^{2/3}\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}\,b^{2/3}\right)\int \frac{\sin(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{4/3}} + \frac{\left((-1)^{2/3}b^{2\cdots}\right.}{}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a} - \frac{\sin(c+dx)}{ax} - \frac{d\sin(c)\text{Si}(dx)}{a} + \frac{\left(b^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+d\cdots\right)}{\sqrt[3]{a}+\sqrt[3]{b}\cdots}}{3a^{4/3}}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a} + \frac{\sqrt[3]{b}\,\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} + \frac{(-1)^{2/3}\sqrt[3]{b}\,\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-\cdots\right.}{3a^{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.26, size = 233, normalized size = 0.61

$$\frac{6dx\cos(c)\text{Ci}(dx) - ix\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\text{Ci}(d(x-\#1))-i\text{Ci}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-\#1))-\sin(c+d\#1)\text{Si}(d(x-\#1))}{\#1}\&\right] + ix\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\text{Ci}(d(x-\#1))+i\text{Ci}(d(x-\#1))\sin(c+d\#1)+i\cos(c+d\#1)\text{Si}(d(x-\#1))-\sin(c+d\#1)\text{Si}(d(x-\#1))}{\#1}\&\right] - 6\sin(c+dx) - 6dx\sin(c)\text{Si}(dx)}{6ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^3)),x]
```

```
[Out] (6*d*x*Cos[c]*CosIntegral[d*x] - I*x*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*
CosIntegral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c
 + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#
1 & ] + I*x*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] +
 I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x
 - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1 & ] - 6*Sin[c + d*x] -
6*d*x*Sin[c]*SinIntegral[d*x])/(6*a*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 116, normalized size = 0.31

method	result

derivativedivides	$d\left(-\frac{\sin(dx+c)}{adx}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{-\mathrm{sinIntegral}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{cosin}}{-\_R1+c}}{3a}\right.$
default	$d\left(-\frac{\sin(dx+c)}{adx}+\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{-\mathrm{sinIntegral}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{cosin}}{-\_R1+c}}{3a}\right.$
risch	$\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-ia\,d^3+ib\,c^3+b\_Z^3-3b\,c^2\_Z\right)}\frac{\mathrm{e}^{\_R1}\,\mathrm{expIntegral}\left(1,-idx-ic+\_R1\right)}{-ic+\_R1}\right)}{6a}-\frac{d\,\mathrm{e}^{ic}\,\mathrm{expIntegral}(1}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-sin(d*x+c)/a/d/x+1/3/a*sum(1/(-_R1+c)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-
_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a*
(-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x^2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.43, size = 454, normalized size = 1.19

$$\frac{6\,ad^3x\mathrm{Ei}(i\,dx)\,e^{(ic)}+6\,ad^3x\mathrm{Ei}(-i\,dx)\,e^{(-ic)}+2i\left(-\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}bx\mathrm{Ei}\left(i\,dx+\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(ic-\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)}-2i\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}bx\mathrm{Ei}\left(-i\,dx+\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(-ic-\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)}-12\,ad^2\sin(dx+c)+\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}\,bx+i\,bx\right)\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-ic\right)}-\left(-\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}\,bx+i\,bx\right)\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)+ic\right)}-\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}\,bx-i\,bx\right)\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)-ic\right)}+\left(-\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}\,bx-i\,bx\right)\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)+ic\right)}}{12\,a^2d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*d^3*x*Ei(I*d*x)*e^(I*c) + 6*a*d^3*x*Ei(-I*d*x)*e^(-I*c) + 2*I*(-I
*a*d^3/b)^(2/3)*b*x*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1
/3)) - 2*I*(I*a*d^3/b)^(2/3)*b*x*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (
I*a*d^3/b)^(1/3)) - 12*a*d^2*sin(d*x + c) + (I*a*d^3/b)^(2/3)*(sqrt(3)*b*x
+ I*b*x)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^
```

```
3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - (-I*a*d^3/b)^(2/3)*(sqrt(3)*b*x + I*b*x
)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(
1/3)*(I*sqrt(3) + 1) + I*c) - (I*a*d^3/b)^(2/3)*(sqrt(3)*b*x - I*b*x)*Ei(-I
*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*
sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(2/3)*(sqrt(3)*b*x - I*b*x)*Ei(I*d*x + 1
/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3
) + 1) + I*c))/(a^2*d^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x**3+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x**2*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)}{x^2\,\left(b\,x^3+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x^2*(a + b*x^3)),x)
```

```
[Out] int(sin(c + d*x)/(x^2*(a + b*x^3)), x)
```

3.101 $\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=408

$$-\frac{d\cos(c+dx)}{2ax}-\frac{d^2\text{Ci}(dx)\sin(c)}{2a}-\frac{b^{2/3}\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}\,b^{2/3}\text{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}$$

```
[Out] -1/2*d*cos(d*x+c)/a/x-1/2*d^2*cos(c)*Si(d*x)/a+1/3*(-1)^(1/3)*b^(2/3)*cos(c
+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3
)-1/3*b^(2/3)*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)-1/
3*(-1)^(2/3)*b^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1
/3)*d/b^(1/3)+d*x)/a^(5/3)-1/2*d^2*Ci(d*x)*sin(c)/a-1/3*b^(2/3)*Ci(a^(1/3)*
d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(5/3)+1/3*(-1)^(1/3)*b^(2/3)*Ci((
-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3
)-1/3*(-1)^(2/3)*b^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2
/3)*a^(1/3)*d/b^(1/3))/a^(5/3)-1/2*sin(d*x+c)/a/x^2
```

Rubi [A]

time = 0.43, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3426, 3378, 3384, 3380, 3383, 3414}

$$-\frac{b^{2/3}\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}\,b^{2/3}\sin\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}}-\frac{\sqrt[3]{-1}\,b^{2/3}\cos\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}+c\right)\text{Si}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{b^{2/3}\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{5/3}}-\frac{d^2\sin(c)\text{CosIntegral}(dx)}{2a}-\frac{d^2\cos(c)\text{Si}(dx)}{2a}-\frac{\sin(c+dx)}{2ax^2}-\frac{d\cos(c+dx)}{2ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(a*x) - (d^2*CosIntegral[d*x]*Sin[c])/(2*a) - (b^(2/3
)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a
^(5/3)) + ((-1)^(1/3)*b^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(5/3)) - ((-1)^(2/3)*b^(
2/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*
a^(1/3)*d)/b^(1/3)])/(3*a^(5/3)) - Sin[c + d*x]/(2*a*x^2) - (d^2*Cos[c]*Sin
Integral[d*x])/(2*a) - ((-1)^(1/3)*b^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b
^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) - (b
^(2/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])
/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*
SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

]

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)}\,dx = \int \left(\frac{\sin(c+dx)}{ax^3} - \frac{b\sin(c+dx)}{a\left(a+bx^3\right)}\right)\,dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{\sin(c+dx)}{2ax^2} - \frac{b\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)} - \frac{\sin(c+}{3a^{2/3}\left(-\sqrt[3]{a}-(\right.}}{a}$$

$$= -\frac{d\cos(c+dx)}{2ax} - \frac{\sin(c+dx)}{2ax^2} + \frac{b\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3a^{5/3}} + \frac{b\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{3a^{5/3}} + \frac{b\int}{}$$

$$= -\frac{d\cos(c+dx)}{2ax} - \frac{\sin(c+dx)}{2ax^2} - \frac{\left(d^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx}{2a} + \frac{\left(b\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\right.}{-\sqrt[3]{}}}{3a^{5/3}}$$

$$= -\frac{d\cos(c+dx)}{2ax} - \frac{d^2\text{Ci}(dx)\sin(c)}{2a} - \frac{b^{2/3}\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{3a^{5/3}} + \frac{\sqrt[3]{-1}\,b^{2/}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.28, size = 253, normalized size = 0.62

$$\frac{-ix^2\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\text{Ci}(d(x-\#1))-i\text{Ci}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-\#1))-\sin(c+d\#1)\text{Si}(d(x-\#1))}{\#1^2}\&\right]+ix^2\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\text{Ci}(d(x-\#1))+i\text{Ci}(d(x-\#1))\sin(c+d\#1)+i\cos(c+d\#1)\text{Si}(d(x-\#1))-\sin(c+d\#1)\text{Si}(d(x-\#1))}{\#1^2}\&\right]-3\left(dx\cos(c+dx)+d^2x^2\text{Ci}(dx)\sin(c)+\sin(c+dx)+d^2x^2\cos(c)\text{Si}(dx)\right)}{6ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-I)*x^2*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I
*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x -
 #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] + I*x^2*RootSum[a +
 b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #
1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]
*SinIntegral[d*(x - #1)])/#1^2 & ] - 3*(d*x*Cos[c + d*x] + d^2*x^2*CosInteg
ral[d*x]*Sin[c] + Sin[c + d*x] + d^2*x^2*Cos[c]*SinIntegral[d*x]))/(6*a*x^2
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 136, normalized size = 0.33

method	result

derivativedivides	$d^2\left(-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{-\mathrm{sinIntegral}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{cosineIntegral}\left(dx-\_\right.}{\_R1^2-2\_R1c+c^2}}{3a}\right.$
default	$d^2\left(-\frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{-\mathrm{sinIntegral}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{cosineIntegral}\left(dx-\_\right.}{\_R1^2-2\_R1c+c^2}}{3a}\right.$
risch	$\frac{id^2\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-ia\,d^3+ib\,c^3+b\_Z^3-3b\,c^2\_Z\right)}\frac{e^{\_R1}\mathrm{expIntegral}\left(1,-idx-ic+\_R1\right)}{-2ic\_R1+\_R1^2-c^2}\right)}{6a}-\frac{id^2e^{ic}\,\mathrm{expIntegral}(}{4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/3/a*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+
c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a*(-1/
2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(
c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x^3), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.41, size = 491, normalized size = 1.20

$$\frac{3i\,ad^3x^2\mathrm{Ei}(i\,dx)\,e^{(ic)}-3i\,ad^3x^2\mathrm{Ei}(-i\,dx)\,e^{(-ic)}+2\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}bx^2\mathrm{Ei}\left(i\,dx+\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(ic-\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)}+2\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}bx^2\mathrm{Ei}\left(-i\,dx+\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(-ic-\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)}-6\,ad^2x\cos(dx+c)+\left(-i\sqrt{3}\,bx^2-bx^2\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-ic\right)}+\left(-i\sqrt{3}\,bx^2-bx^2\right)\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)+ic\right)}+\left(i\sqrt{3}\,bx^2-bx^2\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)-ic\right)}+\left(i\sqrt{3}\,bx^2-bx^2\right)\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)+ic\right)}-6\,ad\sin(dx+c)}{12\,a^2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*I*a*d^3*x^2*Ei(I*d*x)*e^(I*c) - 3*I*a*d^3*x^2*Ei(-I*d*x)*e^(-I*c) +
 2*(-I*a*d^3/b)^(1/3)*b*x^2*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d
^3/b)^(1/3)) + 2*(I*a*d^3/b)^(1/3)*b*x^2*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(
-I*c - (I*a*d^3/b)^(1/3)) - 6*a*d^2*x*cos(d*x + c) + (-I*sqrt(3)*b*x^2 - b*
```

```
x^2)*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*
e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (-I*sqrt(3)*b*x^2 - b*x^2
)*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^
(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (I*sqrt(3)*b*x^2 - b*x^2)*
(I*a*d^3/b)^(1/3)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2
*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (I*sqrt(3)*b*x^2 - b*x^2)*(-I*
a*d^3/b)^(1/3)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-
I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 6*a*d*sin(d*x + c))/(a^2*d*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{\left(c + dx \right)}}{x^{3} \left(a + bx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**3/(b*x**3+a),x)
[Out] Integral(sin(c + d*x)/(x**3*(a + b*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)}{x^3\,\left(b\,x^3+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x^3*(a + b*x^3)),x)
[Out] int(sin(c + d*x)/(x^3*(a + b*x^3)), x)
```

3.102 $\int \frac{x^3 \sin(c+dx)}{\left(a+bx^3\right)^2} \, dx$

Optimal. Leaf size=714

$$-\frac{(-1)^{2/3} d \cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right) \text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}\,b^{5/3}}-\frac{d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{9\sqrt[3]{a}\,b^{5/3}}+\frac{\sqrt[3]{-1}\,d\cos\left(\cdots\right.}{\cdots}$$

```
[Out] -1/9*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(1/3)/b^(5/3)-1
/9*(-1)^(2/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/
3)*d/b^(1/3))/a^(1/3)/b^(5/3)+1/9*(-1)^(1/3)*d*Ci((-1)^(2/3)*a^(1/3)*d/b^(1
/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(5/3)-1/9*(-1)^(1/3)
*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/
a^(2/3)/b^(4/3)+1/9*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(2
/3)/b^(4/3)+1/9*(-1)^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3
)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(4/3)+1/9*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(
c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*d*Si(a^(1/3)*d/b^(1/3)+d*x)*sin(c-
a^(1/3)*d/b^(1/3))/a^(1/3)/b^(5/3)-1/9*(-1)^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b
^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*(-1)^(2
/3)*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1
/3))/a^(1/3)/b^(5/3)+1/9*(-1)^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*si
n(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/9*(-1)^(1/3)*d*Si((-1)^
(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^
(5/3)-1/3*x*sin(d*x+c)/b/(b*x^3+a)
```

Rubi [A]
time = 0.66, antiderivative size = 714, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3424, 3414, 3384, 3380, 3383, 3427}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -1/9*((-1)^(2/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1
)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(a^(1/3)*b^(5/3)) - (d*Cos[c - (a^(1/3)*
d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + (
(-1)^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3
)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + (CosIntegral[(a^(1/3)*d)
/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) - ((-1)^(
1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*
a^(1/3)*d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) + ((-1)^(2/3)*CosIntegral[((-1)^(2
/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a
```

```
^(2/3)*b^(4/3)) - (x*Sin[c + d*x])/(3*b*(a + b*x^3)) + ((-1)^(1/3)*Cos[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
- d*x])/(9*a^(2/3)*b^(4/3)) - ((-1)^(2/3)*d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/
b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(1/3)*b^(5
/3)) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x]
)/(9*a^(2/3)*b^(4/3)) + (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3
)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(2/3)*Cos[c - ((-1)^(2/3)*
a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a
^(2/3)*b^(4/3)) - ((-1)^(1/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Sin
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2}\,dx = -\frac{x\sin(c+dx)}{3b(a+bx^3)} + \frac{\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{3b} + \frac{d\int \frac{x\cos(c+dx)}{a+bx^3}\,dx}{3b}$$

$$= -\frac{x\sin(c+dx)}{3b(a+bx^3)} + \frac{\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)} - \frac{\sin(c}{3a^{2/3}\left(-\sqrt[3]{a}-\right.}\right.}{3b}$$

$$= -\frac{x\sin(c+dx)}{3b(a+bx^3)} - \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{9a^{2/3}b} - \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{9a^{2/3}b} - \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}}{9a^{2/3}b}$$

$$= -\frac{x\sin(c+dx)}{3b(a+bx^3)} - \frac{\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{9a^{2/3}b} - \frac{\left(d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos(}{\sqrt[3]{}}}{9\sqrt[3]{a}\,b^{4/3}}$$

$$= -\frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\operatorname{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}\,b^{5/3}} - \frac{d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\operatorname{Ci}\left(\frac{\sqrt[3]{}}{\sqrt[3]{}}\right.}{9\sqrt[3]{a}\,b^{5/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.15, size = 383, normalized size = 0.54

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] (RootSum[a + b*#1^3 & , (I*Cos[c + d*#1]*CosIntegral[d*(x - #1)] + CosInteg
ral[d*(x - #1)]*Sin[c + d*#1] + Cos[c + d*#1]*SinIntegral[d*(x - #1)] - I*S
in[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #
1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*d*Cos[c + d*#1]*S
inIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^
2 & ] + RootSum[a + b*#1^3 & , ((-I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)]
+ CosIntegral[d*(x - #1)]*Sin[c + d*#1] + Cos[c + d*#1]*SinIntegral[d*(x -
#1)] + I*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegra
l[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c
```

```
 + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1
)]*#1)/#1^2 & ] - (6*b*x*Sin[c + d*x])/(a + b*x^3))/(18*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 1184, normalized size = 1.66

method	result	size
derivativedivides	Expression too large to display	1184
default	Expression too large to display	1184
risch	Expression too large to display	1379

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-d^6*c^3*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d^3)/(a*d^3-b*c^3+3*
b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9/a/d^3/b*sum(1/(_R1^2-2*_R1*c
+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3
*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a/d^3/b*sum(1/(-_RR1+c)*(Si(-d*x+_RR
1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z
*b*c^2+a*d^3-b*c^3)))+sin(d*x+c)*(c^2*d^3/a*(d*x+c)^2-c^3*d^3/a*(d*x+c))/(a
*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+1/3*c^2*d^3/a/b*sum
((c+_R1)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R
1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/3*c^2*d^3/a/b*s
um(_RR1/(-_RR1+c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1
=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+sin(d*x+c)*(-2*c^2*d^3/a
*(d*x+c)^2+3*c^3*d^3/a*(d*x+c)+d^3*c*(a*d^3-b*c^3)/a/b)/(a*d^3-b*c^3+3*b*c^
2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)-2/3*c^2*d^3/a/b*sum(_R1/(_R1^2-2*_R1
*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b
-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/3*d^3*c/a/b^2*sum((-2*_RR1^2*b*c+3*_
RR1*b*c^2+a*d^3-b*c^3)/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(
d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3
))+sin(d*x+c)*(c^2*d^3/a*(d*x+c)^2-1/3*d^3*(a*d^3+5*b*c^3)/a/b*(d*x+c)-2/3*
d^3*c*(a*d^3-b*c^3)/a/b)/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*
x+c)^3)+1/9*d^3/a/b^2*sum((3*_R1*b*c^2+a*d^3-b*c^3)/(_R1^2-2*_R1*c+c^2)*(-S
i(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c
+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*d^3/a/b^2*sum((-3*_RR1^2*b*c^2+_RR1*a*d^3+5*_
RR1*b*c^3+2*a*c*d^3-2*b*c^4)/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR
1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*(cos(c)^2 + sin(c)^2)*d*x^2*sin(d*x + c) + ((d^2*x^3*cos(c) - 3*d*x
^2*sin(c) - 12*x*cos(c))*cos(d*x + c)^2 + (d^2*x^3*cos(c) - 3*d*x^2*sin(c)
- 12*x*cos(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((cos(c)^2 + sin(c)^2)*d^2*
x^3 - 12*(cos(c)^2 + sin(c)^2)*x)*cos(d*x + c) - 2*(((b^2*cos(c)^2 + b^2*si
n(c)^2)*d^3*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^3 + (a^2*cos(c)^2 +
 a^2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^6
 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*
d^3)*sin(d*x + c)^2)*integrate(3*(3*a*d*x*sin(d*x + c) + (a*d^2*x^2 + 10*b*
x^3 - 2*a)*cos(d*x + c))/(b^3*d^3*x^9 + 3*a*b^2*d^3*x^6 + 3*a^2*b*d^3*x^3 +
 a^3*d^3), x) - 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^6 + 2*(a*b*cos(c)^2
 + a*b*sin(c)^2)*d^3*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*cos(d*x + c)^
2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2
)*d^3*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(3*
(3*a*d*x*sin(d*x + c) + (a*d^2*x^2 + 10*b*x^3 - 2*a)*cos(d*x + c))/((b^3*d^
3*x^9 + 3*a*b^2*d^3*x^6 + 3*a^2*b*d^3*x^3 + a^3*d^3)*cos(d*x + c)^2 + (b^3*
d^3*x^9 + 3*a*b^2*d^3*x^6 + 3*a^2*b*d^3*x^3 + a^3*d^3)*sin(d*x + c)^2), x)
+ ((d^2*x^3*sin(c) + 3*d*x^2*cos(c) - 12*x*sin(c))*cos(d*x + c)^2 + (d^2*x^
3*sin(c) + 3*d*x^2*cos(c) - 12*x*sin(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/((
(b^2*cos(c)^2 + b^2*sin(c)^2)*d^3*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3
*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^2*cos(c)^2 +
 b^2*sin(c)^2)*d^3*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^3*x^3 + (a^2*cos
(c)^2 + a^2*sin(c)^2)*d^3)*sin(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.40, size = 670, normalized size = 0.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*a*d*x*sin(d*x + c) + ((b*x^3 - sqrt(3)*(I*b*x^3 + I*a) + a)*(I*a*
d^3/b)^(2/3) - (b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)*(I*a*d^3/b)^(1/3))*Ei(
-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(
I*sqrt(3) + 1) - I*c) + ((b*x^3 - sqrt(3)*(I*b*x^3 + I*a) + a)*(-I*a*d^3/b)
^(2/3) - (b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x
 + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sq
rt(3) + 1) + I*c) + ((b*x^3 - sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a*d^3/b)^(2/
3) - (b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x +
1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3)
 + 1) - I*c) + ((b*x^3 - sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I*a*d^3/b)^(2/3) -
 (b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*
(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) +
 1) + I*c) - 2*((b*x^3 + a)*(-I*a*d^3/b)^(2/3) - (b*x^3 + a)*(-I*a*d^3/b)^(
```

```
1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*((b*x
^3 + a)*(I*a*d^3/b)^(2/3) - (b*x^3 + a)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a
*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a*b^2*d*x^3 + a^2*b*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x**3+a)**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(x^3*sin(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sin\left(c+d\,x\right)}{{\left(b\,x^3+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sin(c + d*x))/(a + b*x^3)^2,x)
[Out] int((x^3*sin(c + d*x))/(a + b*x^3)^2, x)
```

3.103 $\int \frac{x^2 \sin(c+dx)}{\left(a+bx^3\right)^2} \, dx$

Optimal. Leaf size=371

$$-\frac{\sqrt[3]{-1}\, d \cos\left(c+\frac{\sqrt[3]{-1}\, \sqrt[3]{a}\, d}{\sqrt[3]{b}}\right) \text{Ci}\left(\frac{\sqrt[3]{-1}\, \sqrt[3]{a}\, d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}+\frac{d \cos\left(c-\frac{\sqrt[3]{a}\, d}{\sqrt[3]{b}}\right) \text{Ci}\left(\frac{\sqrt[3]{a}\, d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}+\frac{(-1)^{2/3} d \cos\left(\right.}{}$$

```
[Out] 1/9*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/
9*(-1)^(1/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*(-1)^(2/3)*d*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/
3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/9*d*Si(a^(1/3
)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*(-1)^(1/3)*d*
Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a
^(2/3)/b^(4/3)-1/9*(-1)^(2/3)*d*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-
(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/3*sin(d*x+c)/b/(b*x^3+a)
```

Rubi [A]
time = 0.37, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3422, 3415, 3384, 3380, 3383}

$$-\frac{\sqrt[3]{-1}\, d \cos\left(\frac{\sqrt[3]{-1}\, \sqrt[3]{a}\, d}{\sqrt[3]{b}}+c\right) \text{CosIntegral}\left(\frac{\sqrt[3]{-1}\, \sqrt[3]{a}\, d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}+\frac{d \cos\left(c-\frac{\sqrt[3]{a}\, d}{\sqrt[3]{b}}\right) \text{CosIntegral}\left(\frac{\sqrt[3]{a}\, d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}+\frac{(-1)^{2/3} d \cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\, d}{\sqrt[3]{b}}\right) \text{CosIntegral}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\, d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}-\frac{\sqrt[3]{-1}\, d \sin\left(\frac{\sqrt[3]{-1}\, \sqrt[3]{a}\, d}{\sqrt[3]{b}}+c\right) \text{Si}\left(\frac{\sqrt[3]{-1}\, \sqrt[3]{a}\, d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}-\frac{d \sin\left(c-\frac{\sqrt[3]{a}\, d}{\sqrt[3]{b}}\right) \text{Si}\left(xd+\frac{\sqrt[3]{a}\, d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}-\frac{(-1)^{2/3} d \sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}\, d}{\sqrt[3]{b}}\right) \text{Si}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}\, d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}-\frac{\sin(c+dx)}{3b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -1/9*((-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1
)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(a^(2/3)*b^(4/3)) + (d*Cos[c - (a^(1/3)*
d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) + (
(-1)^(2/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3
)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) - Sin[c + d*x]/(3*b*(a + b
*x^3)) - ((-1)^(1/3)*d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(2/3)*b^(4/3)) - (d*Sin[c - (a^
(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3
)) - ((-1)^(2/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1
)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3))
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^2} dx = -\frac{\sin(c+dx)}{3b(a+bx^3)} + \frac{d \int \frac{\cos(c+dx)}{a+bx^3} dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3b(a+bx^3)} + \frac{d \int \left(-\frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cos(}{3a^{2/3}\left(-\sqrt[3]{a}\right.} \right.}{3b}$$

$$= -\frac{\sin(c+dx)}{3b(a+bx^3)} - \frac{d \int \frac{\cos(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x} dx}{9a^{2/3}b} - \frac{d \int \frac{\cos(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x} dx}{9a^{2/3}b} - \frac{d \int \frac{\cos(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{}}}{9a^{2/3}b}$$

$$= -\frac{\sin(c+dx)}{3b(a+bx^3)} - \frac{\left(d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x} dx}{9a^{2/3}b} - \frac{\left(d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right.\right.}{}$$

$$= -\frac{\sqrt[3]{-1}\, d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} + \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Ci}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}\right.}{9a^{2/3}b^{4/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.13, size = 214, normalized size = 0.58

$$\frac{d\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\text{Ci}(d(x-\#1))-i\text{Ci}(d(x-\#1))\sin(c+d\#1)-i\cos(c+d\#1)\text{Si}(d(x-\#1))-\sin(c+d\#1)\text{Si}(d(x-\#1))}{\#1^2}\&\right]+d\text{RootSum}\left[a+b\#1^3\&,\frac{\cos(c+d\#1)\text{Ci}(d(x-\#1))+i\text{Ci}(d(x-\#1))\sin(c+d\#1)+i\cos(c+d\#1)\text{Si}(d(x-\#1))-\sin(c+d\#1)\text{Si}(d(x-\#1))}{\#1^2}\&\right]-\frac{6b\sin(c+dx)}{a+bx^3}}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] (d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*CosInt
egral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] -
 Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] + d*RootSum[a + b*#1^3 & ,
 (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c +
 d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegra
l[d*(x - #1)])/#1^2 & ] - (6*b*Sin[c + d*x])/(a + b*x^3))/(18*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 823, normalized size = 2.22

method	result
derivativedivides	Expression too large to display
default	Expression too large to display

risch	$-\frac{\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-ia\,d^3+ib\,c^3+b\_Z^3-3b\,c^2\_Z\right)}\frac{\left(-3ib\_R1\,c^2+2\_R1^2bc+a\,d^3-b\,c^3-2b\_R1c\right)\mathrm{e}^{-\_R1}\,\mathrm{expInteg}}{2ic\_R1-\_R1^2+c^2}}{18a\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(d^6*c^2*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d^3)/(a*d^3-b*c^3+3*b
*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9/a/d^3/b*sum(1/(_R1^2-2*_R1*c+
c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*
_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a/d^3/b*sum(1/(-_RR1+c)*(Si(-d*x+_RR1
-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*
b*c^2+a*d^3-b*c^3)))+sin(d*x+c)*(-2/3*d^3*c/a*(d*x+c)^2+2/3*c^2*d^3/a*(d*x+
c))/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)-2/9*d^3*c/a/b
*sum((c+_R1)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*si
n(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-2/9*d^3*c/a/b
*sum(_RR1/(-_RR1+c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_R
R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+sin(d*x+c)*(2/3*d^3*c/
a*(d*x+c)^2-c^2*d^3/a*(d*x+c)-1/3*d^3*(a*d^3-b*c^3)/a/b)/(a*d^3-b*c^3+3*b*c
^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9*d^3*c/a/b*sum(_R1/(_R1^2-2*_R1*
c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-
3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*d^3/a/b^2*sum((-2*_RR1^2*b*c+3*_RR1
*b*c^2+a*d^3-b*c^3)/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x
-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + 4*(cos(c)^2 + sin(c)^2)*x*
sin(d*x + c) + ((d*x^2*cos(c) - 4*x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c)
- 4*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^2*cos(c)^2 + b^2*sin(
c)^2)*d^2*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^3 + (a^2*cos(c)^2 + a
^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^6 +
 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^
2)*sin(d*x + c)^2)*integrate(-(3*a*d*x*cos(d*x + c) - 2*(5*b*x^3 - a)*sin(d
*x + c))/(b^3*d^2*x^9 + 3*a*b^2*d^2*x^6 + 3*a^2*b*d^2*x^3 + a^3*d^2), x) +
2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)
*d^2*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c)
^2 + b^2*sin(c)^2)*d^2*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^3 + (a^2
```

```
*cos(c)^2 + a^2*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-(3*a*d*x*cos(d*x
+ c) - 2*(5*b*x^3 - a)*sin(d*x + c))/((b^3*d^2*x^9 + 3*a*b^2*d^2*x^6 + 3*a^
2*b*d^2*x^3 + a^3*d^2)*cos(d*x + c)^2 + (b^3*d^2*x^9 + 3*a*b^2*d^2*x^6 + 3*
a^2*b*d^2*x^3 + a^3*d^2)*sin(d*x + c)^2), x) + ((d*x^2*sin(c) + 4*x*cos(c))
*cos(d*x + c)^2 + (d*x^2*sin(c) + 4*x*cos(c))*sin(d*x + c)^2)*sin(d*x + 2*c
))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d^2*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2
)*d^2*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d^2)*cos(d*x + c)^2 + ((b^2*cos(c
)^2 + b^2*sin(c)^2)*d^2*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d^2*x^3 + (a^
2*cos(c)^2 + a^2*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.41, size = 482, normalized size = 1.30

$$\frac{\left(-i\,bx^3+\sqrt{3}\,(bx^3+a)-i\,a\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}+1\right)-i\,c\right)}+\left(i\,bx^3-\sqrt{3}\,(bx^3+a)+i\,a\right)\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}+1\right)+i\,c\right)}+\left(-i\,bx^3-\sqrt{3}\,(bx^3+a)-i\,a\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}+1\right)-i\,c\right)}+\left(i\,bx^3+\sqrt{3}\,(bx^3+a)+i\,a\right)\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}+1\right)+i\,c\right)}-2\,(i\,bx^3+i\,a)\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\mathrm{Ei}\left(i\,dx+\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(i\,c-\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)}-2\,(-i\,bx^3-i\,a)\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\mathrm{Ei}\left(-i\,dx+\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(-i\,c-\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)}-12\,a\sin\left(dx+c\right)}{36\,(ab^2x^3+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*((-I*b*x^3 + sqrt(3)*(b*x^3 + a) - I*a)*(I*a*d^3/b)^(1/3)*Ei(-I*d*x +
1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3)
 + 1) - I*c) + (I*b*x^3 - sqrt(3)*(b*x^3 + a) + I*a)*(-I*a*d^3/b)^(1/3)*Ei(
I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*
(I*sqrt(3) + 1) + I*c) + (-I*b*x^3 - sqrt(3)*(b*x^3 + a) - I*a)*(I*a*d^3/b)
^(1/3)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b
)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (I*b*x^3 + sqrt(3)*(b*x^3 + a) + I*a)*(-I
*a*d^3/b)^(1/3)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(
-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 2*(I*b*x^3 + I*a)*(-I*a*d^3/b)^
(1/3)*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*(-I*b
*x^3 - I*a)*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a
*d^3/b)^(1/3)) - 12*a*sin(d*x + c))/(a*b^2*x^3 + a^2*b)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sin\left(c+d\,x\right)}{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sin(c + d*x))/(a + b*x^3)^2,x)
```

```
[Out] int((x^2*sin(c + d*x))/(a + b*x^3)^2, x)
```

3.104 $\int \frac{x \sin(c+dx)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=691

$$-\frac{d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9ab}-\frac{d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{9ab}-\frac{d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}}{\sqrt[3]{b}}\right.}{}$$

```
[Out] -1/9*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a/b-1/9*d*Ci((-1)
^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a/b-1/9*d
*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a
/b-1/9*(-1)^(2/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3
)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)-1/9*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/
b^(1/3)+d*x)/a^(4/3)/b^(2/3)+1/9*(-1)^(1/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1
/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)-1/9*Ci(a^(1/3)*d/
b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/9*d*Si(a^(1/3)*d/b^
(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a/b-1/9*(-1)^(2/3)*Ci((-1)^(1/3)*a^(1/3
)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/9*d*
Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a
/b+1/9*(-1)^(1/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(
1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/9*d*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*
sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a/b+1/3*sin(d*x+c)/a/b/x-1/3*sin(d*x+c)
/b/x/(b*x^3+a)
```

Rubi [A]

time = 0.86, antiderivative size = 691, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3424, 3426, 3378, 3384, 3380, 3383, 3427}

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -1/9*(d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(
1/3)*d)/b^(1/3) - d*x])/(a*b) - (d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral
[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) - (d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) - (CosInt
egral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b
^(2/3)) - ((-1)^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin
[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*Cos
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*
d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) + Sin[c + d*x]/(3*a*b*x) - Sin[c + d*x]/(3
*b*x*(a + b*x^3)) + ((-1)^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*Sin
```

```
Integral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) - (d*Si
n[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^
(1/3) - d*x])/(9*a*b) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*
d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) + (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*Si
nIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) + ((-1)^(1/3)*Cos[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])
/(9*a^(4/3)*b^(2/3)) + (d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegr
al[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx &= -\frac{\sin(c+dx)}{3bx\,(a+bx^3)} - \frac{\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx}{3b} + \frac{d\int \frac{\cos(c+dx)}{x(a+bx^3)}\,dx}{3b} \\
&= -\frac{\sin(c+dx)}{3bx\,(a+bx^3)} - \frac{\int \left(\frac{\sin(c+dx)}{ax^2} - \frac{bx\sin(c+dx)}{a(a+bx^3)}\right)dx}{3b} + \frac{d\int \left(\frac{\cos(c+dx)}{ax} - \frac{bx^2\cos(c+dx)}{a(a+bx^3)}\right)dx}{3b} \\
&= -\frac{\sin(c+dx)}{3bx\,(a+bx^3)} + \frac{\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{3a} - \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{3ab} - \frac{d\int \frac{x^2\cos(c+dx)}{a+bx^3}\,dx}{3a} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{3ab} \\
&= \frac{\sin(c+dx)}{3abx} - \frac{\sin(c+dx)}{3bx\,(a+bx^3)} + \frac{\int \left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\right.}\right.}{3a} \\
&= \frac{d\cos(c)\text{Ci}(dx)}{3ab} + \frac{\sin(c+dx)}{3abx} - \frac{\sin(c+dx)}{3bx\,(a+bx^3)} - \frac{d\sin(c)\text{Si}(dx)}{3ab} - \frac{\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{4/3}\sqrt[3]{b}} + \\
&= \frac{\sin(c+dx)}{3abx} - \frac{\sin(c+dx)}{3bx\,(a+bx^3)} - \frac{\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{4/3}\sqrt[3]{b}} - \frac{\left(d\cos\left(c-\frac{\sqrt[3]{a}}{\sqrt[3]{b}}\right.\right.}{} \\
&= -\frac{d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9ab} - \frac{d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right.}{9ab}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.15, size = 408, normalized size = 0.59

$$-\frac{(a+bx^3)\,\text{RootSum}\left[a+b\#1^3\&,\frac{-i\cos(c+d\#1)\text{Ci}(d(x-\#1))-\text{Ci}(d(x-\#1))\sin(c+d\#1)-\cos(c+d\#1)\text{Si}(d(x-\#1))+i\sin(c+d\#1)\text{Si}(d(x-\#1))+d\cos(c+d\#1)\text{Ci}(d(x-\#1))\#1-id\,\text{Ci}(d(x-\#1))\sin(c+d\#1)\#1-id\cos(c+d\#1)\text{Si}(d(x-\#1))\#1-d\sin(c+d\#1)\text{Si}(d(x-\#1))\#1}{\#1}\&\right]+(a+bx^3)\,\text{RootSum}\left[a+b\#1^3\&,\frac{i\cos(c+d\#1)\text{Ci}(d(x-\#1))-\text{Ci}(d(x-\#1))\sin(c+d\#1)-\cos(c+d\#1)\text{Si}(d(x-\#1))-i\sin(c+d\#1)\text{Si}(d(x-\#1))+d\cos(c+d\#1)\text{Ci}(d(x-\#1))\#1+id\,\text{Ci}(d(x-\#1))\sin(c+d\#1)\#1+id\cos(c+d\#1)\text{Si}(d(x-\#1))\#1-d\sin(c+d\#1)\text{Si}(d(x-\#1))\#1}{\#1}\&\right]-6bx^2\sin(c+dx)}{18ab\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -1/18*((a + b*x^3)*RootSum[a + b*#1^3 & , ((-I)*Cos[c + d*#1]*CosIntegral[d
*(x - #1)] - CosIntegral[d*(x - #1)]*Sin[c + d*#1] - Cos[c + d*#1]*SinInteg
ral[d*(x - #1)] + I*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]
*CosIntegral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1
- I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegra
l[d*(x - #1)]*#1)/#1 & ] + (a + b*x^3)*RootSum[a + b*#1^3 & , (I*Cos[c + d*
#1]*CosIntegral[d*(x - #1)] - CosIntegral[d*(x - #1)]*Sin[c + d*#1] - Cos[c
 + d*#1]*SinIntegral[d*(x - #1)] - I*Sin[c + d*#1]*SinIntegral[d*(x - #1)]
+ d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)]*
Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c +
 d*#1]*SinIntegral[d*(x - #1)]*#1)/#1 & ] - 6*b*x^2*Sin[c + d*x])/(a*b*(a +
 b*x^3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 508, normalized size = 0.74 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(sin(d*x+c)*(1/3*d^3/a*(d*x+c)^2-1/3*d^3*c/a*(d*x+c))/(a*d^3-b*c^3+3*
b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+1/9*d^3/a/b*sum((c+_R1)/(_R1^2-2
*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z
^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*d^3/a/b*sum(_RR1/(-_RR1+c)*(Si
(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2
*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-d^6*c*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/
d^3)/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9/a/d^3/b*
sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1))
,_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a/d^3/b*sum(1/(-
_RR1+c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z
^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*c
os(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*
x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)
*d)*cos(d*x + c)^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(a*b*cos(c)^2
 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)*i
ntegrate(1/2*(5*b*x^3 - a)*cos(d*x + c)/(b^3*d*x^9 + 3*a*b^2*d*x^6 + 3*a^2*
```

```
b*d*x^3 + a^3*d), x) + 2*(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(a*b*cos
(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)
^2 + ((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)
*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(5*
b*x^3 - a)*cos(d*x + c)/((b^3*d*x^9 + 3*a*b^2*d*x^6 + 3*a^2*b*d*x^3 + a^3*d
)*cos(d*x + c)^2 + (b^3*d*x^9 + 3*a*b^2*d*x^6 + 3*a^2*b*d*x^3 + a^3*d)*sin(
d*x + c)^2), x) + (x*cos(d*x + c)^2*sin(c) + x*sin(d*x + c)^2*sin(c))*sin(d
*x + 2*c))/(((b^2*cos(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(a*b*cos(c)^2 + a*b*si
n(c)^2)*d*x^3 + (a^2*cos(c)^2 + a^2*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^2*cos
(c)^2 + b^2*sin(c)^2)*d*x^6 + 2*(a*b*cos(c)^2 + a*b*sin(c)^2)*d*x^3 + (a^2*
cos(c)^2 + a^2*sin(c)^2)*d)*sin(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.41, size = 655, normalized size = 0.95

$$\frac{12\,abd^2x^2\sin\left(dx+c\right) - \left(2\,abd^3x^3 + 2\,a^2d^3 - \left(-i\,b^2x^3 - i\,ab - \sqrt{3}\left(b^2x^3+ab\right)\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\right){\rm Ei}\left(-i\,dx + \frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}+1\right)-i\,c\right)} - \left(2\,abd^3x^3 + 2\,a^2d^3 - \left(i\,b^2x^3 + i\,ab + \sqrt{3}\left(b^2x^3+ab\right)\right)\left(-\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\right){\rm Ei}\left(i\,dx + \frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}+1\right)+i\,c\right)} - \left(2\,abd^3x^3 + 2\,a^2d^3 - \left(-i\,b^2x^3 - i\,ab + \sqrt{3}\left(b^2x^3+ab\right)\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\right){\rm Ei}\left(-i\,dx + \frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}+1\right)-i\,c\right)} - \left(2\,abd^3x^3 + 2\,a^2d^3 - \left(i\,b^2x^3 + i\,ab - \sqrt{3}\left(b^2x^3+ab\right)\right)\left(-\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\right){\rm Ei}\left(i\,dx + \frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}+1\right)+i\,c\right)} - 2\left(abd^3x^3 + a^2d^3 + \left(i\,b^2x^3 + i\,ab\right)\left(-\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\right){\rm Ei}\left(i\,dx + \left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(i\,c - \left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)} - 2\left(abd^3x^3 + a^2d^3 + \left(-i\,b^2x^3 - i\,ab\right)\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\right){\rm Ei}\left(-i\,dx + \left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(-i\,c - \left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right)}}{36\left(a^2b^2d^2x^3 + a^3bd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*b*d^2*x^2*sin(d*x + c) - (2*a*b*d^3*x^3 + 2*a^2*d^3 - (-I*b^2*x^
3 - I*a*b - sqrt(3)*(b^2*x^3 + a*b))*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + 1/2*(I*
a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) -
 I*c) - (2*a*b*d^3*x^3 + 2*a^2*d^3 - (I*b^2*x^3 + I*a*b + sqrt(3)*(b^2*x^3
+ a*b))*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) -
 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (2*a*b*d^3*x^3 + 2*
a^2*d^3 - (-I*b^2*x^3 - I*a*b + sqrt(3)*(b^2*x^3 + a*b))*(I*a*d^3/b)^(2/3))
*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3
)*(-I*sqrt(3) + 1) - I*c) - (2*a*b*d^3*x^3 + 2*a^2*d^3 - (I*b^2*x^3 + I*a*b
 - sqrt(3)*(b^2*x^3 + a*b))*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)
^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) -
 2*(a*b*d^3*x^3 + a^2*d^3 + (I*b^2*x^3 + I*a*b)*(-I*a*d^3/b)^(2/3))*Ei(I*d*
x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*(a*b*d^3*x^3 + a^2
*d^3 + (-I*b^2*x^3 - I*a*b)*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3
))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a^2*b^2*d^2*x^3 + a^3*b*d^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*sin(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sin\left(c+d\,x\right)}{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(c + d*x))/(a + b*x^3)^2,x)
```

```
[Out] int((x*sin(c + d*x))/(a + b*x^3)^2, x)
```

3.105 $\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=735

$$\frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}}+\frac{d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}}-\frac{\sqrt[3]{-1}\,d\cos\left(c\right.}{}$$

```
[Out] 1/9*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/
9*(-1)^(2/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))/a^(4/3)/b^(2/3)-1/9*(-1)^(1/3)*d*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/
3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)-2/9*(-1)^(1/3)*
cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a
^(5/3)/b^(1/3)+2/9*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(5/
3)/b^(1/3)+2/9*(-1)^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)
*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)+2/9*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c
-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/9*d*Si(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a
^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)-2/9*(-1)^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^
(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/9*(-1)^(2/
3)*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/
3))/a^(4/3)/b^(2/3)+2/9*(-1)^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin
(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)+1/9*(-1)^(1/3)*d*Si((-1)^(
2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(
2/3)+1/3*sin(d*x+c)/a/b/x^2-1/3*sin(d*x+c)/b/x^2/(b*x^3+a)
```

Rubi [A]
time = 0.82, antiderivative size = 735, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3412, 3426, 3378, 3384, 3380, 3383, 3414, 3427}

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x^3)^2,x]
```

```
[Out] ((-1)^(2/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) + (d*Cos[c - (a^(1/3)*d)/
b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) - ((-1
)^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a
^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) + (2*CosIntegral[(a^(1/3)*d)/
b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) - (2*(-1)^
(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) + (2*(-1)^(2/3)*CosIntegral[((-1)
^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(
```

```
9*a^(5/3)*b^(1/3)) + Sin[c + d*x]/(3*a*b*x^2) - Sin[c + d*x]/(3*b*x^2*(a +
b*x^3)) + (2*(-1)^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) + ((-1)^(2/3)*d
*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)
/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) + (2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinI
ntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (d*Sin[c - (a^(1/
3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3))
+ (2*(-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(
2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) + ((-1)^(1/3)*d*Sin[c -
 ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
 + d*x])/(9*a^(4/3)*b^(2/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Sim
p[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))), x] + (-Dist[
(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Sin[c + d*x])/x^n, x], x]
- Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx = -\frac{\sin(c+dx)}{3bx^2(a+bx^3)} - \frac{2\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx}{3b} + \frac{d\int \frac{\cos(c+dx)}{x^2(a+bx^3)}\,dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx^2(a+bx^3)} - \frac{2\int \left(\frac{\sin(c+dx)}{ax^3} - \frac{b\sin(c+dx)}{a(a+bx^3)}\right)dx}{3b} + \frac{d\int \left(\frac{\cos(c+dx)}{ax^2} - \frac{bx\cos(c+dx)}{a(a+bx^3)}\right)dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx^2(a+bx^3)} + \frac{2\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{3a} - \frac{2\int \frac{\sin(c+dx)}{x^3}\,dx}{3ab} - \frac{d\int \frac{x\cos(c+dx)}{a+bx^3}\,dx}{3a} + \frac{d\int \frac{\cos(c+dx)}{x^2}\,d\ldots}{3ab}$$

$$= -\frac{d\cos(c+dx)}{3abx} + \frac{\sin(c+dx)}{3abx^2} - \frac{\sin(c+dx)}{3bx^2(a+bx^3)} + \frac{2\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{\ldots}{3a^{2/3}\left(-\sqrt[3]{\ldots}\right.}\ldots\right.}{\ldots}$$

$$= \frac{\sin(c+dx)}{3abx^2} - \frac{\sin(c+dx)}{3bx^2(a+bx^3)} - \frac{2\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{9a^{5/3}} - \frac{2\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\,x}\,dx}{9a^{5/3}} - \frac{2\int \ldots}{\ldots}$$

$$= -\frac{d^2\mathrm{Ci}(dx)\sin(c)}{3ab} + \frac{\sin(c+dx)}{3abx^2} - \frac{\sin(c+dx)}{3bx^2(a+bx^3)} - \frac{d^2\cos(c)\mathrm{Si}(dx)}{3ab} + \frac{(d^2\cos(c))\int \frac{\sin(d\ldots)}{x}\ldots}{3ab}$$

$$= \frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} + \frac{d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+\ldots\right)}{9a^{4/3}b^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.16, size = 406, normalized size = 0.55

$$-\frac{(a+bx^3)\,\text{RootSum}\left[a+b\#1^3\&,\frac{-2i\cos(c+d\#1)\text{Ci}(d(x-\#1))-2\text{Ci}(d(x-\#1))\sin(c+d\#1)-2\cos(c+d\#1)\text{Si}(d(x-\#1))+2i\sin(c+d\#1)\text{Si}(d(x-\#1))+d\cos(c+d\#1)\text{Ci}(d(x-\#1))\#1-id\text{Ci}(d(x-\#1))\sin(c+d\#1)\#1-id\cos(c+d\#1)\text{Si}(d(x-\#1))\#1-d\sin(c+d\#1)\text{Si}(d(x-\#1))\#1}{\#1^2}\&\right]+(a+bx^3)\,\text{RootSum}\left[a+b\#1^3\&,\frac{2i\cos(c+d\#1)\text{Ci}(d(x-\#1))-2\text{Ci}(d(x-\#1))\sin(c+d\#1)-2\cos(c+d\#1)\text{Si}(d(x-\#1))-2i\sin(c+d\#1)\text{Si}(d(x-\#1))+d\cos(c+d\#1)\text{Ci}(d(x-\#1))\#1+id\text{Ci}(d(x-\#1))\sin(c+d\#1)\#1+id\cos(c+d\#1)\text{Si}(d(x-\#1))\#1-d\sin(c+d\#1)\text{Si}(d(x-\#1))\#1}{\#1^2}\&\right]-6bx\sin(c+dx)}{18ab\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*x^3)^2,x]
```

```
[Out] -1/18*((a + b*x^3)*RootSum[a + b*#1^3 & , ((-2*I)*Cos[c + d*#1]*CosIntegral
[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c + d*#1]*Si
nIntegral[d*(x - #1)] + (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos
[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c +
 d*#1]*#1 - I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*
SinIntegral[d*(x - #1)]*#1)/#1^2 & ] + (a + b*x^3)*RootSum[a + b*#1^3 & , (
(2*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin
[c + d*#1] - 2*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - (2*I)*Sin[c + d*#1]*
SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*
CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*
(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] - 6*b*x
*Sin[c + d*x])/(a*b*(a + b*x^3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.06, size = 248, normalized size = 0.34

method	result
derivativedivides	$d^5\left(\frac{\sin(dx+c)\left(\frac{dx+c}{3a\,d^3}-\frac{c}{3a\,d^3}\right)}{a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3}+\frac{2\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{-\text{sinInt}}{}\right.}{}\right.$
default	$d^5\left(\frac{\sin(dx+c)\left(\frac{dx+c}{3a\,d^3}-\frac{c}{3a\,d^3}\right)}{a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3}+\frac{2\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)}\frac{-\text{sinInt}}{}\right.}{}\right.$
risch	$-\frac{d^2\left(\sum_{\_R1=\text{RootOf}\left(-3i\_Z^2bc-ia\,d^3+ib\,c^3+b\_Z^3-3b\,c^2\_Z\right)}\frac{\left(i\_R1+c-2i\right)\mathrm{e}^{\_R1}\,\text{expIntegral}\left(1,-idx-ic+\_R1\right)}{2ic\_R1-\_R1^2+c^2}\right)}{18ab}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^5*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d^3)/(a*d^3-b*c^3+3*b*c^2*(d*x+c
)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9/a/d^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-
```

```
d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*
_Z*b*c^2+a*d^3-b*c^3))+1/9/a/d^3/b*sum(1/(-_RR1+c)*(Si(-d*x+_RR1-c)*sin(_RR
1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*x^3 + a)^2, x)
```

Fricas [**C**] Result contains complex when optimal does not.
time = 0.40, size = 669, normalized size = 0.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*d*x*sin(d*x + c) + ((b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a*
d^3/b)^(2/3) + 2*(b*x^3 - sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a*d^3/b)^(1/3))*
Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3
)*(I*sqrt(3) + 1) - I*c) + ((b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I*a*d^
3/b)^(2/3) + 2*(b*x^3 - sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I*a*d^3/b)^(1/3))*E
i(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3
)*(I*sqrt(3) + 1) + I*c) + ((b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)*(I*a*d^3/
b)^(2/3) + 2*(b*x^3 - sqrt(3)*(I*b*x^3 + I*a) + a)*(I*a*d^3/b)^(1/3))*Ei(-I
*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*
sqrt(3) + 1) - I*c) + ((b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)*(-I*a*d^3/b)^(
2/3) + 2*(b*x^3 - sqrt(3)*(I*b*x^3 + I*a) + a)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x
 + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sq
rt(3) + 1) + I*c) - 2*((b*x^3 + a)*(-I*a*d^3/b)^(2/3) + 2*(b*x^3 + a)*(-I*a
*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3))
- 2*((b*x^3 + a)*(I*a*d^3/b)^(2/3) + 2*(b*x^3 + a)*(I*a*d^3/b)^(1/3))*Ei(-I
*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3))))/(a^2*b*d*x^3 + a^3*
d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)}{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*x^3)^2,x)
```

```
[Out] int(sin(c + d*x)/(a + b*x^3)^2, x)
```

3.106 $\int \frac{\sin(c+dx)}{x(a+bx^3)^2}\,dx$

Optimal. Leaf size=693

$$\frac{\sqrt[3]{-1}\,d\cos\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{(-1)^{2/3}d\cos\left(c\right.}{}$$

```
[Out] -1/9*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)+1
/9*(-1)^(1/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/
3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/9*(-1)^(2/3)*d*Ci((-1)^(2/3)*a^(1/3)*d/b^(1
/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)+cos(c)*Si(d*x)
/a^2-1/3*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/
3)+d*x)/a^2-1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^2-1/3*
cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^
2+Ci(d*x)*sin(c)/a^2-1/3*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))
/a^2+1/9*d*Si(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/
3)-1/3*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1
/3))/a^2-1/9*(-1)^(1/3)*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^
(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/3*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3
)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^2+1/9*(-1)^(2/3)*d*Si((-1)^(2/
3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/
3)+1/3*sin(d*x+c)/a/b/x^3-1/3*sin(d*x+c)/b/x^3/(b*x^3+a)
```

Rubi [A]
time = 0.95, antiderivative size = 693, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3424, 3426, 3378, 3384, 3380, 3383, 3427, 3415}

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^3)^2),x]
```

```
[Out] ((-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) - (d*Cos[c - (a^(1/3)*d)/
b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - ((-1
)^(2/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a
^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) + (CosIntegral[d*x]*Sin[c])/a
^2 - (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/
(3*a^2) - (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^
(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^2) - (CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^
(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^2) + Sin[c + d*x
]/(3*a*b*x^3) - Sin[c + d*x]/(3*b*x^3*(a + b*x^3)) + (Cos[c]*SinIntegral[d*
```

```
x])/a^2 + (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) + ((-1)^(1/3)*d*Sin[c + ((-1)^(1/3)*a^(1
/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/
3)*b^(1/3)) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3)
 + d*x])/(3*a^2) + (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/
b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2) + ((-1)^(2/3
)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)
*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
```

```
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^2}\,dx = -\frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)} - \frac{\int \frac{\sin(c+dx)}{x^4(a+bx^3)}\,dx}{b} + \frac{d\int \frac{\cos(c+dx)}{x^3(a+bx^3)}\,dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)} - \frac{\int \left(\frac{\sin(c+dx)}{ax^4} - \frac{b\sin(c+dx)}{a^2x} + \frac{b^2x^2\sin(c+dx)}{a^2(a+bx^3)}\right)dx}{b} + \frac{d\int \left(\frac{\cos(c+dx)}{ax^3} - \frac{b\cos(c}{a(a+}\right.}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^2} - \frac{\int \frac{\sin(c+dx)}{x^4}\,dx}{ab} - \frac{b\int \frac{x^2\sin(c+dx)}{a+bx^3}\,dx}{a^2} - \frac{d\int \frac{\cos(c+dx)}{a+bx^3}\,dx}{3a}$$

$$= -\frac{d\cos(c+dx)}{6abx^2} + \frac{\sin(c+dx)}{3abx^3} - \frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)} - \frac{b\int \left(\frac{\sin(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\text{si}}{3b^{2/3}\left(-\sqrt[3]{-}\right.}\right.}{}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a^2} + \frac{\sin(c+dx)}{3abx^3} + \frac{d^2\sin(c+dx)}{6abx} - \frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\cos(c)\text{Si}(dx)}{a^2} - \frac{\sqrt[3]{b}}{}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a^2} + \frac{\sin(c+dx)}{3abx^3} - \frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\cos(c)\text{Si}(dx)}{a^2} + \frac{d^3\int \frac{\cos(c+dx)}{x}\,dx}{6ab} - \frac{(d^3}{}$$

$$= -\frac{d^3\cos(c)\text{Ci}(dx)}{6ab} + \frac{\sqrt[3]{-1}\,d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}} - dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{d\cos\left(c\right.}{}$$

$$= \frac{\sqrt[3]{-1}\,d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}} - dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right.}{9a^{5/3}\sqrt[3]{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.37, size = 446, normalized size = 0.64

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^3)^2),x]
```

```
[Out] ((-1/2*I)*RootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*
CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x -
#1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)] & ] + (I/2)*RootSum[a + b*#1^3
 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[
c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinInte
```

```
gral[d*(x - #1)] & ] - (a*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegr
al[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*
SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ])/
(6*b) - (a*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)]
+ I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(
x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ])/(6*b) + (a*Cos[
d*x]*Sin[c])/(a + b*x^3) + 3*CosIntegral[d*x]*Sin[c] + (a*Cos[c]*Sin[d*x])/
(a + b*x^3) + 3*Cos[c]*SinIntegral[d*x])/(3*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 233, normalized size = 0.34

method	result
derivativedivides	$\frac{\sin(dx+c)d^3}{3a\left(a d^3-b c^3+3b c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)} - \frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b c^2\_Z+a d^3-b c^3\right)}\left(-\mathrm{sinIntegra}\right.}{}$
default	$\frac{\sin(dx+c)d^3}{3a\left(a d^3-b c^3+3b c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)} - \frac{\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b c^2\_Z+a d^3-b c^3\right)}\left(-\mathrm{sinIntegra}\right.}{}$
risch	$-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-ia d^3+ib c^3+b\_Z^3-3b c^2\_Z\right)}\frac{\left(-ia d^3-6i\_R1bc+3\_R1^2b-3b c^2\right)e^{\_R1}\mathrm{expIntegral}\left(1,-idx\right.}{-2ic\_R1+\_R1^2-c^2}\right.}{18a^2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sin(d*x+c)*d^3/a/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)
^3)-1/3/a^2*sum(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1),_R1=RootOf(
_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a^2*(Si(d*x)*cos(c)+Ci(d*x)*si
n(c))-1/9*d^3/a/b*sum(1/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci
(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^
3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.42, size = 588, normalized size = 0.85

$$\frac{\left(-6ibx^3+\left(ibx^3-\sqrt{3}\left(bx^3+a\right)+ia\right)\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}-6ia\right)\mathrm{Ei}\left(-idx+\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-ic\right)}+\left(6ibx^3+\left(-ibx^3+\sqrt{3}\left(bx^3+a\right)-ia\right)\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}+6ia\right)\mathrm{Ei}\left(idx+\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)+ic\right)}+\left(-6ibx^3+\left(ibx^3+\sqrt{3}\left(bx^3+a\right)+ia\right)\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}-6ia\right)\mathrm{Ei}\left(-idx+\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)-ic\right)}+\left(6ibx^3+\left(-ibx^3-\sqrt{3}\left(bx^3+a\right)-ia\right)\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}+6ia\right)\mathrm{Ei}\left(idx+\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right)+ic\right)}-18\left(ibx^3+ia\right)\mathrm{Ei}\left(idx\right)e^{\left(ic\right)}-18\left(-ibx^3-ia\right)\mathrm{Ei}\left(-idx\right)e^{\left(-ic\right)}-2\left(-3ibx^3+\left(-ibx^3-ia\right)\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}-3ia\right)\mathrm{Ei}\left(idx+\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(ic-\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)}-2\left(3ibx^3+\left(ibx^3+ia\right)\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}+3ia\right)\mathrm{Ei}\left(-idx+\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(-ic-\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)}+12a\sin\left(dx+c\right)}{36\left(a^2bx^3+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*((-6*I*b*x^3 + (I*b*x^3 - sqrt(3)*(b*x^3 + a) + I*a)*(I*a*d^3/b)^(1/3)
 - 6*I*a)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d
^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (6*I*b*x^3 + (-I*b*x^3 + sqrt(3)*(b*x^
3 + a) - I*a)*(-I*a*d^3/b)^(1/3) + 6*I*a)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)
*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (-6*I
*b*x^3 + (I*b*x^3 + sqrt(3)*(b*x^3 + a) + I*a)*(I*a*d^3/b)^(1/3) - 6*I*a)*E
i(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*
(-I*sqrt(3) + 1) - I*c) + (6*I*b*x^3 + (-I*b*x^3 - sqrt(3)*(b*x^3 + a) - I*
a)*(-I*a*d^3/b)^(1/3) + 6*I*a)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3)
 - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 18*(I*b*x^3 + I*
a)*Ei(I*d*x)*e^(I*c) - 18*(-I*b*x^3 - I*a)*Ei(-I*d*x)*e^(-I*c) - 2*(-3*I*b*
x^3 + (-I*b*x^3 - I*a)*(-I*a*d^3/b)^(1/3) - 3*I*a)*Ei(I*d*x + (-I*a*d^3/b)^
(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*(3*I*b*x^3 + (I*b*x^3 + I*a)*(I*a*d
^3/b)^(1/3) + 3*I*a)*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(
1/3)) + 12*a*sin(d*x + c))/(a^2*b*x^3 + a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c+dx\right)}{x\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x**3+a)**2,x)
```

```
[Out] Integral(sin(c + d*x)/(x*(a + b*x**3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)}{x\,{\left(b\,x^3+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x*(a + b*x^3)^2),x)
```

```
[Out] int(sin(c + d*x)/(x*(a + b*x^3)^2), x)
```

3.107 $\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=712

$$\frac{d\cos(c)\text{Ci}(dx)}{a^2}+\frac{d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9a^2}+\frac{d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)}{9a^2}+\text{d co}$$

```
[Out] d*Ci(d*x)*cos(c)/a^2+1/9*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3
))/a^2+1/9*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/3)*
d/b^(1/3))/a^2+1/9*d*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*cos(c-(-1)^(2/3)*
a^(1/3)*d/b^(1/3))/a^2+4/9*(-1)^(2/3)*b^(1/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^
(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)+4/9*b^(1/3)*cos(c-a^(1
/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)-4/9*(-1)^(1/3)*b^(1/3)*cos
(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(7/
3)-d*Si(d*x)*sin(c)/a^2+4/9*b^(1/3)*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)
*d/b^(1/3))/a^(7/3)-1/9*d*Si(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3)
)/a^2+4/9*(-1)^(2/3)*b^(1/3)*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1
)^(1/3)*a^(1/3)*d/b^(1/3))/a^(7/3)-1/9*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d
*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2-4/9*(-1)^(1/3)*b^(1/3)*Ci((-1)^
(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(7/3)-1/
9*d*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3)
)/a^2+1/3*sin(d*x+c)/a/b/x^4-4/3*sin(d*x+c)/a^2/x-1/3*sin(d*x+c)/b/x^4/(b*x
^3+a)
```

Rubi [A]
time = 1.00, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3424, 3426, 3378, 3384, 3380, 3383, 3427}

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^3)^2),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a^2 + (d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^2) + (d*Cos[c - (
a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^2) + (d*Co
s[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^
(1/3) + d*x])/(9*a^2) + (4*b^(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*S
in[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(7/3)) + (4*(-1)^(2/3)*b^(1/3)*CosIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)])/(9*a^(7/3)) - (4*(-1)^(1/3)*b^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d
)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(7/3)) + Sin
```

```
[c + d*x]/(3*a*b*x^4) - (4*Sin[c + d*x])/(3*a^2*x) - Sin[c + d*x]/(3*b*x^4*
(a + b*x^3)) - (d*Sin[c]*SinIntegral[d*x])/a^2 - (4*(-1)^(2/3)*b^(1/3)*Cos[
c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1
/3) - d*x])/(9*a^(7/3)) + (d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^2) + (4*b^(1/3)*Cos[c - (
a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)) - (
d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a
^2) - (4*(-1)^(1/3)*b^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinInte
gral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)) - (d*Sin[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]
)/(9*a^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)^2}\,dx = -\frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)} - \frac{4\int \frac{\sin(c+dx)}{x^5(a+bx^3)}\,dx}{3b} + \frac{d\int \frac{\cos(c+dx)}{x^4(a+bx^3)}\,dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)} - \frac{4\int \left(\frac{\sin(c+dx)}{ax^5} - \frac{b\sin(c+dx)}{a^2x^2} + \frac{b^2x\sin(c+dx)}{a^2(a+bx^3)}\right)dx}{3b} + \frac{d\int \left(\frac{\cos(c+dx)}{ax^4} - \frac{b\cos(\cdots}{a^{\cdots}}\right.}{\cdots}$$

$$= -\frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)} + \frac{4\int \frac{\sin(c+dx)}{x^2}\,dx}{3a^2} - \frac{4\int \frac{\sin(c+dx)}{x^5}\,dx}{3ab} - \frac{(4b)\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{3a^2} - \frac{d\int \frac{\cos(c+\cdots}{x}}{3a^2}$$

$$= -\frac{d\cos(c+dx)}{9abx^3} + \frac{\sin(c+dx)}{3abx^4} - \frac{4\sin(c+dx)}{3a^2x} - \frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)} - \cdots$$

$$= -\frac{d\cos(c)\text{Ci}(dx)}{3a^2} + \frac{\sin(c+dx)}{3abx^4} + \frac{d^2\sin(c+dx)}{18abx^2} - \frac{4\sin(c+dx)}{3a^2x} - \frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)} + \cdots$$

$$= \frac{d^3\cos(c+dx)}{18abx} + \frac{d\cos(c)\text{Ci}(dx)}{a^2} + \frac{\sin(c+dx)}{3abx^4} - \frac{4\sin(c+dx)}{3a^2x} - \frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)} - \frac{d\,\text{s}\cdots}{\cdots}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a^2} + \frac{d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}} - dx\right)}{9a^2} + \frac{d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\cdots}{9a^{\cdots}}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a^2} + \frac{d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}} - dx\right)}{9a^2} + \frac{d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\cdots}{9a^{\cdots}}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a^2} + \frac{d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}} - dx\right)}{9a^2} + \frac{d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\cdots}{9a^{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.49, size = 445, normalized size = 0.62

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -1/3*((3*a + 4*b*x^3)*Cos[d*x]*Sin[c] + (3*a + 4*b*x^3)*Cos[c]*Sin[d*x] - (
x*(a + b*x^3)*(18*d*Cos[c]*CosIntegral[d*x] + RootSum[a + b*#1^3 & , ((-4*I
```

```
)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 4*CosIntegral[d*(x - #1)]*Sin[c +
 d*#1] - 4*Cos[c + d*#1]*SinIntegral[d*(x - #1)] + (4*I)*Sin[c + d*#1]*SinI
ntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - I*d*CosI
ntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*d*Cos[c + d*#1]*SinIntegral[d*(x -
 #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1 & ] + RootSum[a +
 b*#1^3 & , ((4*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 4*CosIntegral[d*
(x - #1)]*Sin[c + d*#1] - 4*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - (4*I)*S
in[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #
1)]*#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*S
inIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1
& ] - 18*d*Sin[c]*SinIntegral[d*x]))/6)/(a^2*x*(a + b*x^3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 284, normalized size = 0.40

method	result
derivativedivides	$d\left(-\frac{\sin(dx+c)\left(\frac{4b(dx+c)^3}{3a^2}-\frac{4cb(dx+c)^2}{a^2}+\frac{4c^2b(dx+c)}{a^2}+\frac{3a\,d^3-4b\,c^3}{3a^2}\right)}{dx\left(a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)}+\frac{4\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\right.}\right.}{}\right.$
default	$d\left(-\frac{\sin(dx+c)\left(\frac{4b(dx+c)^3}{3a^2}-\frac{4cb(dx+c)^2}{a^2}+\frac{4c^2b(dx+c)}{a^2}+\frac{3a\,d^3-4b\,c^3}{3a^2}\right)}{dx\left(a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)}+\frac{4\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\right.}\right.}{}\right.$
risch	$-\frac{d\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-ia\,d^3+ib\,c^3+b\_Z^3-3b\,c^2\_Z\right)}\frac{\left(-ic+\_R1-4\right)e^{-\_R1}\,\mathrm{expIntegral}\left(1,-idx-ic+\_R1\right)}{-ic+\_R1}\right)}{18a^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(-sin(d*x+c)*(4/3*b/a^2*(d*x+c)^3-4*c*b/a^2*(d*x+c)^2+4*c^2*b/a^2*(d*x+c)
+1/3*(3*a*d^3-4*b*c^3)/a^2)/d/x/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^
2+b*(d*x+c)^3)+4/9/a^2*sum(1/(-_R1+c)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+
c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a^2*
sum(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1),_RR1=RootOf(_Z^3*b-3
*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a^2*(-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x^2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.41, size = 726, normalized size = 1.02

$$\frac{\left(abd^3x^4 + a^2d^3x - 2\left(-ib^2x^4 - iabx - \sqrt{3}\left(b^2x^4 + abx\right)\right)\left(\frac{iad^3}{b}\right)^{\frac{2}{3}}\right)\mathrm{Ei}\left(-idx + \frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3} - 1\right)\right)e^{\left(\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right) - ic\right)} + \left(abd^3x^4 + a^2d^3x - 2\left(ib^2x^4 + iabx + \sqrt{3}\left(b^2x^4 + abx\right)\right)\left(-\frac{iad^3}{b}\right)^{\frac{2}{3}}\right)\mathrm{Ei}\left(idx + \frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3} - 1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right) + ic\right)} + \left(abd^3x^4 + a^2d^3x - 2\left(-ib^2x^4 - iabx + \sqrt{3}\left(b^2x^4 + abx\right)\right)\left(\frac{iad^3}{b}\right)^{\frac{2}{3}}\right)\mathrm{Ei}\left(-idx + \frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3} - 1\right)\right)e^{\left(\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right) - ic\right)} + \left(abd^3x^4 + a^2d^3x - 2\left(ib^2x^4 + iabx - \sqrt{3}\left(b^2x^4 + abx\right)\right)\left(-\frac{iad^3}{b}\right)^{\frac{2}{3}}\right)\mathrm{Ei}\left(idx + \frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3} - 1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}+1\right) + ic\right)} + 9\left(abd^3x^4 + a^2d^3x\right)\mathrm{Ei}\left(idx\right)e^{(ic)} + 9\left(abd^3x^4 + a^2d^3x\right)\mathrm{Ei}\left(-idx\right)e^{(-ic)} + \left(abd^3x^4 + a^2d^3x - 4\left(-ib^2x^4 - iabx\right)\left(-\frac{iad^3}{b}\right)^{\frac{2}{3}}\right)\mathrm{Ei}\left(idx + \left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(ic - \left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)} + \left(abd^3x^4 + a^2d^3x - 4\left(ib^2x^4 + iabx\right)\left(\frac{iad^3}{b}\right)^{\frac{2}{3}}\right)\mathrm{Ei}\left(-idx + \left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)e^{\left(-ic - \left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\right)} - 6\left(4abd^2x^3 + 3a^2d^2\right)\sin\left(dx + c\right)}{18\left(a^3bd^2x^4 + a^4d^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/18*((a*b*d^3*x^4 + a^2*d^3*x - 2*(-I*b^2*x^4 - I*a*b*x - sqrt(3)*(b^2*x^4
 + a*b*x))*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3)
 - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (a*b*d^3*x^4 + a^2
*d^3*x - 2*(I*b^2*x^4 + I*a*b*x + sqrt(3)*(b^2*x^4 + a*b*x))*(-I*a*d^3/b)^(
2/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/
b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (a*b*d^3*x^4 + a^2*d^3*x - 2*(-I*b^2*x^4
- I*a*b*x + sqrt(3)*(b^2*x^4 + a*b*x))*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + 1/2*(
I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1)
 - I*c) + (a*b*d^3*x^4 + a^2*d^3*x - 2*(I*b^2*x^4 + I*a*b*x - sqrt(3)*(b^2*
x^4 + a*b*x))*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt
(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + 9*(a*b*d^3*x^
4 + a^2*d^3*x)*Ei(I*d*x)*e^(I*c) + 9*(a*b*d^3*x^4 + a^2*d^3*x)*Ei(-I*d*x)*e
^(-I*c) + (a*b*d^3*x^4 + a^2*d^3*x - 4*(-I*b^2*x^4 - I*a*b*x)*(-I*a*d^3/b)^
(2/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + (a*b*d
^3*x^4 + a^2*d^3*x - 4*(I*b^2*x^4 + I*a*b*x)*(I*a*d^3/b)^(2/3))*Ei(-I*d*x +
 (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) - 6*(4*a*b*d^2*x^3 + 3*a^2
*d^2)*sin(d*x + c))/(a^3*b*d^2*x^4 + a^4*d^2*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)}{x^2\,{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] int(sin(c + d*x)/(x^2*(a + b*x^3)^2), x)
```

3.108 $\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=800

$$-\frac{d\cos(c+dx)}{2a^2x}-\frac{(-1)^{2/3}\sqrt[3]{b}\,d\cos\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}}-\frac{\sqrt[3]{b}\,d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}\right.}{9a^{7/3}}$$

```
[Out] -1/9*b^(1/3)*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(7/3)-1
/9*(-1)^(2/3)*b^(1/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/
3)*a^(1/3)*d/b^(1/3))/a^(7/3)+1/9*(-1)^(1/3)*b^(1/3)*d*Ci((-1)^(2/3)*a^(1/3
)*d/b^(1/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(7/3)-1/2*d*cos(d*x+
c)/a^2/x-1/2*d^2*cos(c)*Si(d*x)/a^2+5/9*(-1)^(1/3)*b^(2/3)*cos(c+(-1)^(1/3)
*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)-5/9*b^(2/
3)*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)-5/9*(-1)^(2/3
)*b^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/
3)+d*x)/a^(8/3)-1/2*d^2*Ci(d*x)*sin(c)/a^2-5/9*b^(2/3)*Ci(a^(1/3)*d/b^(1/3)
+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(8/3)+1/9*b^(1/3)*d*Si(a^(1/3)*d/b^(1/3)+d
*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(7/3)+5/9*(-1)^(1/3)*b^(2/3)*Ci((-1)^(1/3)*a
^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(8/3)+1/9*(-1)^
(2/3)*b^(1/3)*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^(1/3)*a^(1
/3)*d/b^(1/3))/a^(7/3)-5/9*(-1)^(2/3)*b^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/
3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(8/3)-1/9*(-1)^(1/3)*b^(1/3)*
d*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/
a^(7/3)+1/3*sin(d*x+c)/a/b/x^5-5/6*sin(d*x+c)/a^2/x^2-1/3*sin(d*x+c)/b/x^5/
(b*x^3+a)
```

Rubi [A]
time = 1.15, antiderivative size = 800, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3424, 3426, 3378, 3384, 3380, 3383, 3414, 3427}

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/2*(d*Cos[c + d*x])/(a^2*x) - ((-1)^(2/3)*b^(1/3)*d*Cos[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^
(7/3)) - (b^(1/3)*d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^
(1/3) + d*x])/(9*a^(7/3)) + ((-1)^(1/3)*b^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/
3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3
)) - (d^2*CosIntegral[d*x]*Sin[c])/(2*a^2) - (5*b^(2/3)*CosIntegral[(a^(1/3
)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(8/3)) + (5*(-1)^(1/
```

```
3)*b^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^
(1/3)*a^(1/3)*d)/b^(1/3)]])/(9*a^(8/3)) - (5*(-1)^(2/3)*b^(2/3)*CosIntegral[
((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)]])/(9*a^(8/3)) + Sin[c + d*x]/(3*a*b*x^5) - (5*Sin[c + d*x])/(6*a^2*x^2) -
 Sin[c + d*x]/(3*b*x^5*(a + b*x^3)) - (d^2*Cos[c]*SinIntegral[d*x])/(2*a^2)
 - (5*(-1)^(1/3)*b^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(8/3)) - ((-1)^(2/3)*b^(1/3)*
d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d
)/b^(1/3) - d*x])/(9*a^(7/3)) - (5*b^(2/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*Sin
Integral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(8/3)) + (b^(1/3)*d*Sin[c - (a^(1
/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)) - (5*(-
1)^(2/3)*b^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^
(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(8/3)) - ((-1)^(1/3)*b^(1/3)*d*Sin[c
- ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3
) + d*x])/(9*a^(7/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)} - \frac{5\int \frac{\sin(c+dx)}{x^6(a+bx^3)}\,dx}{3b} + \frac{d\int \frac{\cos(c+dx)}{x^5(a+bx^3)}\,dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)} - \frac{5\int \left(\frac{\sin(c+dx)}{ax^6} - \frac{b\sin(c+dx)}{a^2x^3} + \frac{b^2\sin(c+dx)}{a^2(a+bx^3)}\right)dx}{3b} + \frac{d\int \left(\frac{\cos(c+dx)}{ax^5} - \frac{b\cos(}{a^{}}\right.}{}$$

$$= -\frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)} + \frac{5\int \frac{\sin(c+dx)}{x^3}\,dx}{3a^2} - \frac{5\int \frac{\sin(c+dx)}{x^6}\,dx}{3ab} - \frac{(5b)\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{3a^2} - \frac{d\int \frac{\cos(c+}{x^2}}{3a^2}$$

$$= -\frac{d\cos(c+dx)}{12abx^4} + \frac{d\cos(c+dx)}{3a^2x} + \frac{\sin(c+dx)}{3abx^5} - \frac{5\sin(c+dx)}{6a^2x^2} - \frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)} - \cdots$$

$$= -\frac{d\cos(c+dx)}{2a^2x} + \frac{\sin(c+dx)}{3abx^5} + \frac{d^2\sin(c+dx)}{36abx^3} - \frac{5\sin(c+dx)}{6a^2x^2} - \frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)} + \cdots$$

$$= \frac{d^3\cos(c+dx)}{72abx^2} - \frac{d\cos(c+dx)}{2a^2x} + \frac{d^2\text{Ci}(dx)\sin(c)}{3a^2} + \frac{\sin(c+dx)}{3abx^5} - \frac{5\sin(c+dx)}{6a^2x^2} - \cdots$$

$$= -\frac{d\cos(c+dx)}{2a^2x} - \frac{(-1)^{2/3}\sqrt[3]{b}\,d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}} - \cdots$$

$$= -\frac{d\cos(c+dx)}{2a^2x} - \frac{(-1)^{2/3}\sqrt[3]{b}\,d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}} - \cdots$$

$$= -\frac{d\cos(c+dx)}{2a^2x} + \frac{d^5\cos(c)\text{Ci}(dx)}{72ab} - \frac{(-1)^{2/3}\sqrt[3]{b}\,d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{}}{\sqrt[3]{b}}\right.}{9a^{7/3}}$$

$$= -\frac{d\cos(c+dx)}{2a^2x} - \frac{(-1)^{2/3}\sqrt[3]{b}\,d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}} - \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.60, size = 470, normalized size = 0.59

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x^3)^2),x]
```

```
[Out] (RootSum[a + b*#1^3 & , ((-5*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 5*C
osIntegral[d*(x - #1)]*Sin[c + d*#1] - 5*Cos[c + d*#1]*SinIntegral[d*(x - #
1)] + (5*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosInte
gral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*d*Co
s[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x -
 #1)]*#1)/#1^2 & ] + RootSum[a + b*#1^3 & , ((5*I)*Cos[c + d*#1]*CosIntegra
l[d*(x - #1)] - 5*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 5*Cos[c + d*#1]*S
inIntegral[d*(x - #1)] - (5*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Co
s[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c
+ d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]
*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] - (3*(3*a*d*x*Cos[c + d*x] + 3*b*d*x^
4*Cos[c + d*x] + 3*d^2*x^2*(a + b*x^3)*CosIntegral[d*x]*Sin[c] + 3*a*Sin[c
+ d*x] + 5*b*x^3*Sin[c + d*x] + 3*d^2*x^2*(a + b*x^3)*Cos[c]*SinIntegral[d*
x]))/(x^2*(a + b*x^3)))/(18*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.33, size = 388, normalized size = 0.48

method	result
risch	$-\frac{i d^2 \left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2 bc-ia\,d^3+ib\,c^3+b\_Z^3-3b\,c^2\_Z\right)} \frac{\left(-ic+\_R1-5\right) e^{\_R1}\,\mathrm{expIntegral}\left(1,-idx-ic+\_R1\right)}{-2ic\_R1+\_R1^2-c^2}\right)}{18a^2} -$
derivativedivides	$d^2 \left(-\frac{b\,d^3 \left(\frac{\sin(dx+c)\left(\frac{dx+c}{3a\,d^3}-\frac{c}{3a\,d^3}\right)}{a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3}+\frac{2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)} \frac{-\sin\mathrm{Inte}}{}\right.}{}\right.}{}\right.$
default	$d^2 \left(-\frac{b\,d^3 \left(\frac{\sin(dx+c)\left(\frac{dx+c}{3a\,d^3}-\frac{c}{3a\,d^3}\right)}{a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3}+\frac{2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+a\,d^3-b\,c^3\right)} \frac{-\sin\mathrm{Inte}}{}\right.}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(-1/a*b*d^3*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d^3)/(a*d^3-b*c^3+3*
b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+2/9/a/d^3/b*sum(1/(_R1^2-2*_R1*c
```

```
+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3
*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a/d^3/b*sum(1/(-_RR1+c)*(Si(-d*x+_RR
1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z
*b*c^2+a*d^3-b*c^3)))-1/3/a^2*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*co
s(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^
3-b*c^3))+1/a^2*(-1/2*sin(d*x+c)/d^2/x^2-1/2*cos(d*x+c)/d/x-1/2*Si(d*x)*cos
(c)-1/2*Ci(d*x)*sin(c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x^3), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.44, size = 908, normalized size = 1.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(((b^2*x^5 + a*b*x^2 - sqrt(3)*(I*b^2*x^5 + I*a*b*x^2))*(I*a*d^3/b)^(
2/3) + 5*(b^2*x^5 + a*b*x^2 + sqrt(3)*(I*b^2*x^5 + I*a*b*x^2))*(I*a*d^3/b)^
(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/
b)^(1/3)*(I*sqrt(3) + 1) - I*c) + ((b^2*x^5 + a*b*x^2 - sqrt(3)*(I*b^2*x^5
+ I*a*b*x^2))*(-I*a*d^3/b)^(2/3) + 5*(b^2*x^5 + a*b*x^2 + sqrt(3)*(I*b^2*x^
5 + I*a*b*x^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*s
qrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + ((b^2*x^5 +
 a*b*x^2 - sqrt(3)*(-I*b^2*x^5 - I*a*b*x^2))*(I*a*d^3/b)^(2/3) + 5*(b^2*x^5
 + a*b*x^2 + sqrt(3)*(-I*b^2*x^5 - I*a*b*x^2))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x
 + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt
(3) + 1) - I*c) + ((b^2*x^5 + a*b*x^2 - sqrt(3)*(-I*b^2*x^5 - I*a*b*x^2))*(
-I*a*d^3/b)^(2/3) + 5*(b^2*x^5 + a*b*x^2 + sqrt(3)*(-I*b^2*x^5 - I*a*b*x^2)
)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^
(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + 9*(-I*a*b*d^3*x^5 - I*a^2
*d^3*x^2)*Ei(I*d*x)*e^(I*c) + 9*(I*a*b*d^3*x^5 + I*a^2*d^3*x^2)*Ei(-I*d*x)*
e^(-I*c) - 2*((b^2*x^5 + a*b*x^2)*(-I*a*d^3/b)^(2/3) + 5*(b^2*x^5 + a*b*x^2
)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^
(1/3)) - 2*((b^2*x^5 + a*b*x^2)*(I*a*d^3/b)^(2/3) + 5*(b^2*x^5 + a*b*x^2)*(
I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3
```

```
)) + 18*(a*b*d^2*x^4 + a^2*d^2*x)*cos(d*x + c) + 6*(5*a*b*d*x^3 + 3*a^2*d)*
sin(d*x + c))/(a^3*b*d*x^5 + a^4*d*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**3/(b*x**3+a)**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)}{x^3\,{\left(b\,x^3+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x^3*(a + b*x^3)^2),x)
[Out] int(sin(c + d*x)/(x^3*(a + b*x^3)^2), x)
```

3.109 $\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=772

$$\frac{d\cos(c+dx)}{18ab^2x} - \frac{d\cos(c+dx)}{18b^2x\,(a+bx^3)} + \frac{\mathrm{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} + \frac{d^2\mathrm{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{54ab^2} -$$

```
[Out] 1/18*d*cos(d*x+c)/a/b^2/x-1/18*d*cos(d*x+c)/b^2/x/(b*x^3+a)-1/27*(-1)^(1/3)
*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/
a^(5/3)/b^(4/3)+1/54*d^2*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)
*a^(1/3)*d/b^(1/3)+d*x)/a/b^2+1/27*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^
(1/3)+d*x)/a^(5/3)/b^(4/3)+1/54*d^2*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b
^(1/3)+d*x)/a/b^2+1/27*(-1)^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-
1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)+1/54*d^2*cos(c-(-1)^(2/3)*a
^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a/b^2+1/27*Ci(a^(1/3
)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/54*d^2*Ci(a^(1/
3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a/b^2-1/27*(-1)^(1/3)*Ci((-1)^(1
/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4
/3)+1/54*d^2*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*
d/b^(1/3))/a/b^2+1/27*(-1)^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c
-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/54*d^2*Ci((-1)^(2/3)*a^(1/
3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a/b^2+1/18*sin(d*x+c)
/a/b^2/x^2-1/6*x*sin(d*x+c)/b/(b*x^3+a)^2-1/18*sin(d*x+c)/b^2/x^2/(b*x^3+a)
```

Rubi [A]
time = 1.72, antiderivative size = 772, normalized size of antiderivative = 1.00, number of steps used = 71, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3424, 3412, 3426, 3378, 3384, 3380, 3383, 3414, 3427, 3425}

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x) - (d*Cos[c + d*x])/(18*b^2*x*(a + b*x^3)) + (
CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(27*a^
(5/3)*b^(4/3)) + (d^2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/
3)*d)/b^(1/3)])/(54*a*b^2) - ((-1)^(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)
/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(5/3)*b^(4/3
)) + (d^2*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(
1/3)*a^(1/3)*d)/b^(1/3)])/(54*a*b^2) + ((-1)^(2/3)*CosIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(5
/3)*b^(4/3)) + (d^2*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c
```

```
 - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(54*a*b^2) + Sin[c + d*x]/(18*a*b^2*x^2
) - (x*Sin[c + d*x])/(6*b*(a + b*x^3)^2) - Sin[c + d*x]/(18*b^2*x^2*(a + b*
x^3)) + ((-1)^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-
1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Cos[c + ((-
1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d
*x])/(54*a*b^2) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(
1/3) + d*x])/(27*a^(5/3)*b^(4/3)) + (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIn
tegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a*b^2) + ((-1)^(2/3)*Cos[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])
/(27*a^(5/3)*b^(4/3)) + (d^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinInt
egral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a*b^2)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Sim
p[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))), x] + (-Dist[
(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Sin[c + d*x])/x^n, x], x]
- Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3425

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^3}\,dx = -\frac{x\sin(c+dx)}{6b\,(a+bx^3)^2} + \frac{\int \frac{\sin(c+dx)}{(a+bx^3)^2}\,dx}{6b} + \frac{d\int \frac{x\cos(c+dx)}{(a+bx^3)^2}\,dx}{6b}$$

$$= -\frac{d\cos(c+dx)}{18b^2x\,(a+bx^3)} - \frac{x\sin(c+dx)}{6b\,(a+bx^3)^2} - \frac{\sin(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx}{9b^2} - \frac{d^2\int \frac{\sin(c+dx)}{x(a+bx^3)}}{18b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x\,(a+bx^3)} - \frac{x\sin(c+dx)}{6b\,(a+bx^3)^2} - \frac{\sin(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{\int \left(\frac{\sin(c+dx)}{ax^3} - \frac{b\sin(c+dx)}{a(a+bx^3)}\right) dx}{9b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x\,(a+bx^3)} - \frac{x\sin(c+dx)}{6b\,(a+bx^3)^2} - \frac{\sin(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{9ab^2} + \frac{\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{9ab}$$

$$= -\frac{d\cos(c+dx)}{18b^2x\,(a+bx^3)} + \frac{\sin(c+dx)}{18ab^2x^2} - \frac{x\sin(c+dx)}{6b\,(a+bx^3)^2} - \frac{\sin(c+dx)}{18b^2x^2\,(a+bx^3)} + \frac{\int \left(-\frac{\sin}{3a^{2/3}\left(-\sqrt[3]{\ldots}\right.}\right.}{}$$

$$= \frac{d\cos(c+dx)}{18ab^2x} - \frac{d\cos(c+dx)}{18b^2x\,(a+bx^3)} - \frac{d^2\operatorname{Ci}(dx)\sin(c)}{18ab^2} + \frac{\sin(c+dx)}{18ab^2x^2} - \frac{x\sin(c+dx)}{6b\,(a+bx^3)^2} - $$

$$= \frac{d\cos(c+dx)}{18ab^2x} - \frac{d\cos(c+dx)}{18b^2x\,(a+bx^3)} - \frac{d^2\operatorname{Ci}(dx)\sin(c)}{18ab^2} + \frac{\sin(c+dx)}{18ab^2x^2} - \frac{x\sin(c+dx)}{6b\,(a+bx^3)^2} - $$

$$= \frac{d\cos(c+dx)}{18ab^2x} - \frac{d\cos(c+dx)}{18b^2x\,(a+bx^3)} + \frac{\operatorname{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}} + dx\right)\sin\left(c - \frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} + \frac{d^2\operatorname{Ci}\left(\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}} \ldots\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.43, size = 457, normalized size = 0.59

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (I*RootSum[a + b*#1^3 & , (2*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - (2*I)*
CosIntegral[d*(x - #1)]*Sin[c + d*#1] - (2*I)*Cos[c + d*#1]*SinIntegral[d*(
x - #1)] - 2*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d^2*Cos[c + d*#1]*CosI
ntegral[d*(x - #1)]*#1^2 - I*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2
 - I*d^2*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2 - d^2*Sin[c + d*#1]*Sin
```

```
Integral[d*(x - #1)]*#1^2)/#1^2 & ] - I*RootSum[a + b*#1^3 & , (2*Cos[c + d
*#1]*CosIntegral[d*(x - #1)] + (2*I)*CosIntegral[d*(x - #1)]*Sin[c + d*#1]
+ (2*I)*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - 2*Sin[c + d*#1]*SinIntegral
[d*(x - #1)] + d^2*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1^2 + I*d^2*CosIn
tegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 + I*d^2*Cos[c + d*#1]*SinIntegral[d*(
x - #1)]*#1^2 - d^2*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2)/#1^2 & ] +
(6*b*x*(d*x*(a + b*x^3)*Cos[c + d*x] + (-2*a + b*x^3)*Sin[c + d*x]))/(a + b
*x^3)^2)/(108*a*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.34, size = 2035, normalized size = 2.64

method	result	size
risch	Expression too large to display	1337
derivativedivides	Expression too large to display	2035
default	Expression too large to display	2035

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(-d^9*c^3*(-1/18*sin(d*x+c)*(8*a*c*d^3-8*a*d^3*(d*x+c)-5*b*c^4+20*b*c
^3*(d*x+c)-30*b*c^2*(d*x+c)^2+20*b*c*(d*x+c)^3-5*b*(d*x+c)^4)/a^2/d^6/(a*d^
3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)^2-1/18*cos(d*x+c)*(c^2
-2*(d*x+c)*c+(d*x+c)^2)/a^2/d^6/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^
2+b*(d*x+c)^3)-1/54/a^2/d^6/b*sum((_R1^2-2*_R1*c+c^2-10)/(_R1^2-2*_R1*c+c^2
)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^
2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a^2/d^6/b*sum(1/(-_RR1+c)*(Si(-d*x+_RR1-
c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b
*c^2+a*d^3-b*c^3)))-1/6*sin(d*x+c)*c^2*d^3*(a*c^2*d^3+6*c*d^3*(d*x+c)*a-7*a
*d^3*(d*x+c)^2-c^5*b+10*b*c^3*(d*x+c)^2-20*b*c^2*(d*x+c)^3+15*b*c*(d*x+c)^4
-4*b*(d*x+c)^5)/a^2/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^
3)^2+1/6*cos(d*x+c)*c^2*d^3*(a*d^3-b*c^3+2*b*c^2*(d*x+c)-b*c*(d*x+c)^2)/a^2
/b/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+1/18*c^2*d^3/a
^2/b^2*sum((-_R1^2*b*c+2*_R1*b*c^2+a*d^3-b*c^3+4*_R1*b+6*b*c)/(_R1^2-2*_R1*
c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-
3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a^2*d^3/b*c^2*sum((c+2*_RR1)/(-_RR1
+c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b
-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6*sin(d*x+c)*d^3*c*(3*a^2*d^6-6*a*b*
c^3*d^3+20*a*b*c^2*d^3*(d*x+c)-14*a*b*c*d^3*(d*x+c)^2+3*b^2*c^6-20*b^2*c^5*
(d*x+c)+50*b^2*c^4*(d*x+c)^2-60*b^2*c^3*(d*x+c)^3+35*b^2*c^2*(d*x+c)^4-8*b^
2*c*(d*x+c)^5)/a^2/b/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)
^3)^2-1/6*cos(d*x+c)*d^3*c*(a*c*d^3+a*d^3*(d*x+c)-b*c^4+2*b*c^3*(d*x+c)-b*c
^2*(d*x+c)^2)/a^2/b/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^
3)-1/18*d^3*c/a^2/b^2*sum((-_R1^2*b*c^2+_R1*a*d^3+2*_R1*b*c^3+a*c*d^3-b*c^4
+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R
```

```
1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9*d^
3*c/a^2/b^2*sum((-4*_RR1^2*b*c+5*_RR1*b*c^2+a*d^3-b*c^3)/(_RR1^2-2*_RR1*c+c
^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b
-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18*sin(d*x+c)*d^3*(7*a^2*c*d^6+2*a^2
*d^6*(d*x+c)-14*a*b*c^4*d^3+38*a*b*c^3*d^3*(d*x+c)-27*a*b*c^2*d^3*(d*x+c)^2
+4*a*b*c*d^3*(d*x+c)^3-a*b*d^3*(d*x+c)^4+7*b^2*c^7-40*b^2*c^6*(d*x+c)+90*b^
2*c^5*(d*x+c)^2-100*b^2*c^4*(d*x+c)^3+55*b^2*c^3*(d*x+c)^4-12*b^2*c^2*(d*x+
c)^5)/a^2/b/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)^2+1/1
8*cos(d*x+c)*d^3*(a*c^2*d^3+c*d^3*(d*x+c)*a+a*d^3*(d*x+c)^2-c^5*b+2*b*c^4*(
d*x+c)-b*c^3*(d*x+c)^2)/a^2/b/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+
b*(d*x+c)^3)+1/54*d^3/a^2/b^2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R
1*b*c^4+a*c^2*d^3-b*c^5+12*_R1*b*c^2+2*a*d^3-2*b*c^3)/(_R1^2-2*_R1*c+c^2)*(
-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b
*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*d^3*c/a^2/b^2*sum((-2*_RR1^2*b*c+3*_RR1*b*c
^2+a*d^3-b*c^3)/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR
1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*(cos(c)^2 + sin(c)^2)*d*x^2*sin(d*x + c) + ((d^2*x^3*cos(c) - 6*d*x
^2*sin(c) - 42*x*cos(c))*cos(d*x + c)^2 + (d^2*x^3*cos(c) - 6*d*x^2*sin(c)
- 42*x*cos(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + ((cos(c)^2 + sin(c)^2)*d^2*
x^3 - 42*(cos(c)^2 + sin(c)^2)*x)*cos(d*x + c) - 2*(((b^3*cos(c)^2 + b^3*si
n(c)^2)*d^3*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^6 + 3*(a^2*b*co
s(c)^2 + a^2*b*sin(c)^2)*d^3*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*cos(d
*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^9 + 3*(a*b^2*cos(c)^2 + a*
b^2*sin(c)^2)*d^3*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^3 + (a^3*
cos(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate(3/2*(18*a*d*x*sin(d
*x + c) + (3*a*d^2*x^2 + 112*b*x^3 - 14*a)*cos(d*x + c))/(b^4*d^3*x^12 + 4*
a*b^3*d^3*x^9 + 6*a^2*b^2*d^3*x^6 + 4*a^3*b*d^3*x^3 + a^4*d^3), x) - 2*(((b
^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d
^3*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^3 + (a^3*cos(c)^2 + a^3*
sin(c)^2)*d^3)*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^9 + 3*
(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c
)^2)*d^3*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x + c)^2)*integrate
(3/2*(18*a*d*x*sin(d*x + c) + (3*a*d^2*x^2 + 112*b*x^3 - 14*a)*cos(d*x + c)
)/((b^4*d^3*x^12 + 4*a*b^3*d^3*x^9 + 6*a^2*b^2*d^3*x^6 + 4*a^3*b*d^3*x^3 +
a^4*d^3)*cos(d*x + c)^2 + (b^4*d^3*x^12 + 4*a*b^3*d^3*x^9 + 6*a^2*b^2*d^3*x
^6 + 4*a^3*b*d^3*x^3 + a^4*d^3)*sin(d*x + c)^2), x) + ((d^2*x^3*sin(c) + 6*
```

```
d*x^2*cos(c) - 42*x*sin(c))*cos(d*x + c)^2 + (d^2*x^3*sin(c) + 6*d*x^2*cos(
c) - 42*x*sin(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin
(c)^2)*d^3*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^3*x^6 + 3*(a^2*b*cos
(c)^2 + a^2*b*sin(c)^2)*d^3*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^3)*cos(d*
x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^3*x^9 + 3*(a*b^2*cos(c)^2 + a*b
^2*sin(c)^2)*d^3*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^3*x^3 + (a^3*c
os(c)^2 + a^3*sin(c)^2)*d^3)*sin(d*x + c)^2)
```

Fricas [**C**] Result contains complex when optimal does not.
time = 0.42, size = 890, normalized size = 1.15

$$[illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*((I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 + (b^3*x^6 + 2*a*b^
2*x^3 + a^2*b + sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(I*a*d^3/b)^
(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/
b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I
*a^3*d^3 + (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*
x^3 + I*a^2*b))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*s
qrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (I*a*b^2*d^
3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 + (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sq
rt(3)*(-I*b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x
+ 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(
3) + 1) - I*c) + (-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3 + (b^3*x
^6 + 2*a*b^2*x^3 + a^2*b + sqrt(3)*(-I*b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b))*
(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1
/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + (-I*a*b^2*d^3*x^6 - 2*I*a^2
*b*d^3*x^3 - I*a^3*d^3 - 2*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*(-I*a*d^3/b)^(1/
3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + (I*a*b^2*
d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 - 2*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1
/3)) + 6*(a*b^2*d^2*x^5 + a^2*b*d^2*x^2)*cos(d*x + c) + 6*(a*b^2*d*x^4 - 2*
a^2*b*d*x)*sin(d*x + c))/(a^2*b^4*d*x^6 + 2*a^3*b^3*d*x^3 + a^4*b^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] integrate(x^3*sin(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sin\left(c+d\,x\right)}{{\left(b\,x^3+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*sin(c + d*x))/(a + b*x^3)^3,x)
[Out] int((x^3*sin(c + d*x))/(a + b*x^3)^3, x)
```

3.110 $\int \frac{x^2 \sin(c+dx)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=777

$$\frac{d\cos(c+dx)}{18ab^2x^2}-\frac{d\cos(c+dx)}{18b^2x^2\,(a+bx^3)}-\frac{\sqrt[3]{-1}\,d\cos\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}}+\frac{d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\cdots}{27a^{5/}\cdots}$$

```
[Out] 1/27*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1
/27*(-1)^(1/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1
/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/27*(-1)^(2/3)*d*Ci((-1)^(2/3)*a^(1/3)*d/b^
(1/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/18*d*cos(d
*x+c)/a/b^2/x^2-1/18*d*cos(d*x+c)/b^2/x^2/(b*x^3+a)-1/54*(-1)^(2/3)*d^2*cos
(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4
/3)/b^(5/3)-1/54*d^2*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(
4/3)/b^(5/3)+1/54*(-1)^(1/3)*d^2*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1
)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(5/3)-1/54*d^2*Ci(a^(1/3)*d/b^(1/3
)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)-1/27*d*Si(a^(1/3)*d/b^(1/3)
+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/54*(-1)^(2/3)*d^2*Ci((-1)^
(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^
(5/3)+1/27*(-1)^(1/3)*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^(1
/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/54*(-1)^(1/3)*d^2*Ci((-1)^(2/3)*a^
(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(5/3)-1/
27*(-1)^(2/3)*d*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/
3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/6*sin(d*x+c)/b/(b*x^3+a)^2
```

Rubi [A]
time = 0.93, antiderivative size = 777, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3422, 3413, 3427, 3378, 3384, 3380, 3383, 3415, 3426}

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^2) - (d*Cos[c + d*x])/(18*b^2*x^2*(a + b*x^3))
 - ((-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^
(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^(4/3)) + (d*Cos[c - (a^(1/3)
*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) +
 ((-1)^(2/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2
/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*CosIntegral[(a^(
1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(54*a^(4/3)*b^(5/3)) -
 ((-1)^(2/3)*d^2*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c +
```

```
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^(4/3)*b^(5/3)) + ((-1)^(1/3)*d^2*Cos
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*
d)/b^(1/3)])/(54*a^(4/3)*b^(5/3)) - Sin[c + d*x]/(6*b*(a + b*x^3)^2) + ((-1
)^(2/3)*d^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)
*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(4/3)*b^(5/3)) - ((-1)^(1/3)*d*Sin[c + ((
-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[
(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(4/3)*b^(5/3)) - (d*Sin[c - (a^(1/3)*d)/b
^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) + ((-1
)^(1/3)*d^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)
*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(4/3)*b^(5/3)) - ((-1)^(2/3)*d*Sin[c - ((
-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) +
d*x])/(27*a^(5/3)*b^(4/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3413

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Sim
p[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1))), x] + (-Dist[
(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Cos[c + d*x])/x^n, x], x]
+ Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3422

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))),
x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2\sin(c+dx)}{(a+bx^3)^3}\,dx = -\frac{\sin(c+dx)}{6b\,(a+bx^3)^2} + \frac{d\int \frac{\cos(c+dx)}{(a+bx^3)^2}\,dx}{6b}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{\sin(c+dx)}{6b\,(a+bx^3)^2} - \frac{d\int \frac{\cos(c+dx)}{x^3(a+bx^3)}\,dx}{9b^2} - \frac{d^2\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx}{18b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{\sin(c+dx)}{6b\,(a+bx^3)^2} - \frac{d\int \left(\frac{\cos(c+dx)}{ax^3} - \frac{b\cos(c+dx)}{a(a+bx^3)}\right)dx}{9b^2} - \frac{d^2\int \left(\frac{\sin(c+dx)}{ax^2}\right.}{1}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{\sin(c+dx)}{6b\,(a+bx^3)^2} - \frac{d\int \frac{\cos(c+dx)}{x^3}\,dx}{9ab^2} + \frac{d\int \frac{\cos(c+dx)}{a+bx^3}\,dx}{9ab} - \frac{d^2\int \frac{\sin(c+dx)}{x^2}}{18ab^2}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^2} - \frac{d\cos(c+dx)}{18b^2x^2\,(a+bx^3)} + \frac{d^2\sin(c+dx)}{18ab^2x} - \frac{\sin(c+dx)}{6b\,(a+bx^3)^2} + \frac{d\int \left(-\frac{\cos}{3a^{2/3}\left(-\sqrt[3]{}\right.}\right.}{}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^2} - \frac{d\cos(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{\sin(c+dx)}{6b\,(a+bx^3)^2} - \frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{27a^{5/3}b} - \frac{d\int \frac{\cos}{-\sqrt[3]{a}+}}{27}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^2} - \frac{d\cos(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{d^3\cos(c)\mathrm{Ci}(dx)}{18ab^2} - \frac{\sin(c+dx)}{6b\,(a+bx^3)^2} + \frac{d^3\sin(c)\mathrm{Si}(dx)}{18ab^2}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^2} - \frac{d\cos(c+dx)}{18b^2x^2\,(a+bx^3)} - \frac{\sqrt[3]{-1}\,d\cos\left(c+\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{a}\,d}{\sqrt[3]{b}} - \right.}{27a^{5/3}b^{4/3}}$$

<u>**Mathematica**</u> [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.31, size = 449, normalized size = 0.58

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (I*d*RootSum[a + b*#1^3 & , ((-2*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] -
 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c + d*#1]*SinIntegral[d*(x
 - #1)] + (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*Cos
Integral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*
d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*
```

```
(x - #1)]*#1)/#1^2 & ] - I*d*RootSum[a + b*#1^3 & , ((2*I)*Cos[c + d*#1]*Co
sIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c +
 d*#1]*SinIntegral[d*(x - #1)] - (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)
] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)
]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c
 + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] + (6*b*Cos[d*x]*(d*x*(a + b*x
^3)*Cos[c] - 3*a*Sin[c]))/(a + b*x^3)^2 - (6*b*(3*a*Cos[c] + d*x*(a + b*x^3
)*Sin[c])*Sin[d*x])/(a + b*x^3)^2)/(108*a*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 1396, normalized size = 1.80

method	result
risch	$-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-iad^3+ibc^3+b\_Z^3-3bc^2\_Z\right)}\frac{\left(i\_R1ad^3+2i\_R1bc^3-\_R1^2bc^2-8i\_R1^2bc-acd^3-2\right.}{}\right.}{108a^2b^2}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(d^9*c^2*(-1/18*sin(d*x+c)*(8*a*c*d^3-8*a*d^3*(d*x+c)-5*b*c^4+20*b*c^
3*(d*x+c)-30*b*c^2*(d*x+c)^2+20*b*c*(d*x+c)^3-5*b*(d*x+c)^4)/a^2/d^6/(a*d^3
-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)^2-1/18*cos(d*x+c)*(c^2-
2*(d*x+c)*c+(d*x+c)^2)/a^2/d^6/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2
+b*(d*x+c)^3)-1/54/a^2/d^6/b*sum((_R1^2-2*_R1*c+c^2-10)/(_R1^2-2*_R1*c+c^2)
*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2
*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a^2/d^6/b*sum(1/(-_RR1+c)*(Si(-d*x+_RR1-c
)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*
c^2+a*d^3-b*c^3)))+1/9*sin(d*x+c)*d^3*c*(a*c^2*d^3+6*c*d^3*(d*x+c)*a-7*a*d^
3*(d*x+c)^2-c^5*b+10*b*c^3*(d*x+c)^2-20*b*c^2*(d*x+c)^3+15*b*c*(d*x+c)^4-4*
b*(d*x+c)^5)/a^2/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)^
2-1/9*cos(d*x+c)*d^3*c*(a*d^3-b*c^3+2*b*c^2*(d*x+c)-b*c*(d*x+c)^2)/a^2/b/(a
*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)-1/27*d^3*c/a^2/b^2*
sum((-_R1^2*b*c+2*_R1*b*c^2+a*d^3-b*c^3+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*
(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*
b*c+3*_Z*b*c^2+a*d^3-b*c^3))-2/27/a^2*d^3/b*c*sum((c+2*_RR1)/(-_RR1+c)*(Si(
-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*
b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18*sin(d*x+c)*d^3*(3*a^2*d^6-6*a*b*c^3*d^3+2
0*a*b*c^2*d^3*(d*x+c)-14*a*b*c*d^3*(d*x+c)^2+3*b^2*c^6-20*b^2*c^5*(d*x+c)+5
0*b^2*c^4*(d*x+c)^2-60*b^2*c^3*(d*x+c)^3+35*b^2*c^2*(d*x+c)^4-8*b^2*c*(d*x+
c)^5)/a^2/b/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)^2+1/1
8*cos(d*x+c)*d^3*(a*c*d^3+a*d^3*(d*x+c)-b*c^4+2*b*c^3*(d*x+c)-b*c^2*(d*x+c)
```

```
^2)/a^2/b/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+1/54*d^
3/a^2/b^2*sum((-_R1^2*b*c^2+_R1*a*d^3+2*_R1*b*c^3+a*c*d^3-b*c^4+8*_R1*b*c+2
*b*c^2)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1
)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/27*d^3/a^2/b^2*s
um((-4*_RR1^2*b*c+5*_RR1*b*c^2+a*d^3-b*c^3)/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+
_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3
*_Z*b*c^2+a*d^3-b*c^3)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*d*x^2*cos(d*x + c) + 7*(cos(c)^2 + sin(c)^2)*x*
sin(d*x + c) + ((d*x^2*cos(c) - 7*x*sin(c))*cos(d*x + c)^2 + (d*x^2*cos(c)
- 7*x*sin(c))*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^3*cos(c)^2 + b^3*sin(
c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*cos(
c)^2 + a^2*b*sin(c)^2)*d^2*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*cos(d*x
 + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2 + a*b^
2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^3 + (a^3*co
s(c)^2 + a^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-1/2*(9*a*d*x*cos(d*x
 + c) - 7*(8*b*x^3 - a)*sin(d*x + c))/(b^4*d^2*x^12 + 4*a*b^3*d^2*x^9 + 6*a
^2*b^2*d^2*x^6 + 4*a^3*b*d^2*x^3 + a^4*d^2), x) + 2*(((b^3*cos(c)^2 + b^3*s
in(c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*c
os(c)^2 + a^2*b*sin(c)^2)*d^2*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*cos(
d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2 + a
*b^2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^3 + (a^3
*cos(c)^2 + a^3*sin(c)^2)*d^2)*sin(d*x + c)^2)*integrate(-1/2*(9*a*d*x*cos(
d*x + c) - 7*(8*b*x^3 - a)*sin(d*x + c))/((b^4*d^2*x^12 + 4*a*b^3*d^2*x^9 +
 6*a^2*b^2*d^2*x^6 + 4*a^3*b*d^2*x^3 + a^4*d^2)*cos(d*x + c)^2 + (b^4*d^2*x
^12 + 4*a*b^3*d^2*x^9 + 6*a^2*b^2*d^2*x^6 + 4*a^3*b*d^2*x^3 + a^4*d^2)*sin(
d*x + c)^2), x) + ((d*x^2*sin(c) + 7*x*cos(c))*cos(d*x + c)^2 + (d*x^2*sin(
c) + 7*x*cos(c))*sin(d*x + c)^2)*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin(
c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*cos(
c)^2 + a^2*b*sin(c)^2)*d^2*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d^2)*cos(d*x
 + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d^2*x^9 + 3*(a*b^2*cos(c)^2 + a*b^
2*sin(c)^2)*d^2*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d^2*x^3 + (a^3*co
s(c)^2 + a^3*sin(c)^2)*d^2)*sin(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.42, size = 935, normalized size = 1.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*(((-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 +
a^2))*(I*a*d^3/b)^(2/3) - 2*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 - sqrt(3)*(b^2
*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/
3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + ((I*
b^2*x^6 + 2*I*a*b*x^3 + I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(-I*a*
d^3/b)^(2/3) - 2*(-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 + sqrt(3)*(b^2*x^6 + 2*a
*b*x^3 + a^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sq
rt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + ((-I*b^2*x^6
 - 2*I*a*b*x^3 - I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(
2/3) - 2*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 +
a^2))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))
*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + ((I*b^2*x^6 + 2*I*a*b*x
^3 + I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(-I*a*d^3/b)^(2/3) - 2*(-
I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(-I*
a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(
-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 2*((I*b^2*x^6 + 2*I*a*b*x^3 + I
*a^2)*(-I*a*d^3/b)^(2/3) + 2*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2)*(-I*a*d^3/b)
^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*((-
I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2)*(I*a*d^3/b)^(2/3) + 2*(-I*b^2*x^6 - 2*I*a*
b*x^3 - I*a^2)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c -
(I*a*d^3/b)^(1/3)) - 36*a^2*sin(d*x + c) + 12*(a*b*d*x^4 + a^2*d*x)*cos(d*x
 + c))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sin(c+d\,x)}{\left(b\,x^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*sin(c + d*x))/(a + b*x^3)^3,x)
[Out] int((x^2*sin(c + d*x))/(a + b*x^3)^3, x)
```

3.111 $\int \frac{x \sin(c+dx)}{(a+bx^3)^3} dx$

Optimal. Leaf size=1141

$$\frac{d\cos(c+dx)}{18ab^2x^3} - \frac{d\cos(c+dx)}{18b^2x^3(a+bx^3)} - \frac{2d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{27a^2b} - \frac{2d\cos\left(c-\frac{\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\text{Ci}}{27a^2b}$$

```
[Out] -2/27*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^2/b-2/27*d*Ci(
(-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2/b
-2/27*d*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(
1/3))/a^2/b+1/18*d*cos(d*x+c)/a/b^2/x^3-1/18*d*cos(d*x+c)/b^2/x^3/(b*x^3+a)
-2/27*(-1)^(2/3)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)
*d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)-1/54*(-1)^(1/3)*d^2*cos(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)-2/27*cos
(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)+1/54*d^2*co
s(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)+2/27*(-1)^
(1/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d
*x)/a^(7/3)/b^(2/3)+1/54*(-1)^(2/3)*d^2*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))
*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)-2/27*Ci(a^(1/3)*d/b^(
1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)+1/54*d^2*Ci(a^(1/3)*d/b^
(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+2/27*d*Si(a^(1/3)*d/b^(
1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^2/b-2/27*(-1)^(2/3)*Ci((-1)^(1/3)*a^(1
/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)-1/54
*(-1)^(1/3)*d^2*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/
3)*d/b^(1/3))/a^(5/3)/b^(4/3)+2/27*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*
sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2/b+2/27*(-1)^(1/3)*Ci((-1)^(2/3)*a^(
1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)+1/5
4*(-1)^(2/3)*d^2*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1
/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+2/27*d*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*
sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^2/b-1/18*sin(d*x+c)/a/b^2/x^4+2/9*sin
(d*x+c)/a^2/b/x-1/6*sin(d*x+c)/b/x/(b*x^3+a)^2+1/18*sin(d*x+c)/b^2/x^4/(b*x
^3+a)
```

Rubi [A]
time = 1.97, antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 89, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3424, 3426, 3378, 3384, 3380, 3383, 3427, 3425, 3414}

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^3) - (d*Cos[c + d*x])/(18*b^2*x^3*(a + b*x^3))
 - (2*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(
1/3)*d)/b^(1/3) - d*x])/(27*a^2*b) - (2*d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosI
ntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b) - (2*d*Cos[c - ((-1)^(2/3)*a
^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a
^2*b) - (2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/
3)])/(27*a^(7/3)*b^(2/3)) + (d^2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - (a^(1/3)*d)/b^(1/3)])/(54*a^(5/3)*b^(4/3)) - (2*(-1)^(2/3)*CosIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/
3)])/(27*a^(7/3)*b^(2/3)) - ((-1)^(1/3)*d^2*CosIntegral[((-1)^(1/3)*a^(1/3)
*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^(5/3)*b^(
4/3)) + (2*(-1)^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(7/3)*b^(2/3)) + ((-1)^(2/3)*d^
2*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)])/(54*a^(5/3)*b^(4/3)) - Sin[c + d*x]/(18*a*b^2*x^4) + (2*S
in[c + d*x])/(9*a^2*b*x) - Sin[c + d*x]/(6*b*x*(a + b*x^3)^2) + Sin[c + d*x
]/(18*b^2*x^4*(a + b*x^3)) + (2*(-1)^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b
^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(7/3)*b^(2
/3)) + ((-1)^(1/3)*d^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*Sin[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
- d*x])/(27*a^2*b) - (2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d
)/b^(1/3) + d*x])/(27*a^(7/3)*b^(2/3)) + (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*
SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) + (2*d*Sin[c -
 (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b) +
(2*(-1)^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/
3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(7/3)*b^(2/3)) + ((-1)^(2/3)*d^2*Cos[c
- ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3
) + d*x])/(54*a^(5/3)*b^(4/3)) + (2*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3425

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx = -\frac{\sin(c+dx)}{6bx(a+bx^3)^2} - \frac{\int \frac{\sin(c+dx)}{x^2(a+bx^3)^2}\,dx}{6b} + \frac{d\int \frac{\cos(c+dx)}{x(a+bx^3)^2}\,dx}{6b}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^3(a+bx^3)} - \frac{\sin(c+dx)}{6bx(a+bx^3)^2} + \frac{\sin(c+dx)}{18b^2x^4(a+bx^3)} + \frac{2\int \frac{\sin(c+dx)}{x^5(a+bx^3)}\,dx}{9b^2} - \frac{d\int \frac{\cos(c+}{x^4(a+b}}{18b^}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^3(a+bx^3)} - \frac{\sin(c+dx)}{6bx(a+bx^3)^2} + \frac{\sin(c+dx)}{18b^2x^4(a+bx^3)} + \frac{2\int \left(\frac{\sin(c+dx)}{ax^5} - \frac{b\sin(c+dx)}{a^2x^2} + \right.}{9b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^3(a+bx^3)} - \frac{\sin(c+dx)}{6bx(a+bx^3)^2} + \frac{\sin(c+dx)}{18b^2x^4(a+bx^3)} + \frac{2\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{9a^2} + \frac{2\int \frac{\sin(c+}{x^5}}{9ab}$$

$$= \frac{2d\cos(c+dx)}{27ab^2x^3} - \frac{d\cos(c+dx)}{18b^2x^3(a+bx^3)} - \frac{\sin(c+dx)}{18ab^2x^4} + \frac{d^2\sin(c+dx)}{36ab^2x^2} + \frac{2\sin(c+dx)}{9a^2bx} -$$

$$= \frac{d\cos(c+dx)}{18ab^2x^3} + \frac{d^3\cos(c+dx)}{36ab^2x} - \frac{d\cos(c+dx)}{18b^2x^3(a+bx^3)} + \frac{2d\cos(c)\text{Ci}(dx)}{9a^2b} - \frac{\sin(c+dx)}{18ab^2x^4} -$$

$$= \frac{d\cos(c+dx)}{18ab^2x^3} - \frac{d^3\cos(c+dx)}{108ab^2x} - \frac{d\cos(c+dx)}{18b^2x^3(a+bx^3)} - \frac{\sin(c+dx)}{18ab^2x^4} + \frac{2\sin(c+dx)}{9a^2bx} -$$

$$= \frac{d\cos(c+dx)}{18ab^2x^3} - \frac{d\cos(c+dx)}{18b^2x^3(a+bx^3)} - \frac{2d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{27a^2b} -$$

$$= \frac{d\cos(c+dx)}{18ab^2x^3} - \frac{d\cos(c+dx)}{18b^2x^3(a+bx^3)} - \frac{2d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{27a^2b} -$$

$$= \frac{d\cos(c+dx)}{18ab^2x^3} - \frac{d\cos(c+dx)}{18b^2x^3(a+bx^3)} - \frac{2d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}} - dx\right)}{27a^2b} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.41, size = 698, normalized size = 0.61

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -1/108*(RootSum[a + b*#1^3 & , ((-I)*a*d^2*Cos[c + d*#1]*CosIntegral[d*(x -
 #1)] - a*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - a*d^2*Cos[c + d*#1]*S
inIntegral[d*(x - #1)] + I*a*d^2*Sin[c + d*#1]*SinIntegral[d*(x - #1)] - (4
*I)*b*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - 4*b*CosIntegral[d*(x - #1)
]*Sin[c + d*#1]*#1 - 4*b*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + (4*I)*b
*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + 4*b*d*Cos[c + d*#1]*CosIntegral
[d*(x - #1)]*#1^2 - (4*I)*b*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 -
(4*I)*b*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2 - 4*b*d*Sin[c + d*#1]*
SinIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + RootSum[a + b*#1^3 & , (I*a*d^2*Co
s[c + d*#1]*CosIntegral[d*(x - #1)] - a*d^2*CosIntegral[d*(x - #1)]*Sin[c +
 d*#1] - a*d^2*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - I*a*d^2*Sin[c + d*#1
]*SinIntegral[d*(x - #1)] + (4*I)*b*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#
1 - 4*b*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - 4*b*Cos[c + d*#1]*SinInt
egral[d*(x - #1)]*#1 - (4*I)*b*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + 4
*b*d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1^2 + (4*I)*b*d*CosIntegral[d*(
x - #1)]*Sin[c + d*#1]*#1^2 + (4*I)*b*d*Cos[c + d*#1]*SinIntegral[d*(x - #1
)]*#1^2 - 4*b*d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2)/#1^2 & ] - (6*b
*Cos[d*x]*(a*d*(a + b*x^3)*Cos[c] + b*x^2*(7*a + 4*b*x^3)*Sin[c]))/(a + b*x
^3)^2 - (6*b*(b*x^2*(7*a + 4*b*x^3)*Cos[c] - a*d*(a + b*x^3)*Sin[c])*Sin[d*
x])/(a + b*x^3)^2)/(a^2*b^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 847, normalized size = 0.74 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(-1/18*sin(d*x+c)*d^3*(a*c^2*d^3+6*c*d^3*(d*x+c)*a-7*a*d^3*(d*x+c)^2-
c^5*b+10*b*c^3*(d*x+c)^2-20*b*c^2*(d*x+c)^3+15*b*c*(d*x+c)^4-4*b*(d*x+c)^5)
/a^2/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)^2+1/18*cos(d
*x+c)*d^3*(a*d^3-b*c^3+2*b*c^2*(d*x+c)-b*c*(d*x+c)^2)/a^2/b/(a*d^3-b*c^3+3*
b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)+1/54*d^3/a^2/b^2*sum((-_R1^2*b*c
+2*_R1*b*c^2+a*d^3-b*c^3+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c
)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+
a*d^3-b*c^3))+1/27*d^3/a^2/b*sum((c+2*_RR1)/(-_RR1+c)*(Si(-d*x+_RR1-c)*sin(
_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*
d^3-b*c^3))-d^9*c*(-1/18*sin(d*x+c)*(8*a*c*d^3-8*a*d^3*(d*x+c)-5*b*c^4+20*b
*c^3*(d*x+c)-30*b*c^2*(d*x+c)^2+20*b*c*(d*x+c)^3-5*b*(d*x+c)^4)/a^2/d^6/(a*
d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)^2-1/18*cos(d*x+c)*(c
^2-2*(d*x+c)*c+(d*x+c)^2)/a^2/d^6/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c
)^2+b*(d*x+c)^3)-1/54/a^2/d^6/b*sum((_R1^2-2*_R1*c+c^2-10)/(_R1^2-2*_R1*c+c
^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_
Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a^2/d^6/b*sum(1/(-_RR1+c)*(Si(-d*x+_RR
1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z
*b*c^2+a*d^3-b*c^3))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((cos(c)^2 + sin(c)^2)*x*cos(d*x + c) + (x*cos(d*x + c)^2*cos(c) + x*c
os(c)*sin(d*x + c)^2)*cos(d*x + 2*c) + 2*(((b^3*cos(c)^2 + b^3*sin(c)^2)*d*
x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b
*sin(c)^2)*d*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^3*
cos(c)^2 + b^3*sin(c)^2)*d*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^6
+ 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)
*d)*sin(d*x + c)^2)*integrate(1/2*(8*b*x^3 - a)*cos(d*x + c)/(b^4*d*x^12 +
4*a*b^3*d*x^9 + 6*a^2*b^2*d*x^6 + 4*a^3*b*d*x^3 + a^4*d), x) + 2*(((b^3*cos
(c)^2 + b^3*sin(c)^2)*d*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^6 + 3
*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)
*cos(d*x + c)^2 + ((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^9 + 3*(a*b^2*cos(c)^2
+ a*b^2*sin(c)^2)*d*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^3 + (a^3*
cos(c)^2 + a^3*sin(c)^2)*d)*sin(d*x + c)^2)*integrate(1/2*(8*b*x^3 - a)*cos
(d*x + c)/((b^4*d*x^12 + 4*a*b^3*d*x^9 + 6*a^2*b^2*d*x^6 + 4*a^3*b*d*x^3 +
a^4*d)*cos(d*x + c)^2 + (b^4*d*x^12 + 4*a*b^3*d*x^9 + 6*a^2*b^2*d*x^6 + 4*a
^3*b*d*x^3 + a^4*d)*sin(d*x + c)^2), x) + (x*cos(d*x + c)^2*sin(c) + x*sin(
d*x + c)^2*sin(c))*sin(d*x + 2*c))/(((b^3*cos(c)^2 + b^3*sin(c)^2)*d*x^9 +
3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^6 + 3*(a^2*b*cos(c)^2 + a^2*b*sin(c
)^2)*d*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*cos(d*x + c)^2 + ((b^3*cos(c)
^2 + b^3*sin(c)^2)*d*x^9 + 3*(a*b^2*cos(c)^2 + a*b^2*sin(c)^2)*d*x^6 + 3*(a
^2*b*cos(c)^2 + a^2*b*sin(c)^2)*d*x^3 + (a^3*cos(c)^2 + a^3*sin(c)^2)*d)*si
n(d*x + c)^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.46, size = 1319, normalized size = 1.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/216*((8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3 + 4*(I*b^3*x^6 + 2*
I*a*b^2*x^3 + I*a^2*b + sqrt(3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*(I*a*d^3/b
)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(3)*(I*a*b^2*d^3
*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(
I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1)
 - I*c) + (8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3 + 4*(-I*b^3*x^6 -
```

```
 2*I*a*b^2*x^3 - I*a^2*b - sqrt(3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*(-I*a*d
^3/b)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(3)*(I*a*b^2
*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1
/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3
) + 1) + I*c) + (8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3 + 4*(I*b^3*
x^6 + 2*I*a*b^2*x^3 + I*a^2*b - sqrt(3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*(I
*a*d^3/b)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(3)*(-I*
a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3))*(I*a*d^3/b)^(1/3))*Ei(-I*d*
x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqr
t(3) + 1) - I*c) + (8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3 + 4*(-I*
b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b + sqrt(3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
)*(-I*a*d^3/b)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(3)
*(-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3))*(-I*a*d^3/b)^(1/3))*Ei
(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*
(-I*sqrt(3) + 1) + I*c) + 2*(4*a*b^2*d^3*x^6 + 8*a^2*b*d^3*x^3 + 4*a^3*d^3
+ 4*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b)*(-I*a*d^3/b)^(2/3) + (a*b^2*d^3*x
^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)
^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + 2*(4*a*b^2*d^3*x^6 + 8*a^2*b*d^3*x^3
 + 4*a^3*d^3 + 4*(-I*b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b)*(I*a*d^3/b)^(2/3) +
 (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x +
 (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) - 12*(a^2*b*d^3*x^3 + a^3*
d^3)*cos(d*x + c) - 12*(4*a*b^2*d^2*x^5 + 7*a^2*b*d^2*x^2)*sin(d*x + c))/(a
^3*b^3*d^2*x^6 + 2*a^4*b^2*d^2*x^3 + a^5*b*d^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x**3+a)**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
[Out] integrate(x*sin(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sin\left(c+d\,x\right)}{{\left(b\,x^3+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(c + d*x))/(a + b*x^3)^3,x)
```

```
[Out] int((x*sin(c + d*x))/(a + b*x^3)^3, x)
```

3.112 $\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=1161

$$\frac{d\cos(c+dx)}{18ab^2x^4}-\frac{d\cos(c+dx)}{18a^2bx}-\frac{d\cos(c+dx)}{18b^2x^4(a+bx^3)}+\frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}b^{2/3}}+\cdots$$

```
[Out] 1/9*(-1)^(2/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*cos(c+(-1)^(1/3)*a^(1
/3)*d/b^(1/3))/a^(7/3)/b^(2/3)-1/9*(-1)^(1/3)*d*Ci((-1)^(2/3)*a^(1/3)*d/b^(
1/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)+1/9*d*Ci(a^(1
/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)+1/18*d*cos(d*x+
c)/a/b^2/x^4-1/18*d*cos(d*x+c)/a^2/b/x-1/18*d*cos(d*x+c)/b^2/x^4/(b*x^3+a)+
5/27*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)/b^(1/3)+5/2
7*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1/3)-1/9*si
n(d*x+c)/a/b^2/x^5+5/18*sin(d*x+c)/a^2/b/x^2-1/6*sin(d*x+c)/b/x^2/(b*x^3+a)
^2+1/9*sin(d*x+c)/b^2/x^5/(b*x^3+a)-5/27*(-1)^(1/3)*cos(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)/b^(1/3)-1/54*d^2
*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/
a^2/b+1/9*(-1)^(1/3)*d*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3
)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)-1/54*d^2*cos(c-a^(1/3)*d/b^(1/3))*Si(a
^(1/3)*d/b^(1/3)+d*x)/a^2/b+5/27*(-1)^(2/3)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1
/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(8/3)/b^(1/3)-1/54*d^2*cos(c-(-
1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^2/b-1/54
*d^2*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^2/b-1/9*d*Si(a^(1
/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)-5/27*(-1)^(1/3)
*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a
^(8/3)/b^(1/3)-1/54*d^2*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/
3)*a^(1/3)*d/b^(1/3))/a^2/b+5/27*(-1)^(2/3)*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)
+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1/3)-1/54*d^2*Ci((-1)^
(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^2/b-1/9*
(-1)^(2/3)*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c+(-1)^(1/3)*a^(1/3)
*d/b^(1/3))/a^(7/3)/b^(2/3)
```

Rubi [A]
time = 2.08, antiderivative size = 1161, normalized size of antiderivative = 1.00, number of steps used = 99, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3412, 3424, 3426, 3378, 3384, 3380, 3383, 3414, 3427, 3425}

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x^3)^3,x]
```

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^4) - (d*Cos[c + d*x])/(18*a^2*b*x) - (d*Cos[c
+ d*x])/(18*b^2*x^4*(a + b*x^3)) + ((-1)^(2/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(7/3)
*b^(2/3)) + (d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3)
 + d*x])/(9*a^(7/3)*b^(2/3)) - ((-1)^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)*b^(
2/3)) + (5*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/
3)])/(27*a^(8/3)*b^(1/3)) - (d^2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - (a^(1/3)*d)/b^(1/3)])/(54*a^2*b) - (5*(-1)^(1/3)*CosIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a
^(8/3)*b^(1/3)) - (d^2*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Si
n[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^2*b) + (5*(-1)^(2/3)*CosIntegr
al[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(
1/3)])/(27*a^(8/3)*b^(1/3)) - (d^2*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/
3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(54*a^2*b) - Sin[c + d*x
]/(9*a*b^2*x^5) + (5*Sin[c + d*x])/(18*a^2*b*x^2) - Sin[c + d*x]/(6*b*x^2*(
a + b*x^3)^2) + Sin[c + d*x]/(9*b^2*x^5*(a + b*x^3)) + (5*(-1)^(1/3)*Cos[c
+ ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3
) - d*x])/(27*a^(8/3)*b^(1/3)) + (d^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3
)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^2*b) + ((-1)^(2
/3)*d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/
3)*d)/b^(1/3) - d*x])/(9*a^(7/3)*b^(2/3)) + (5*Cos[c - (a^(1/3)*d)/b^(1/3)]
*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (d^2*Cos[c
- (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^2*b) -
 (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9
*a^(7/3)*b^(2/3)) + (5*(-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*S
inIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (d
^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*
d)/b^(1/3) + d*x])/(54*a^2*b) + ((-1)^(1/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d
)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)*b^
(2/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
```

```
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Sim
p[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1))), x] + (-Dist[
(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Sin[c + d*x])/x^n, x], x]
- Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3425

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+bx^3)^3}\,dx = -\frac{\sin(c+dx)}{6bx^2(a+bx^3)^2} - \frac{\int \frac{\sin(c+dx)}{x^3(a+bx^3)^2}\,dx}{3b} + \frac{d\int \frac{\cos(c+dx)}{x^2(a+bx^3)^2}\,dx}{6b}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^4(a+bx^3)} - \frac{\sin(c+dx)}{6bx^2(a+bx^3)^2} + \frac{\sin(c+dx)}{9b^2x^5(a+bx^3)} + \frac{5\int \frac{\sin(c+dx)}{x^6(a+bx^3)}\,dx}{9b^2} - \frac{d\int \frac{\cos(c+}{x^5(a+b}}{9b^2} \ldots$$

$$= -\frac{d\cos(c+dx)}{18b^2x^4(a+bx^3)} - \frac{\sin(c+dx)}{6bx^2(a+bx^3)^2} + \frac{\sin(c+dx)}{9b^2x^5(a+bx^3)} + \frac{5\int \left(\frac{\sin(c+dx)}{ax^6} - \frac{b\sin(c+dx)}{a^2x^3} + \ldots\right.}{9b^2} \ldots$$

$$= -\frac{d\cos(c+dx)}{18b^2x^4(a+bx^3)} - \frac{\sin(c+dx)}{6bx^2(a+bx^3)^2} + \frac{\sin(c+dx)}{9b^2x^5(a+bx^3)} + \frac{5\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{9a^2} + \frac{5\int \frac{\sin(c+d}{x^6}}{9ab^2} \ldots$$

$$= \frac{d\cos(c+dx)}{12ab^2x^4} - \frac{d\cos(c+dx)}{3a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4(a+bx^3)} - \frac{\sin(c+dx)}{9ab^2x^5} + \frac{d^2\sin(c+dx)}{54ab^2x^3} + \ldots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^4} + \frac{d^3\cos(c+dx)}{108ab^2x^2} - \frac{d\cos(c+dx)}{18a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4(a+bx^3)} + \frac{d^2\mathrm{Ci}(dx)\sin(c)}{18a^2b} \ldots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^4} - \frac{d^3\cos(c+dx)}{216ab^2x^2} - \frac{d\cos(c+dx)}{18a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4(a+bx^3)} - \frac{5d^2\mathrm{Ci}(dx)\sin(c)}{18a^2b} \ldots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^4} - \frac{d\cos(c+dx)}{18a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4(a+bx^3)} + \frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{C}\ldots}{9a^{7/3}b^{2/3}} \ldots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^4} - \frac{d\cos(c+dx)}{18a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4(a+bx^3)} + \frac{d^5\cos(c)\mathrm{Ci}(dx)}{108ab^2} + \frac{(-1)^{2/3}d\cos\Big(\ldots}{\ldots} \ldots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^4} - \frac{d\cos(c+dx)}{18a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4(a+bx^3)} - \frac{d^5\cos(c)\mathrm{Ci}(dx)}{216ab^2} + \frac{(-1)^{2/3}d\cos\Big(\ldots}{\ldots} \ldots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^4} - \frac{d\cos(c+dx)}{18a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4(a+bx^3)} + \frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{C}\ldots}{9a^{7/3}b^{2/3}} \ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 0.32, size = 675, normalized size = 0.58

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*x^3)^3,x]
```

```
[Out] (((-I)*RootSum[a + b*#1^3 & , (-10*Cos[c + d*#1]*CosIntegral[d*(x - #1)] +
(10*I)*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + (10*I)*Cos[c + d*#1]*SinInte
gral[d*(x - #1)] + 10*Sin[c + d*#1]*SinIntegral[d*(x - #1)] - (6*I)*d*Cos[c
 + d*#1]*CosIntegral[d*(x - #1)]*#1 - 6*d*CosIntegral[d*(x - #1)]*Sin[c + d
*#1]*#1 - 6*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + (6*I)*d*Sin[c + d*
#1]*SinIntegral[d*(x - #1)]*#1 + d^2*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*
#1^2 - I*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 - I*d^2*Cos[c + d*#
1]*SinIntegral[d*(x - #1)]*#1^2 - d^2*Sin[c + d*#1]*SinIntegral[d*(x - #1)]
*#1^2)/#1^2 & ])/b + (I*RootSum[a + b*#1^3 & , (-10*Cos[c + d*#1]*CosIntegr
al[d*(x - #1)] - (10*I)*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - (10*I)*Cos[
c + d*#1]*SinIntegral[d*(x - #1)] + 10*Sin[c + d*#1]*SinIntegral[d*(x - #1)
] + (6*I)*d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - 6*d*CosIntegral[d*(x
 - #1)]*Sin[c + d*#1]*#1 - 6*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - (
6*I)*d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + d^2*Cos[c + d*#1]*CosInte
gral[d*(x - #1)]*#1^2 + I*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 +
I*d^2*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2 - d^2*Sin[c + d*#1]*SinInt
egral[d*(x - #1)]*#1^2)/#1^2 & ])/b - (6*x*Cos[d*x]*(d*x*(a + b*x^3)*Cos[c]
 - (8*a + 5*b*x^3)*Sin[c]))/(a + b*x^3)^2 + (6*x*((8*a + 5*b*x^3)*Cos[c] +
d*x*(a + b*x^3)*Sin[c])*Sin[d*x])/(a + b*x^3)^2)/(108*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 392, normalized size = 0.34

method	result
risch	$-\frac{i d^2 \left(\sum_{\_R1 = \mathrm{RootOf}\left(-3i\_Z^2 bc - ia\,d^3 + ib\,c^3 + b\_Z^3 - 3b\,c^2\_Z\right)} \frac{\left(-2ic\_R1 + \_R1^2 - c^2 + 6ic - 6\_R1 + 10\right) e^{\_R1} \mathrm{expIntegral}}{-2ic\_R1 + \_R1^2 - c^2} \right.}{108a^2 b}$
derivativedivides	$d^8 \left(-\frac{\sin(dx+c)\left(8ac\,d^3 - 8a\,d^3(dx+c) - 5b\,c^4 + 20b\,c^3(dx+c) - 30b\,c^2(dx+c)^2 + 20bc(dx+c)^3 - 5b(dx+c)^4\right)}{18a^2 d^6 \left(a\,d^3 - b\,c^3 + 3b\,c^2(dx+c) - 3bc(dx+c)^2 + b(dx+c)^3\right)^2} - \frac{}{18a^2 d^6 (a} \right.$
default	$d^8 \left(-\frac{\sin(dx+c)\left(8ac\,d^3 - 8a\,d^3(dx+c) - 5b\,c^4 + 20b\,c^3(dx+c) - 30b\,c^2(dx+c)^2 + 20bc(dx+c)^3 - 5b(dx+c)^4\right)}{18a^2 d^6 \left(a\,d^3 - b\,c^3 + 3b\,c^2(dx+c) - 3bc(dx+c)^2 + b(dx+c)^3\right)^2} - \frac{}{18a^2 d^6 (a} \right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^8*(-1/18*sin(d*x+c)*(8*a*c*d^3-8*a*d^3*(d*x+c)-5*b*c^4+20*b*c^3*(d*x+c)-3
0*b*c^2*(d*x+c)^2+20*b*c*(d*x+c)^3-5*b*(d*x+c)^4)/a^2/d^6/(a*d^3-b*c^3+3*b*
c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)^2-1/18*cos(d*x+c)*(c^2-2*(d*x+c)*c
+(d*x+c)^2)/a^2/d^6/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^
3)-1/54/a^2/d^6/b*sum((_R1^2-2*_R1*c+c^2-10)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+
_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b
*c^2+a*d^3-b*c^3))+1/9/a^2/d^6/b*sum(1/(-_RR1+c)*(Si(-d*x+_RR1-c)*sin(_RR1)
+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b
*c^3)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*x^3 + a)^3, x)
```

Fricas [**C**] Result contains complex when optimal does not.
time = 0.44, size = 1223, normalized size = 1.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*((-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3 + 3*(b^3*x^6 + 2*a
*b^2*x^3 + a^2*b - sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(I*a*d^3/
b)^(2/3) + 5*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(3)*(-I*b^3*x^6 - 2*I*a*b
^2*x^3 - I*a^2*b))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I
*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (I*a*b^2*d
^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 + 3*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b -
 sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(-I*a*d^3/b)^(2/3) + 5*(b^3
*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(3)*(-I*b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b)
)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e
^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (-I*a*b^2*d^3*x^6 - 2*I*a
^2*b*d^3*x^3 - I*a^3*d^3 + 3*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(3)*(-I*b
^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b))*(I*a*d^3/b)^(2/3) + 5*(b^3*x^6 + 2*a*b^2
*x^3 + a^2*b - sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(I*a*d^3/b)^(
1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)
^(1/3)*(-I*sqrt(3) + 1) - I*c) + (I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a
^3*d^3 + 3*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(3)*(-I*b^3*x^6 - 2*I*a*b^2
*x^3 - I*a^2*b))*(-I*a*d^3/b)^(2/3) + 5*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sq
rt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x +
```

```
 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt
(3) + 1) + I*c) + (I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 - 6*(b^3
*x^6 + 2*a*b^2*x^3 + a^2*b)*(-I*a*d^3/b)^(2/3) - 10*(b^3*x^6 + 2*a*b^2*x^3
+ a^2*b)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*
d^3/b)^(1/3)) + (-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3 - 6*(b^3*
x^6 + 2*a*b^2*x^3 + a^2*b)*(I*a*d^3/b)^(2/3) - 10*(b^3*x^6 + 2*a*b^2*x^3 +
a^2*b)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3
/b)^(1/3)) - 6*(a*b^2*d^2*x^5 + a^2*b*d^2*x^2)*cos(d*x + c) + 6*(5*a*b^2*d*
x^4 + 8*a^2*b*d*x)*sin(d*x + c))/(a^3*b^3*d*x^6 + 2*a^4*b^2*d*x^3 + a^5*b*d
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)}{{\left(b\,x^3+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*x^3)^3,x)
```

```
[Out] int(sin(c + d*x)/(a + b*x^3)^3, x)
```

3.113 $\int \frac{\sin(c+dx)}{x(a+bx^3)^3}\,dx$

Optimal. Leaf size=1163

$$\frac{d\cos(c+dx)}{18ab^2x^5}-\frac{d\cos(c+dx)}{18a^2bx^2}-\frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)}+\frac{4\sqrt[3]{-1}\,d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}-dx\right)}{27a^{8/3}\sqrt[3]{b}}-\ 4$$

```
[Out] -1/6*sin(d*x+c)/a/b^2/x^6+1/3*sin(d*x+c)/a^2/b/x^3-1/6*sin(d*x+c)/b/x^3/(b*
x^3+a)^2+1/6*sin(d*x+c)/b^2/x^6/(b*x^3+a)+cos(c)*Si(d*x)/a^3+Ci(d*x)*sin(c)
/a^3-4/27*d*Ci(a^(1/3)*d/b^(1/3)+d*x)*cos(c-a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1
/3)+1/18*d*cos(d*x+c)/a/b^2/x^5-1/18*d*cos(d*x+c)/a^2/b/x^2-1/18*d*cos(d*x+
c)/b^2/x^5/(b*x^3+a)+1/54*d^2*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)
+d*x)/a^(7/3)/b^(2/3)+1/54*d^2*Ci(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^
(1/3))/a^(7/3)/b^(2/3)+4/27*d*Si(a^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(
1/3))/a^(8/3)/b^(1/3)+4/27*(-1)^(1/3)*d*Ci((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x
)*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1/3)-4/27*(-1)^(2/3)*d*Ci(
(-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(8/
3)/b^(1/3)-1/54*(-1)^(1/3)*d^2*cos(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^
(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)+1/54*(-1)^(2/3)*d^2*Ci((-1)^(1
/3)*a^(1/3)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2
/3)-1/54*(-1)^(1/3)*d^2*Ci((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/
3)*a^(1/3)*d/b^(1/3))/a^(7/3)/b^(2/3)+4/27*(-1)^(2/3)*d*Si((-1)^(2/3)*a^(1/
3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1/3)-1/3*c
os(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^
3-1/3*cos(c-a^(1/3)*d/b^(1/3))*Si(a^(1/3)*d/b^(1/3)+d*x)/a^3-1/3*cos(c-(-1)
^(2/3)*a^(1/3)*d/b^(1/3))*Si((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^3-1/3*Ci(a
^(1/3)*d/b^(1/3)+d*x)*sin(c-a^(1/3)*d/b^(1/3))/a^3-1/3*Ci((-1)^(1/3)*a^(1/3
)*d/b^(1/3)-d*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^3-1/3*Ci((-1)^(2/3)*
a^(1/3)*d/b^(1/3)+d*x)*sin(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^3+1/54*(-1)^(2
/3)*d^2*cos(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3
)+d*x)/a^(7/3)/b^(2/3)-4/27*(-1)^(1/3)*d*Si(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d
*x)*sin(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(8/3)/b^(1/3)
```

Rubi [A]
time = 2.43, antiderivative size = 1163, normalized size of antiderivative = 1.00, number of steps used = 110, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3424, 3426, 3378, 3384, 3380, 3383, 3427, 3415, 3425}

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^3)^3),x]
```

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^5) - (d*Cos[c + d*x])/(18*a^2*b*x^2) - (d*Cos[
c + d*x])/(18*b^2*x^5*(a + b*x^3)) + (4*(-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^
(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^
(8/3)*b^(1/3)) - (4*d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/
b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (4*(-1)^(2/3)*d*Cos[c - ((-1)^(2/3)*
a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*
a^(8/3)*b^(1/3)) + (CosIntegral[d*x]*Sin[c])/a^3 - (CosIntegral[(a^(1/3)*d)
/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^3) + (d^2*CosIntegral[(a
^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(54*a^(7/3)*b^(2/3))
 - (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)])/(3*a^3) + ((-1)^(2/3)*d^2*CosIntegral[((-1)^(1/3)*a^(1/
3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^(7/3)*b
^(2/3)) - (CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^3) - ((-1)^(1/3)*d^2*CosIntegral[((-1)^(2/3
)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(54*a^
(7/3)*b^(2/3)) - Sin[c + d*x]/(6*a*b^2*x^6) + Sin[c + d*x]/(3*a^2*b*x^3) -
Sin[c + d*x]/(6*b*x^3*(a + b*x^3)^2) + Sin[c + d*x]/(6*b^2*x^6*(a + b*x^3))
 + (Cos[c]*SinIntegral[d*x])/a^3 + (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]
*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^3) - ((-1)^(2/3)*d
^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*
d)/b^(1/3) - d*x])/(54*a^(7/3)*b^(2/3)) + (4*(-1)^(1/3)*d*Sin[c + ((-1)^(1/
3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(
27*a^(8/3)*b^(1/3)) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)
/b^(1/3) + d*x])/(3*a^3) + (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a
^(1/3)*d)/b^(1/3) + d*x])/(54*a^(7/3)*b^(2/3)) + (4*d*Sin[c - (a^(1/3)*d)/b
^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (Cos
[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(
1/3) + d*x])/(3*a^3) - ((-1)^(1/3)*d^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(7/3)*b^(2/3))
 + (4*(-1)^(2/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1
)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
```

```
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3424

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sin[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3425

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cos[c + d*x]/(b*n*(p + 1)))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3426

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3427

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
```

```
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^3}\,dx = -\frac{\sin(c+dx)}{6bx^3\left(a+bx^3\right)^2} - \frac{\int \frac{\sin(c+dx)}{x^4\left(a+bx^3\right)^2}\,dx}{2b} + \frac{d\int \frac{\cos(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx^3\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{6b^2x^6\left(a+bx^3\right)} + \frac{\int \frac{\sin(c+dx)}{x^7\left(a+bx^3\right)}\,dx}{b^2} - \frac{d\int \frac{\cos(c+dx)}{x^6(a+bx}}{6b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx^3\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{6b^2x^6\left(a+bx^3\right)} + \frac{\int \left(\frac{\sin(c+dx)}{ax^7} - \frac{b\sin(c+dx)}{a^2x^4} + b\right.}{b}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx^3\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{6b^2x^6\left(a+bx^3\right)} + \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^3} + \frac{\int \frac{\sin(c+dx)}{x^7}}{ab^2}$$

$$= \frac{4d\cos(c+dx)}{45ab^2x^5} - \frac{2d\cos(c+dx)}{9a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6ab^2x^6} + \frac{d^2\sin(c+dx)}{72ab^2x^4} +$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} + \frac{d^3\cos(c+dx)}{216ab^2x^3} - \frac{d\cos(c+dx)}{18a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} + \frac{\mathrm{Ci}(dx)\sin(c)}{a^3} -$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} - \frac{d^3\cos(c+dx)}{360ab^2x^3} - \frac{d\cos(c+dx)}{18a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} + \frac{d^3\cos(c)\mathrm{Ci}(dx)}{18a^2b}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} - \frac{d\cos(c+dx)}{18a^2bx^2} - \frac{d^5\cos(c+dx)}{432ab^2x} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} - \frac{d^3\cos(c)\mathrm{Ci}(dx)}{6a^2b}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} - \frac{d\cos(c+dx)}{18a^2bx^2} + \frac{d^5\cos(c+dx)}{720ab^2x} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} + \frac{4\sqrt[3]{-1}\,d\cos\left(c\right.}{}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} - \frac{d\cos(c+dx)}{18a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} + \frac{4\sqrt[3]{-1}\,d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{C}}{27a^{8/3}\sqrt[3]{b}}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} - \frac{d\cos(c+dx)}{18a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} + \frac{4\sqrt[3]{-1}\,d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{C}}{27a^{8/3}\sqrt[3]{b}}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} - \frac{d\cos(c+dx)}{18a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} + \frac{4\sqrt[3]{-1}\,d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}\,d}{\sqrt[3]{b}}\right)\mathrm{C}}{27a^{8/3}\sqrt[3]{b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order

4 in optimal.
time = 0.51, size = 2109, normalized size = 1.81

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^3)^3),x]
```

```
[Out] (-6*a^2*b*d*x*Cos[c + d*x] - 6*a*b^2*d*x^4*Cos[c + d*x] - (18*I)*b*(a + b*x
^3)^2*RootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*CosI
ntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #1)]
 - Sin[c + d*#1]*SinIntegral[d*(x - #1)] & ] + (18*I)*b*(a + b*x^3)^2*RootS
um[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(
x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c +
d*#1]*SinIntegral[d*(x - #1)] & ] - 6*a^3*d*RootSum[a + b*#1^3 & , (Cos[c +
 d*#1]*CosIntegral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] -
I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x -
#1)])/#1^2 & ] - 12*a^2*b*d*x^3*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIn
tegral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*
#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 &
 ] - 6*a*b^2*d*x^6*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x -
 #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegr
al[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] - 6*a^3*d*
RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegr
al[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Si
n[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] - 12*a^2*b*d*x^3*RootSum[a +
b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1
)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*
SinIntegral[d*(x - #1)])/#1^2 & ] - 6*a*b^2*d*x^6*RootSum[a + b*#1^3 & , (C
os[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c + d*
#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d
*(x - #1)])/#1^2 & ] - I*a^3*d*RootSum[a + b*#1^3 & , ((-2*I)*Cos[c + d*#1]
*CosIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[
c + d*#1]*SinIntegral[d*(x - #1)] + (2*I)*Sin[c + d*#1]*SinIntegral[d*(x -
#1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x -
#1)]*Sin[c + d*#1]*#1 - I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Si
n[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] - (2*I)*a^2*b*d*x^3*RootSu
m[a + b*#1^3 & , ((-2*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 2*CosInteg
ral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c + d*#1]*SinIntegral[d*(x - #1)] + (
2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*
(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*d*Cos[c + d
*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#
1)/#1^2 & ] - I*a*b^2*d*x^6*RootSum[a + b*#1^3 & , ((-2*I)*Cos[c + d*#1]*Co
sIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c +
 d*#1]*SinIntegral[d*(x - #1)] + (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)
```

```
] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)
]*Sin[c + d*#1]*#1 - I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c
 + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] + I*a^3*d*RootSum[a + b*#1^3
& , ((2*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)
]*Sin[c + d*#1] - 2*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - (2*I)*Sin[c + d
*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 +
 I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegr
al[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] +
(2*I)*a^2*b*d*x^3*RootSum[a + b*#1^3 & , ((2*I)*Cos[c + d*#1]*CosIntegral[d
*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c + d*#1]*SinI
ntegral[d*(x - #1)] - (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c
 + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c + d
*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*Si
nIntegral[d*(x - #1)]*#1)/#1^2 & ] + I*a*b^2*d*x^6*RootSum[a + b*#1^3 & , (
(2*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin
[c + d*#1] - 2*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - (2*I)*Sin[c + d*#1]*
SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*
CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*
(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] + 108*a
^2*b*CosIntegral[d*x]*Sin[c] + 216*a*b^2*x^3*CosIntegral[d*x]*Sin[c] + 108*
b^3*x^6*CosIntegral[d*x]*Sin[c] + 54*a^2*b*Sin[c + d*x] + 36*a*b^2*x^3*Sin[
c + d*x] + 108*a^2*b*Cos[c]*SinIntegral[d*x] + 216*a*b^2*x^3*Cos[c]*SinInte
gral[d*x] + 108*b^3*x^6*Cos[c]*SinIntegral[d*x])/(108*a^3*b*(a + b*x^3)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.54, size = 363, normalized size = 0.31

method	result
derivativedivides	$\frac{\sin(dx+c)d^3\left(3a\,d^3-2b\,c^3+6b\,c^2(dx+c)-6bc(dx+c)^2+2b(dx+c)^3\right)}{6a^2\left(a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)^2}-\frac{\cos(dx+c)d^4x}{18\left(a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx-\right.}$
default	$\frac{\sin(dx+c)d^3\left(3a\,d^3-2b\,c^3+6b\,c^2(dx+c)-6bc(dx+c)^2+2b(dx+c)^3\right)}{6a^2\left(a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx+c)^3\right)^2}-\frac{\cos(dx+c)d^4x}{18\left(a\,d^3-b\,c^3+3b\,c^2(dx+c)-3bc(dx+c)^2+b(dx-\right.}$
risch	$-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(-3i\_Z^2bc-ia\,d^3+ib\,c^3+b\_Z^3-3b\,c^2\_Z\right)}\frac{\left(i\_R1 a\,d^3+ac\,d^3-8ia\,d^3-36i\_R1bc+18\_R1^2b-18b\,c^2\right)e}{-2ic\_R1+\_R1^2-c^2}\right.}{108b\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sin(d*x+c)*d^3*(3*a*d^3-2*b*c^3+6*b*c^2*(d*x+c)-6*b*c*(d*x+c)^2+2*b*(d*
x+c)^3)/a^2/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3)^2-1/1
```

```
8*cos(d*x+c)*d^4*x/(a*d^3-b*c^3+3*b*c^2*(d*x+c)-3*b*c*(d*x+c)^2+b*(d*x+c)^3
)/a^2+1/54/b/a^3*sum((a*d^3+18*_R1*b-18*b*c)/(-_R1+c)*(-Si(-d*x+_R1-c)*cos(
_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-
b*c^3))+1/a^3*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-4/27*d^3/a^2/b*sum(1/(_RR1^2-
2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=Roo
tOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^3*x), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.47, size = 1125, normalized size = 0.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*((-36*I*b^2*x^6 - 72*I*a*b*x^3 - 36*I*a^2 + (I*b^2*x^6 + 2*I*a*b*x^3
+ I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(2/3) - 8*(-I*b^
2*x^6 - 2*I*a*b*x^3 - I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3
/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*
d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (36*I*b^2*x^6 + 72*I*a*b*x^3 + 36*I*a
^2 + (-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2
))*(-I*a*d^3/b)^(2/3) - 8*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 - sqrt(3)*(b^2*x
^6 + 2*a*b*x^3 + a^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3
)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (-36
*I*b^2*x^6 - 72*I*a*b*x^3 - 36*I*a^2 + (I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 - s
qrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(2/3) - 8*(-I*b^2*x^6 - 2*I
*a*b*x^3 - I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(1/3))*
Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)
*(-I*sqrt(3) + 1) - I*c) + (36*I*b^2*x^6 + 72*I*a*b*x^3 + 36*I*a^2 + (-I*b^
2*x^6 - 2*I*a*b*x^3 - I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(-I*a*d^
3/b)^(2/3) - 8*(I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*
x^3 + a^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3
) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 108*(I*b^2*x^6
+ 2*I*a*b*x^3 + I*a^2)*Ei(I*d*x)*e^(I*c) - 108*(-I*b^2*x^6 - 2*I*a*b*x^3 -
I*a^2)*Ei(-I*d*x)*e^(-I*c) - 2*(-18*I*b^2*x^6 - 36*I*a*b*x^3 - 18*I*a^2 + (
-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2)*(-I*a*d^3/b)^(2/3) + 8*(-I*b^2*x^6 - 2*I*
```

```
a*b*x^3 - I*a^2)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c
- (-I*a*d^3/b)^(1/3)) - 2*(18*I*b^2*x^6 + 36*I*a*b*x^3 + 18*I*a^2 + (I*b^2*
x^6 + 2*I*a*b*x^3 + I*a^2)*(I*a*d^3/b)^(2/3) + 8*(I*b^2*x^6 + 2*I*a*b*x^3 +
 I*a^2)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^
3/b)^(1/3)) - 12*(a*b*d*x^4 + a^2*d*x)*cos(d*x + c) + 36*(2*a*b*x^3 + 3*a^2
)*sin(d*x + c))/(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{x(a+bx^3)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x**3+a)**3,x)
[Out] Integral(sin(c + d*x)/(x*(a + b*x**3)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac")
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^3*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)}{x\,(b\,x^3+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(x*(a + b*x^3)^3),x)
[Out] int(sin(c + d*x)/(x*(a + b*x^3)^3), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).___name___ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
          return expnType(expn.operands()[0])    #expnType(expn.args[0])
       elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
          if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```